# Labor market and search through personal contacts.

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# Literature review

- **Social Networks and Labor markets:**
  - Calvo-Armengol and Jackson (2004) study the correlation of employment statuses and wages of connected workers.
  - Calvo-Armengol (2004), Galeotti and Merlino (2011) consider endogenous network formation.
  - Calvo-Armengol & Zenou (2005) consider regular network in the framework of Mortensen-Pissarides model.
  - Ioannides and Soetevent (2006) perform a numerical analysis for the case of Poisson random network.

# Motivation

- Previous literature imposes various simplifying assumptions:
  - Only one worker initially becomes aware about a job offer.
  - Job offers can be transmitted only to immediate contacts.
  - Offer is relayed at random.
  - Firm behavior and wages are exogenous.
- The structure of job contact network is also an open question and varies from one study to another

# The model

**Workers:**

- A large number $N$ of ex-ante identical workers that are embedded into an undirected network of personal contacts.
- The network is characterized by a socialization level $s$ of workers that has cost $cs$.
- With some probability a worker learns about a vacancy directly from an employer. The worker may accept the offer or pass it to her contacts.
- An employed worker produces output $y$ and receives wage $w_t$.
- With probability $\delta$ an employed worker looses a job.

# The model cont'd

**Firms and wage:**

- A firm can open a vacancy. We refer to $v_t = V_t/N$ as vacancy rate.
- The cost of having an unfilled vacancy is $\gamma$.
- A wage is bargained according to the Nash bargaining process.

# Assumptions on Matching Function

- Matching function $m(s, v, u)$ depends on socialization level $s$, vacancy rate $v$ and unemployment rate $u$.
- We require the resulting matching function to satisfy the following four properties:

(A1) $m(s, v, u)$ is positive and increasing in both $u$ and $v$.

(A2) $m(s, v, u) \leq \min(u, v)$, $m(s, 1, u) = u$ and $m(s, v, 1) = v$.

(A3) $\frac{m(s,v,u)}{v}$ is decreasing in $v$ and $\frac{m(s,v,u)}{u}$ is decreasing in $u$.

(A4) $m(s, v, u)$ is increasing in the socialization level $s$.

# Worker's Problem

The stream of discounted utility of employed worker $I_{E,t}$ and of unemployed $I_{U,t}$ are given by:

$$I_{E,t} = w_t - cs + \frac{1}{1+r}[(1-\delta)I_{E,t+1} + \delta I_{U,t+1}]$$

$$I_{U,t} = -cs + \frac{1}{1+r}\left[\left(1 - \frac{1}{u_t}m(s, v_t, u_t)\right) I_{U,t+1} + \frac{1}{u_t}m(s, v_t, u_t)I_{E,t}\right]$$

where $r$ is the discount factor.

# Firm's Problem

We denote by $I_{F,t}$ and $I_{V,t}$ the expected inter-temporal profits generated by a filled job, and a vacancy respectively:

$$I_{F,t} = y - w_t + \frac{1}{1+r}\left[(1-\delta)I_{F,t+1} + \delta I_{V,t+1}\right]$$

$$I_{V,t} = -\gamma + \frac{1}{1+r}\left[\left(1 - \frac{1}{v_t}m(s, v_t, u_t)\right) I_{V,t+1} + \frac{1}{v_t}m(s, v_t, u_t)I_{F,t}\right]$$

# Labor Market Turnover

- At the beginning of each period $t$, the proportion $m(s, v_{t-1}, u_{t-1})$ of workers start to work.
- At the end of each period with probability $\delta$ an employed worker loses a job and becomes unemployed.

$$u_t = u_{t-1} - m(s, v_{t-1}, u_{t-1}) + \delta(1 - u_{t-1} + m(s, v_{t-1}, u_{t-1}))$$

- In the steady state:

$$m(s, v, u) = \frac{\delta}{1-\delta}(1 - u)$$

# Wage

- Workers' wage is determined according to the generalized Nash bargaining process, with worker's bargaining power being denoted by $\beta \in [0, 1]$:

$$w = \arg\max_{w} \{(I_E - I_U)^{\beta}(I_F - I_V)^{1-\beta}\}$$

- One can show that in the steady state:

$$w = \beta \left( y + \gamma \frac{v}{u} \right)$$

# Existence and Uniqueness of the Equilibrium.

**Proposition**

*For any $s$ there is a unique labor market equilibrium $\{u^*(s), v^*(s), w^*(s)\}$ if $\frac{\gamma(r+\beta+\delta)}{(1-\beta)} < Y < \frac{\gamma(r+\beta+\delta)}{\delta(1-\beta)}$. Moreover, functions $u^*(s), v^*(s)$, and $w^*(s)$ are continuous.*

- The first part of the condition, $\frac{\gamma(r+\beta+\delta)}{(1-\beta)} < Y$ implies that $Y$ is sufficiently high and firms want to hire workers when $u = 1$.
- Part $Y < \frac{\gamma(r+\beta+\delta)}{\delta(1-\beta)}$ puts upper bound on productivity not allowing $v$ to explode.

# Unemployment and Vacancy Rates

We relate $u^*(s)$ and $v^*(s)$ to workers' socialization level and productivity:

## Proposition

*In the equilibrium the following holds:*

*(i)* $u^*(s)$ *decreases in socialization level of workers* $s$ *and productivity* $y$.

*(ii)* $v^*(s)$ *increases in the productivity* $y$, *while* $v^*(s)$ *decreases in* $s$ *if* $u^*(s) < \bar{u}_v$ *and increases otherwise, where* $\bar{u}_v = \frac{\sqrt{\beta\delta(1-\delta)(\delta+r)}-\beta\delta}{(1-\delta)(\delta+r)-\beta\delta}$.

# Market Tightness and Wage

- Market tightness is the ratio of the number of vacancies to number of unemployed workers.
- The market tightness indicates which side of the market is better-off.

**Proposition**

*The equilibrium market tightness* $\frac{v^*(s)}{u^*(s)}$ *and the wage* $w^*(s)$ *are increasing in the socialization level* $s$.

# An Example of Matching Function

- To illustrate an application of our model we consider the network formation mechanism a la Galeotti and Merlino (2010)
- Each worker $i$ selects a socialization level, $s_i \geq 0$. Let $s = (s_1, \dots, s_n)$ be a profile of socialization levels.
- A probability that a link between $i$ and $j$ is present at time period $t$ is given by:

$$g_{ij}(s) = \rho(s) s_i s_j,$$

where

$$\rho(s) = \begin{cases} \left(\sum_{j=1}^{n} s_j\right)^{-1}, \text{ if } s \neq 0 \\ 0, \text{ otherwise} \end{cases}$$

# Probability to get at least one job-offer through contacts.

The matching function in this case is:

$$m(s,v,u) = u[v + (1-v)P^s(s,v,u)] = u\left[1 - (1-v)e^{-\frac{(1-u)v}{u}(1-e^{-us})}\right]$$

**Lemma**

*The matching function $m(s,v,u)$ satisfies conditions (A1)-(A4) and is concave in $u$, $v$ and $s$.*

# Conclusion

- We formulated four properties that a matching function should satisfy.
- Using these properties we showed that result obtained in previous studies about unemployment rate holds in more general setup.
- Our framework allowed us to get new results concerning the impact of network of personal contacts on vacancy rate, market tightness and wage.