Continuous time option pricing with scheduled jumps in the underlying asset

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Abstract

This paper introduces a new model of continuous time option pricing, which explicitly accounts for scheduled jumps caused by quarterly earnings announcements in the underlying stock. We present the stock price process as the product of a geometric brownian motion and scheduled jump process with a uniform jump size. This simple specification allows for obtaining a closed-form analytical solution for the European call option price. Empirical tests using a vast number of options with different strikes and maturities on several US stocks during 1999-2008 show significant superiority of our model over Black-Scholes in terms of fitting option prices. Moreover, the suggested model turns out to be no less precise as the Merton (1976) model with unscheduled jumps. Considering the parsimony and computational simplicity of our model compared to Merton (1976), we deem it preferable for application in the pricing of options on individual securities.

JEL Codes: *G00, G12, G13*

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1 Introduction

Several recent papers emphasize significant impact of regular announcements on asset price dynamics. In particular, Beber and Brandt (2006), Piazzesi (2005) and Johannes (2004) find that scheduled macroeconomic announcements cause jumps in bond prices, and Pattel and Wolfson (1981, 1984), Maheu and McCurdy (2004) and Dubinsky and Johannes (2006) find evidence of jumps in stock prices on the dates of quarterly earnings announcements. Such effects must have an impact on option pricing. In fact, Pattel and Wolfson (1979, 1981) find increasing option implied volatility prior to the earnings announcements and decreasing right after, Dubinsky and Johannes (2006) support these findings and report also decreasing term structure of option implied volatility around earnings announcement dates. This evidence calls for a modification of standard option pricing models in order to explicitly account for the phenomenon of scheduled jumps. An attempt has been undertaken in Dubinsky and Johannes (2006), where Heston (1993) model has been augmented by a number of scheduled jumps, however it provides only for a numerical solution and is not fully supported empirically (one of the pivotal parameters, volatility of volatility, is insignificant for all considered stocks).

Our paper closes this research gap, providing an analytical closed-form solution for the price of a European call option on a stock underlying scheduled jumps. We achieve this result modelling the underlying stock price dynamics as a product of a geometric brownian motion and a jump process with a uniform jump size. This means that we introduce only one additional parameter to the classical stock price dynamics specification, which leads to Black-Scholes (1973, further BS) option prices. However, fitting our model and BS to observed option prices on a large spectrum of strikes and maturities provides for indisputable superiority of our model. Similar empirical comparison to a considerably higher parameterized model - Merton (1976) model with unlimited number of unscheduled jumps - shows that our model is in no way inferior in terms of pricing errors.

Thus we contribute to the literature providing a model most adequately reflecting discontinuities in stock prices.

The justification for modeling discontinuities in stock price is market efficiency. When investors receive new information in a perfectly efficient market, they immediately price in this information, which results in instantaneous stock price change. Despite perfectly efficient market exists only in theory, the stock exchanges of developed countries are efficient enough so that discontinuity is a reasonable approximation for stock price at the arrival of new information. In the seminal paper of Merton (1976) discontinuities in stock prices are normally distributed jumps, happening due to arrival of news at the market. In his model, the timing of jumps is random and their number follows a compensated Poisson process.

While randomly timed jumps help to model stock dynamics much better compared to BS model, they still do not reveal the whole picture of stock market. In fact, the most significant jumps in stock prices are caused by scheduled events, the timing of which is non-random [1]. Such events include earning reports, shareholder meetings, monetary authorities meetings, etc. . The presence of jumps is significant for option pricing, since other things being equal, the introduction of jumps during the life of an option increases the variation of stock price at maturity. This variation influences expected payoff of an option and, therefore, its price. Particularly, for European call option larger variation in S_T increases the value of this option. We think that scheduled jumps are more significant than unscheduled, first, because they have a larger impact on stock prices (we show in our estimations that average volatility of scheduled jump across different subsamples is around 8 percent, while that of unscheduled is only 2 percent) and second, because they are directly observable. For instance, one can clearly see that at 15-00 GMT, 15 April, the stock price of company XXX skyrocketed, because positive quarter results were disclosed to the market. In contrast, unscheduled jumps are somewhat vague in definition and difficult to observe. For instance, if Russian traders knew that local news was coming today, but international traders were unaware - that local news cannot be defined as 100 percent unscheduled. The study of jumps in general is important, because it relates to the fundamental problems of assymmetric information between economic agents and market efficiency. At the time of any event the stock price faces a jump, because there is a flow of information from one agent to another. For instance, at earning announcement the information flows from company to market participants. The sharp change in stock price at the announcement means that investors did not know something about the company, particularly they were uncertain about its earning figures. Such kind

[1] Dubinsky and Johannes (2006) argue that the timing of scheduled event is non-random in the vicinity of an announcement. For instance, the date of earning announcement is disclosed by a company a few months before the announcement, so since then it becomes a certainty. The announcement date can be a random variable in a large time span before the announcement, but still the standard deviation of this time is negligible for regular announcements. Thus, modelling the announcement time as certainty is appropriate.

of uncertainty is known as 'fundamental uncertainty'. It is studied in great detail by Dubinsky and Johannes (2006). They state that flows of information under 'fundamental uncertainty' greatly influence the stock price dynamics. It means that they also influence the price of an option on that stock and have to be accounted in pricing formulas.

Following the literature on option pricing with jumps, we think that the main disadvantage of existing models is their analytical and computational complexity. We believe that it is not appropriate to mix three models (stochastic volatility, unscheduled jumps and scheduled jumps) altogether as done by Abraham and Taylor (1993, 1997),Dubinsky and Johannes (2006) and Boes and Drost (2007). Such a model looses parsimony and becomes troublesome to estimate. Rather, we suggest to replace unscheduled jumps with scheduled ones in the model (of course combined with geometric brownian motion). Furthermore, the models mentioned above are complicated by assuming normally distributed jumps. This assumption results either in double integrals from normal pdf or infinite sequences (as in Merton (1976)) in solutions for option prices. As our main goal is to make the model as simple as possible, we assume uniformly discributed jumps. This results in a nice closed form solution for option price represented by a finite sum of normal densities with different coefficients, which makes the option price easily computable.

The complexity of Merton's model also stems from the use of randomly timed jumps. Random timing calls for the introduction of Poisson process, which drives the number of jumps per period. We make the model simpler by replacing unscheduled jumps with scheduled ones, effectively cutting one source of uncertainty - the time of jump. What several existing papers such as Abraham and Taylor (1993, 1997) and Boes and Drost (2007) do is they just add scheduled jumps to unscheduled and estimate them altogether. Our paper looks at the model from another angle: we suggest to replace unscheduled jumps with scheduled ones. We claim that the explanatory power of scheduled events is strong enough to produce a correctly specified pricing model. The simple version of our model, presented in this paper, which contains only one jump, has two free parameters. Merton's model has four. It is reasonable to say that Merton's model will provide a better fit to market option prices than our model. Surprisingly, in the majority of subsamples this is not the case. Not only our empirical tests show that Merton's model is not significantly better than the ours, but also in many subsamples Merton's model actually provides a poorer fit than our model. Theoretically, it is possible as neither of them nests the other. The result that a simpler model can provide a better fit than a more complex model is an evidence for corrrect specification in our model. Such results lead us to advice the financial practitioners and researchers to use scheduled jumps instead of unscheduled in face of the former's parsimony, analytical and computational simplicity.

Another problem in option pricing with jumps is that martingale measure is non-unique, so the choice of specific measure has to be justified. In this paper we use 'martingale measure','equivalent martingale measure' and 'risk-neutral measure' interchangeably. In case of no dividends, martingale measure is that probability measure, under which the discounted stock price process $e^{-rt}S_t$ is a martingale. Many papers disregard the need to justify the choice of martingale measure and to construct it analytically. For instance, the papers by Abraham and Taylor (1993, 1997) did not receive much attention among researchers due to vague martingale measure specification. We note that martingale measure choice is important, because option price, being the payoff expectation in this martingale measure, is completely determined by the measure choice. It means that different measures result in different option prices. Thus, risk-neutral measure choice is the major issue in developing an option pricing model. We overcome this issue by assuming that jump process has uniform distribution in risk-neutral measure and show that its amplitude in this measure has to be the same as in market measure so that these probabilities remain equivalent. The construction of risk-neutral measure is also important, because this measure is unobservable. It is an abstract object created by researcher to determite the value of his/her option. It means that a good research paper will carefully explain how this object is constructed and justify the particular construction. Furthermore, there is a very important option pricing convention that market measure and risk-neutral measure have to be equivalent (see Broadie, Chernov and Johannes (2007, 2009)). Equivalence intuitively means that risk-neutral investors have the same view as risk-averse ones on events with zero probability. If it is not the case, then market and risk-neutral worlds will be of different possibilities, which raises a need for other option pricing models than we have now. Measure equivalence requirement means that researcher has to show that his/her way of risk-neutral measure construction preserves equivalence between two measures. We preserve equivalence by using Radon-Nikodym derivative in measure construction. We address the issue of choosing a particular martingale measure by analytically describing the set of all possible equivalent martingale measures and choosing a particular one from this set by minimizing the error between our model prices and market prices.

The rest of the paper is structured as follows. In section 2 we develop our model, moving step-by step from underlying dynamics under market measure to the risk-neutral measure and solving for the risk-neutral expectation of the option's payoff. Section 3 reports the results of the empirical comparison of fit of BS, Merton (1976) and our model and provides some discussion. Section 4 concludes.

2 Model Specification

2.1 Black-Scholes model and Merton's Jump-Diffusion

The Black-Scholes model assumes that the distribution of possible stock prices at any finite interval is lognormal. To be more precise, stock price S_t follows a geometric brownian motion of the form

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right) \tag{1}$$

with expected stock return μ and volatility σ. BS assumptions are restrictive in a way that σ is constant and the path of stock price is almost surely continuous.

Merton (1976) relaxes the continuity assumption. He models discontinuities in stock prices as normally distributed jumps, happening due to arrival of news at the market. In his model, the timing of jumps is random and their number follows a compensated Poisson process. Thus, the stock price at time t follows the process

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2} - \lambda k\right) t + \sigma W_t\right) Y(n) \tag{2}$$

where $k * 100$ is the expected percentage change in stock price if random jump occurs and λ is the mean rate of such jumps per period.

$$Y(N) = \begin{cases} 1, & \text{if } N = 0 \\ \sum_{j=1}^{N} Y_j, & \text{if } N \geq 1 \end{cases} \tag{3}$$

Note than Y_j is normally distributed with mean $k+1$ and variance δ^2. Also, N is Poisson distributed with mean λt.

The introduction of Poisson jumps in the stock price makes the derivation of European call option value quite complex, as well as the calculation of option prices when the formula is derived. The simplest possible solution for option price, derived in Merton's paper is an infinite sequence of BS option prices with different parameters. Namely, the option price at time zero $V(0)$ is given by

$$V(0) = \sum_{n=0}^{\infty} weight_n BS_n \tag{4}$$

In this equation BS_n is the Black-Scholes value of an imaginary option with the same S,K,T as for the real one, but different r and σ. namely, $r_n = r - \lambda k + \frac{nlog(1+k)}{T}$ and $\sigma_n^2 = \sigma^2 + \frac{n\delta^2}{T}$. Each imaginary BS_n is weighted by $weight_n$ where

$$weight_n = \frac{e^{(-\lambda(1+k)T)} \left(\lambda \left(1+k\right) T\right)^n}{n!}$$

2.2 Market measure

In this section we explain the specification of our model. Its main difference from Merton's is that we replace unscheduled jumps, the number of which is Poisson distributed and the size of which is Normally distributed, by scheduled jumps happening at deterministic points in time with uniformly distributed magnitude. Jumps, both in our model and in Merton's one help to capture fat tails in the distribution of S_T (the stock price at option expiration), because they make extreme stock prices more probable. The market at time $t \in [0; T]$ is described by a probability space $(\Omega, \mathcal{F}_t, P)$. Our model uses the same basic assumptions as the Black-Scholes one, except the fact that we relax the almost surely continuity of stock price process. We model discontinuities by $\mathcal{F}_t$ - adapted jump process ξ_t, which multiplicatively enters the stock price dynamics. In the simple[2] version of our model we assume the following analytical form of ξ_t:

$$\xi_t = \begin{cases} 1, & \text{if } t < t_1 \\ \Psi\left(1\right), & \text{if } t = t_1 \\ \xi_{t_1}, & \text{if } t > t_1, \end{cases} \tag{5}$$

where $0 < t_1 < T$. $\Psi(1)$ means any almost surely nonnegative random variable with expected value equal to 1. That is, the process ξ_t represents only one jump at a fixed time t_1, the amplitude of which is randomly distributed. For $t \neq t_1$ the process is deterministic, fixed at unity before the jump and "frozen" at the realised size of jump after t_1. The restriction that $E\left(\Psi\left(1\right)\right) = 1$ is carefully justified. This restriction results in $E\left(\xi_{t_1} \mid \mathcal{F}_o\right) = 1$, which means that on average the expected stock return on jump date is zero. This implies semi-strong form market efficiency with respect to scheduled events (earning announcements in particular). Thus we assume market efficiency by making the specification above. In face of this fact, our model specification is applicable only to efficient stock markets. Thus, we use data from NYSE and NASDAQ, because a significant

[2]We refer to the simple version of the model as that which contains only one scheduled jump. The incorporation of N jumps is the topic for further research

amount of event studies indicate semi-strong efficiency on these exchanges (see Pettit (1972)).

Stock price in our model is described by the following differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t + S_{t-} d\xi_t \tag{6}$$

Where ξ_t is independent of W_t. As you can see, the first two terms $\mu S_t dt + \sigma S_t dW_t$ are the same as in the paper of Black and Scholes (1973), while our innovation is the jump ξ_t, which multiplicatively influences the stock value S_{-t} at the time of jump. This specification means that ξ_t is measured in fractions of unity and the value of ξ_t equal to unity means no change in stock price. Our specification is slightly different from Merton's one, as he places the jump term in the power of exponent. We dont's use log-uniform distribution, since it complicates calculations without actually improving the model. The solution to this equation for the stock price, derived in Appendix 6.3, is

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right) \times \xi_t \tag{7}$$

As you can see, ξ_t is constant before t_1, then it randomly jumps at t_1 and afterwards is constant again. So the only uncertainty in the process ξ_t is at time t_1. We will also be using a more concise notation

$$S_t = D_t J_t \tag{8}$$

where $D_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right)$ stands for the diffusion part of stock process and $J_t = \xi_t$ stands for the jump part.

To price an option we need to specify the distribution of $\Psi(1)$. As our main goal is to make the model as simple as possible, we assume that the jump is uniformly distributed. Since its mean is unity, the distribution can be written as:

$$\Psi(1) \sim \text{Uniform}\,(1 - b; 1 + b) \tag{9}$$

We have to impose a restriction $b < 1$, which guarantees that stock price is almost-surely nonnegative. Technically, this is the only distribution which allows to avoid double integrals or infinite sequences in the solution for option price. The reader clearly understands that uniform distribution is only an approximation to the real jump distribution. It is worth to make such an approximation only if the approximation error is negligible, concerning the main purpose of the research. It turns out that in our case the error from approximation is indeed negligible. That happens, because we are only interested in the distribution of S_T at maturity for option pricing. This distribution is a sum of many distributions over small time intervals. The distribution of the jump is reflected in stock distribution over only one time interval, which washes out among other time intervals to form S_T. As a result of this, it is not important to model the jump distribution shape correctly for option pricing purpose. What matters is the volatility of jump distribution, which directly influences the option price through the volatility of S_T. Uniform distribution has no problems with capturing the jump volatility correctly (we do not speak about time-varying volatility here). Now, the reader can clearly see that uniform distribution has enormous computational benefit and negligible approximation error.

2.3 Risk-neutral measure

In the framework of stock with no dividends, the risk-neutral probability space is defined as that in which the discounted stock price $e^{-rt}S_t$ is a martingale. We will denote the risk-neutral space as (Ω, F_t, Q), keeping in mind that it differs from the market space only in measure Q, while the space of elementary outcomes Ω and filtration $\mathcal{F}_t$ are the same. Q is also called equivalent martingale measure, because we impose the following consistent link between measures P and Q: for any event $A \in \mathcal{F}_t$, $Q(A) = 0 \Leftrightarrow P(A) = 0$ and $Q(A) = 1 \Leftrightarrow P(A) = 1$. It means that an impossible event in P is still impossible in Q. According to no arbitrage condition, the value of an option is the expectation of its payoff under risk-neutral probability. That is, for European call option $V(0) = E^Q\left(max\left(S - K; 0\right)\right)$. When stock price is driven by (1), the measure Q is unique, but when the path is discontinuous as in (7), an infinite amount of risk-neutral measures exist. This results in infinitely many possible option prices and often the lowest option price V_0^{Low} and the highest possible price V_0^{High} are characterised. A researcher then has to choose the most appropriate Q, which determines a particular option price $V_0^{\text{Low}} \le V_0 \le V_0^{\text{High}}$.

We will choose the appropriate Q out of an uncountable set of risk-neutral measures by assuming certain distributions of D_t and ξ_t. We state that under Q the diffusion is $D_t^Q = S_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right)t + \sigma W_t^Q\right)$ and the jump process is given by:

$$\xi_t^Q = \begin{cases} 1, & \text{if } t < t_1 \\ \text{Uniform}\left(1 - a; 1 + a\right), & \text{if } t = t_1 \\ \xi_{t_1}, & \text{if } t > t_1 \end{cases} \tag{10}$$

where $a < 1$, but it need not be equal to b (the jump amplitude under P). The assumption above excludes risk-neutral measures under which the distributions of D_t ξ_t are different from what we assumed. Here we maintained a common principle of risk-neutral measure construction used in Black and Scholes (1973), Merton (1976), Dubinsky and Johannes (2006), Boes and Drost (2007). The principle is that the type of distribution does not change when we transfer from market to risk-neutral measure, while the only thing that changes is the distribution's parameters. Preserving the distribution type is beneficial in a way that it allows to compare similar parameters in measures P and Q, which is often examined in literature and referred to as risk premia. Despite the fact that we preserved uniform distribution in Q, we are still faced with an uncountable set of possible risk-neutral measures, where each measure corresponds to a different amplitude of uniform jump a. Denote this set by (Ω, F_t, Q_a) In fact, the only possible choice of a to preserve the equivalence of P and Q, given that under both measures the jump is uniformly distributed, is $a = b$. If $a \neq b$, then $\exists A \in \mathcal{F}_t : P(A) = 0 \cap Q(A) > 0$ -or- $P(A) > 0 \cap Q(A) = 0$, which means that P and Q are not equivalent. For instance, if $a > b$, then the events resulting in $1 - a < \xi_T < 1 - b$ have zero probability under P, but non-zero probability under Q (see Appendix 6.4 for more precise explanation). Given the argument above, we will use $a = b$ in our model, but for consistency in notations we will use letter a, when referred to ξ_t under Q and letter b when referred to ξ_t under P.

Given the fact that only the market measure is observable, we need to specify how the unobservable risk-neutral measure Q_a is constructed from the market measure P. For this we employ Radon-Nikodym derivative denoted by $\frac{dQ}{dP}$. If a random variable Z defined on a probability space (Ω, F_t, P) satisfies the following 2 properties: 1)P(Z$\geq$ 0) = 1 and 2)E^P $(Z) = 1$, then for $\forall A \in \mathcal{F}_t$,

$$Q(A) = \int_A Z(\omega) dP(\omega) \tag{11}$$

is a probability measure. Such variable Z with properties 1 and 2 is called Radon-Nikodym derivative $\frac{dQ}{dP}$. As you can see, the risk-neutral measure can be constructed by specifying variable $Z = \frac{dQ}{dP}$ and taking Lebesgue integral from Z for each event $A \in \mathcal{F}_t$. Furthermore, when Q is constructed as in (11), then expectations under P and Q are linked in the following way:

$$E^Q(Z) = E^P(XZ) \tag{12}$$

for any nonnegative random variable X.

For measure Q constructed as in (11) to be an equivalent martingale (risk-neutral) measure, the process $e^{-rt} S_t$ must be a martingale under Q. As D_t and ξ_t are independent, we will find such Z_0 and Z_1 satisfying conditions 1 and 2 that $e^{-rt}D_t$ is a martingale under $Q(A) = \int_A Z_0(\omega)dP(\omega)$ and ξ_t is a martingale under $Q(A) = \int_A Z_1(\omega)dP(\omega)$ and then obtain $Z = Z_0 Z_1$. Under $Z = Z_0 Z_1$ constructed in such a way, the process $e^{-rt}S_t = e^{-rt}D_t\xi_t$ will be a martingale (see Appendix 6.5 for a proof).

The expression for Z_0 is well-known from many stochastic calculus textbooks, so we just state it without explanation.

$$Z_0 = e^{-\frac{\mu-r}{\sigma}W_T - \frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 T} \tag{13}$$

The expression for Z_1 is $Z_1 = 1$. Please see Appendix 6.3 for the formal derivation of Z_1, but the intuition is straightforward. We showed above that proper value of uniform jump amplitude under measure Q is such that $a = b$. The fact that $a = b$ preserves the equivalence of Q and P. In effectively means that we keep both type of jump distribution and its parameter the same in both measures Q and P. Completely identical distributions mean that $Z_1 = 1$

2.4 Hedging premium

We showed above that $a = b$ is necessary for Q and P to be equivalent, given that jump is uniform under both measures. This means that the difference between market jump parameters and risk-neutral ones, which is often referred to as 'risk premium' is zero. Despite this fact we introduce a premium of another functional form to account for market incompleteness and impossibility of perfect hedging. As you know, the principle of a perfect hedge of an option with replicating portfolio consisting of underlying stock and bonds is the basis of risk-neutral pricing (Black and Scholes, 1973). When jumps are introduced, perfect hedge may not work at times of jumps. It means that option price may differ from the value of potential replicating portfolio, since we don't have perfect replication at jumps. Merton (1976) addresses this issue by simply assuming that jump risk is diversifiable. This assumption implies that a properly constructed broad portfolio replicates the option even at jumps. However, this assumption is quite restrictive, since scheduled news for a particular stock have low correlation with other securities, which means that it is problematic to find such securities that offset the news impact on the stock under consideration. We relax Merton's assumption and say that perfect hedge is possible in our model at any time except t_1 due to stock price discontinuity there. This problem causes us to make an adjustment to option pricing formula in order to account for continuous hedge impossibility at t_1. We use the following logic: risk averse investors might set aside additional capital for replicating portfolio so that it is used at time $t_1 + \epsilon$ to readjust perfect hedge after the jump. This additional capital makes replicating portfolio more expensive, therefore its size serves as a premium to call option price. We model this premium as $\rho \times aE^Q(S_{t_1})$, where ρ is some non-negative constant, which will be estimated from option prices. The premium is added to risk-neutral expectation to obtain option's value. Thus the price of option is $V(0) + \rho \times aE^Q(S_{t_1})$, where $V(0) = E^Q\left(e^{-rt} \times max\left(S - K; 0\right)\right)$. Most papers on jump pricing introduce premium in another form. In Dubinsky and Johannes (2006), Broadie, Chernov and Johannes (2007), Boes and Drost (2007) premium is the difference between parameters in P and Q. Our premium is different in a way that it is added with + to the option value, because we are simply restricted to keeping the same jump parameters in both measures.

2.5 The derivation of option pricing formula

The price price of a call at moment t=0 with strike price K and maturity at T (neglecting hedge premium) is $V(0) = e^{-rT}E^Q\left(\max\left(S_T - K; 0\right)\right)$, where E^Q is the expectation with respect to risk-neutral measure.

$$V(0) = e^{-rT}E^Q\left(\left(S_T - K\right) \times 1_{(S_T > K)} + 0 \times 1_{(S_T < K)}\right) = \\ = e^{-rT}\left(E^Q\left(S_T \times 1_{(S_T>K)}\right) - KE^Q\left(1_{(S_T>K)}\right)\right) \quad (14)$$

By taking the expectation under Q (see Appendix), we find that

$$E^Q\left(1_{\{S_T>K\}}\right) = \frac{a+1}{2a}\left(\Phi\left\{d_2\right\} - \Phi\left\{d_1\right\}\right) - \\ - \frac{K}{2aS_0}e^{\sigma^2 T - rT}\left(\Phi\left\{d_2 + \sigma\sqrt{T}\right\} - \Phi\left\{d_1 + \sigma\sqrt{T}\right\}\right) + \Phi\left\{-d_2\right\} \quad (15)$$

$$E^Q\left(S_T \times 1_{(S_T>K)}\right) = S_0 e^{rT}\Phi\left\{-d_2 + \sigma\sqrt{T}\right\} + S_0 e^{rT}\frac{(1+a)^2}{4a}\left(\Phi\left\{d_2 - \sigma\sqrt{T}\right\} - \\ - \Phi\left\{d_1 - \sigma\sqrt{T}\right\}\right) - \frac{K^2}{4aS_0}e^{\sigma^2 T - rT}\left(\Phi\left\{d_2 + \sigma\sqrt{T}\right\} - \Phi\left\{d_1 + \sigma\sqrt{T}\right\}\right) \quad (16)$$

Where $d_1 = \frac{ln\frac{K}{(1+a)S_0} + \frac{\sigma^2}{2}T - rT}{\sigma\sqrt{T}}$ and $d_2 = \frac{ln\frac{K}{(1-a)S_0} + \frac{\sigma^2}{2}T - rT}{\sigma\sqrt{T}}$

Combining the terms above, we get

$$
\begin{aligned}
V(0) = S_0 \frac{(1+a)^2}{4a} & \left(\Phi\left\{ d_2 - \sigma\sqrt{T} \right\} - \Phi\left\{ d_1 - \sigma\sqrt{T} \right\} \right) + \\
& + \frac{K^2}{4aS_0} e^{\sigma^2 T - 2rT} \left(\Phi\left\{ d_2 + \sigma\sqrt{T} \right\} - \Phi\left\{ d_1 + \sigma\sqrt{T} \right\} \right) + \\
& + S_0 \Phi\left\{ -d_2 + \sigma\sqrt{T} \right\} - K e^{-rT} \frac{a+1}{2a} \left(\Phi\{d_2\} - \Phi\{d_1\} \right) - K e^{-rT} \Phi\{-d_2\} \qquad (17)
\end{aligned}
$$

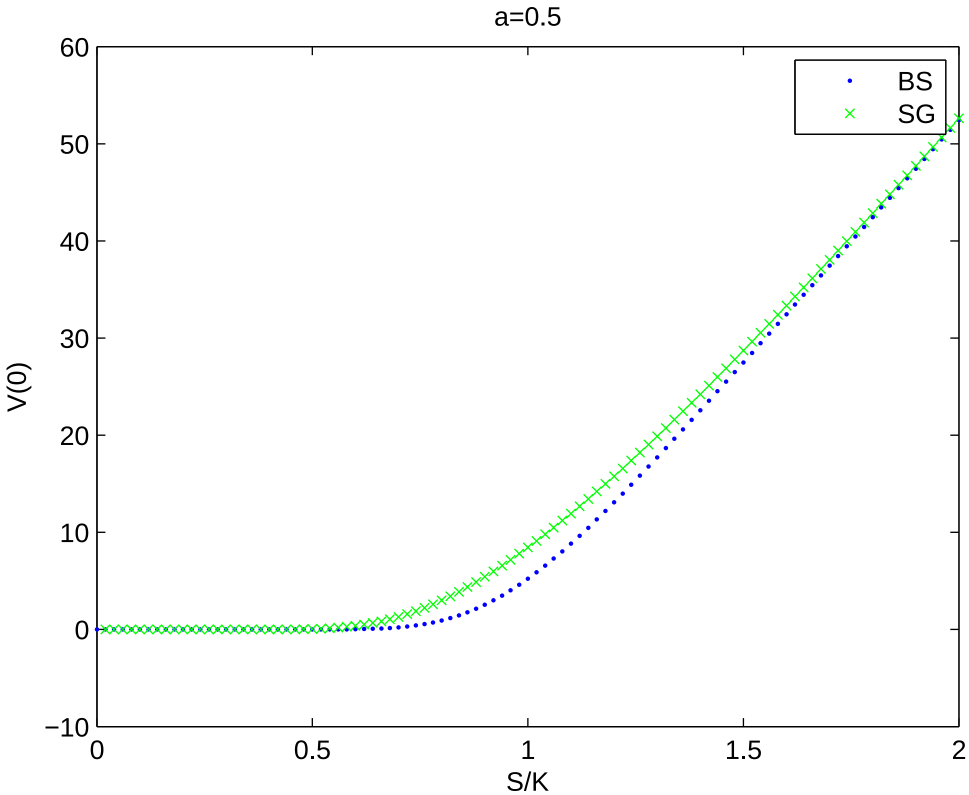
If we account for hedging premium, then the price of an option is given by

$$
V_o + \rho \times a E^Q \left(S_{t_1} \right) = V_0 + \rho a e^{r t_1} S_0 \qquad (18)
$$

2.6 Comparative Statics

The formula obtained above is linked to BS in the following way: $\lim_{a\to0}(SG) = BS$, which means that BS model is a limiting restriction of our model when a approaches zero. Technically, the BS model cannot be obtained from SG by directly setting $a = 0$, because we face division by zero in this case. As you can see from the formula above, the intuitive difference between our model and Black-Scholes is that the former makes extreme values of S_T more probable though the impact of instantaneous jump (keeping σ the same in both models). Thus if $a > 0$ the fair price of a call from our model will be higher than that from Black-Scholes for the same values of σ. Calculations show that this difference is quite large for out-of-the-money options, while for in-the-money options our model converges to Black-Scholes relatively fast with increase in moneyness. The comparative statics for BS and SG are illustrated below. As you can see from the graphs, for moneyness lower than 0.5, both models are almost identical. This stems from the fact that for deep OTM options (for instance with $\frac{S}{K} < 0.5$), the introduction of jump with maximum amplitude 0.5 will still not make the option expire in the money. For larger moneyness the jump starts to influence option payoff significantly, since extreme values of jump can make the option expire in the money. Thus, we face difference in our model and BS model for moneyness from 0.5 to 1.5. When we look at moneyness more than 1.5, the option price of BS starts to converge to $S - K \times e^{-rt}$. The same convergence is seen for SG model, as it should be for any model since the lower bound on option price is $S - K \times e^{-rt}$. For large values of moneyness the option price (which does not depend on particular model) converges to its lower limit.

Figure 1: Theoretical prices of SG and BS for $a = 0.5$. $\sigma = 0.2$, $K = 50$, $T = 1$, $r = 0.05$



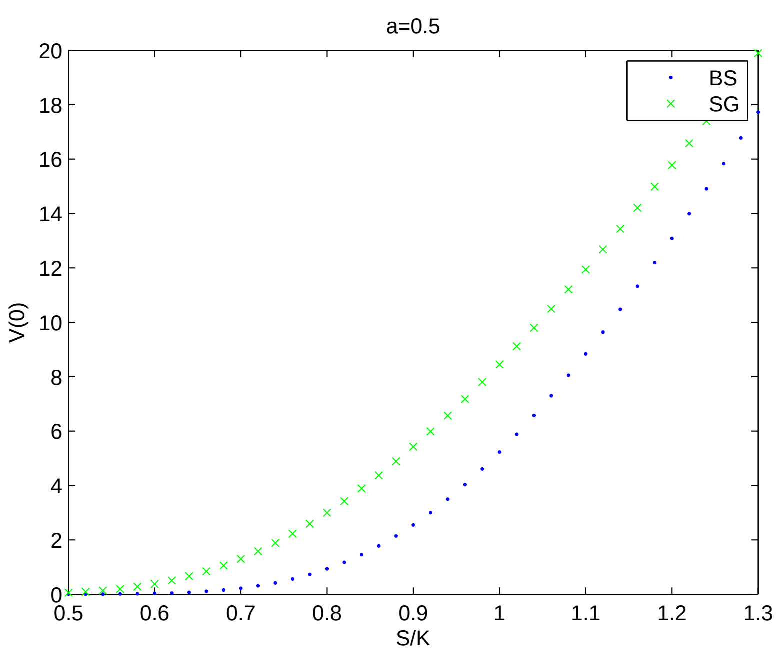
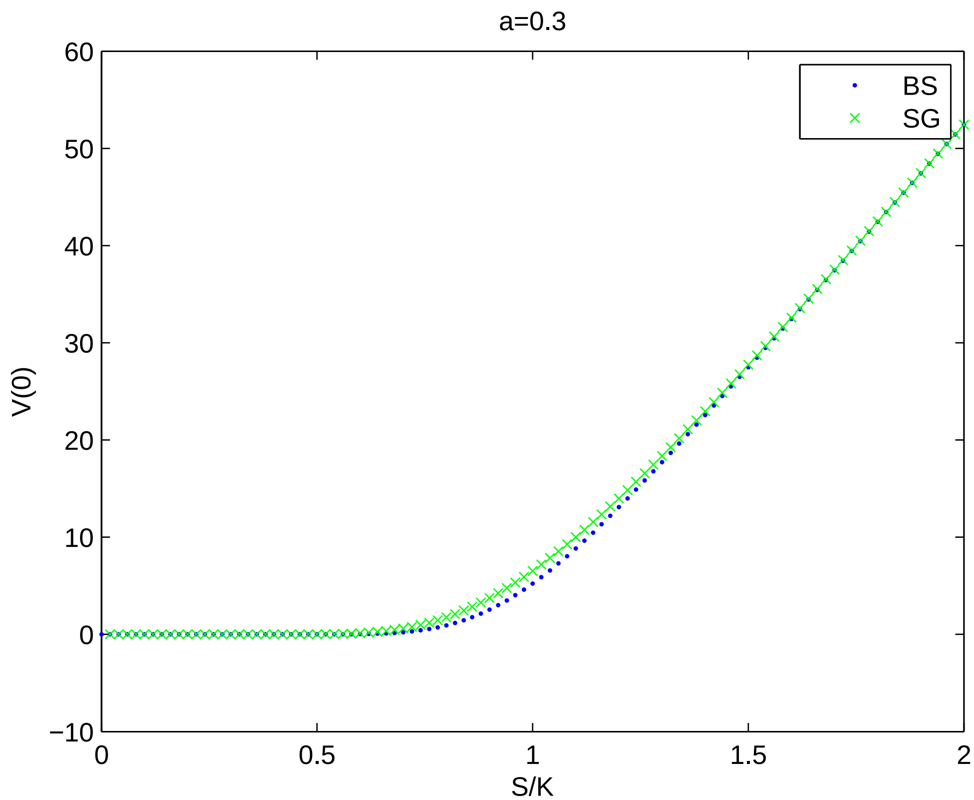


Figure 2: Theoretical prices of SG and BS for $a = 0.3$. $\sigma = 0.2$, $K = 50$, $T = 1$, $r = 0.05$



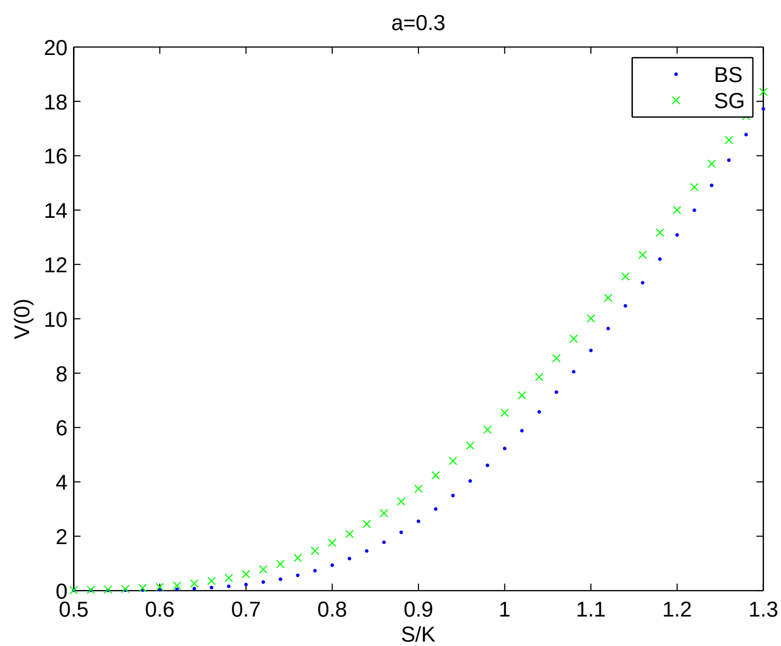
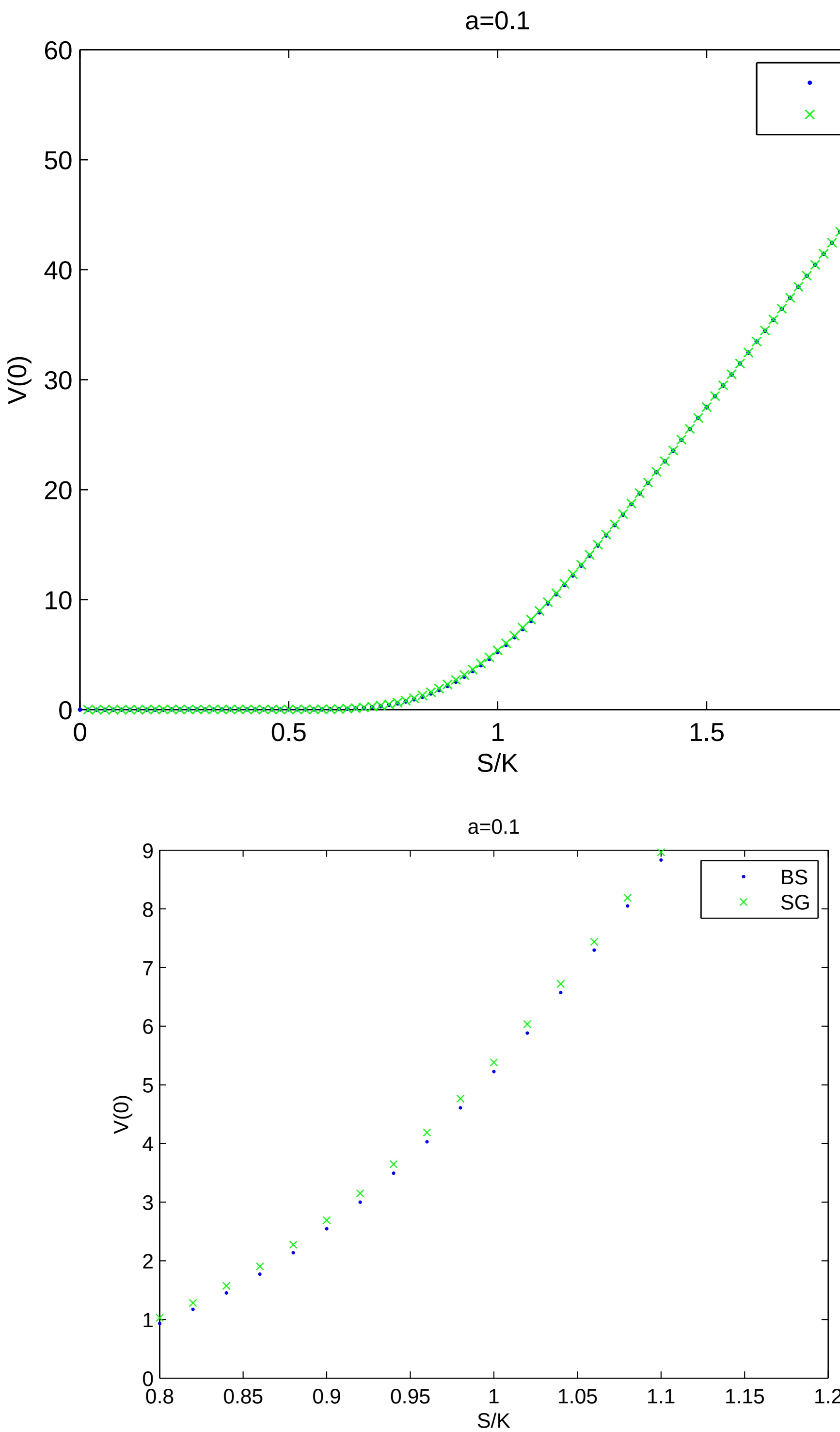


Figure 3: Theoretical prices of SG and BS for $a = 0.1$. $\sigma = 0.2$, $K = 50$, $T = 1$, $r = 0.05$



3 Emprical Results

3.1 Data Description

For model estimation we choose European call options on the stocks of five low dividend companies traded on NYSE or NASDAQ: Apple Computer (AAPL), Microsoft (MSFT), Cisco Systems (CSCO), Intel Corporation (INTC) and Advanced Micro Devices (AMD). Most researchers do not use more than 5-6 companies for estimation (Dubinsky and Johannes (2006), Macbeth(1979)), so that a mild amount of machine time is taken for calculations. Choosing low dividend stocks allows us to neglect any early excercise effects and use American calls equivalently with European calls for estimation. Furthermore, we choose the U.S. stock market, because it is one fo the most efficient with respect to new information and stock paths on that market at earning announcements (EA) closely resemble 'jumps'. The time span we use is 1999-2008, we estimate parameters for each year separately. Having 10 years and 5 companies results in 50 subsamples, on which the models are estimated. As we explicitly account for stock price discontinuities at EA, we need their precise dates. Those dates are collected directly from companies' websites and www.fulldisclosure.com. When EA happens before the market opens, the timing of jump is set at the moment when market opens on the same day. When EA happens after the market closes, the jump time is set at the moment when market opens the next day. The option prices are take from OptionMetrics. This database reports the daily best closing bid and offer prices together with all relevant information about an option contract. Below is the table with descriptive statistics of our sample.

Among all levels of moneyness, we consider OTM ($\frac{S}{K} < 0.97$) and ATM ($0.97 \leq \frac{S}{K} \leq 1.03$) options because our model shows the largest difference from Blak-Scholes model for these levels of moneyness. Moreover, we intend to show that our model solves some problems faced by Black-Scholes model in fitting OTM options, which are often considered as the hardest in pricing. Thus, we intend to show that our model will help to significantly reduce the pricing error on OTM options. As the basic version of our model incorporates one jump, then we use only those option contracts, which cover one EA during their life span. Among these options we eliminate those quotes which are given closer than 3 days to the EA and which mature 3 days or closer to EA in order to eliminate any disturbances arising when the EA date is near. We also clear our sample for the following: 1) contracts with implied volatility less that 0.05 and more than 1; 2) contracts, which

Figure 4: Sample Descriptive Statistics for AMD (figures in dollars)

The table shows our sample statistics (in dollars) after clearing for mispricing and selecting options which cover only one earning announcement in their life. Effectively, the data in the table is the one used directly in model estimation

		T(days)			
Moneyness		< 30	$[30; 60]$	> 60	Subtotal N
< 0.97	price	0.76	0.89	1.51	
	spread	0.08	0.08	0.11	
	N	(152)	(1421)	(129)	(1702)
$[0.97; 1.03]$	price	1.65	2.01	2.80	
	spread	0.08	0.10	0.13	
	N	(47)	(382)	(37)	(466)
> 1.03	price	11.43	7.94	11.63	
	spread	0.17	0.19	0.23	
	N	(702)	(3305)	(359)	(4376)
Subtotal N		(901)	(5108)	(535)	(6544)

Figure 5: Sample Descriptive Statistics for CSCO (figures in dollars)

		T(days)			
Moneyness		< 30	$[30; 60]$	> 60	Subtotal N
< 0.97	price	0.53	1.04	1.14	
	spread	0.06	0.08	0.09	
	N	(318)	(1090)	(44)	(1452)
$[0.97; 1.03]$	price	1.80	2.48	2.92	
	spread	0.08	0.10	0.11	
	N	(164)	(482)	(20)	(666)
> 1.03	price	9.31	9.35	9.27	
	spread	0.15	0.16	0.16	
	N	(1958)	(5669)	(241)	(7868)
Subtotal N		(2440)	(7241)	(305)	(9986)

allow the profit on put-call parity arbitrage to be more than 1.5 of bid-ask spread on the corresponding call; 3) call options with market price violating theoretical lower and up-

Figure 6: Sample Descriptive Statistics for AAPL (figures in dollars)

Moneyness		T(days) < 30	[30; 60]	> 60	Subtotal N
< 0.97	price	0.79	2.38	2.39	
	spread	0.09	0.13	0.14	
	N	(190)	(1848)	(116)	(2154)
[0.97; 1.03]	price	3.17	6.39	5.11	
	spread	0.09	0.17	0.18	
	N	(53)	(659)	(42)	(754)
> 1.03	price	20.34	22.5	17.6	
	spread	0.22	0.26	0.25	
	N	(567)	(5200)	(306)	(6073)
Subtotal N		(810)	(7707)	(464)	(8981)

Figure 7: Sample Descriptive Statistics for MSFT (figures in dollars)

Moneyness		T(days) < 30	[30; 60]	> 60	Subtotal N
< 0.97	price	0.55	1.41	2.80	
	spread	0.08	0.12	0.17	
	N	(63)	(805)	(32)	(900)
[0.97; 1.03]	price	2.32	3.32	4.93	
	spread	0.11	0.14	0.19	
	N	(39)	(491)	(18)	(548)
> 1.03	price	41.9	18.74	18.40	
	spread	0.24	0.19	0.20	
	N	(791)	(5861)	(239)	(6891)
Subtotal N		(893)	(7157)	(289)	(8339)

per bounds (S_t as the upper bound and $S_t - e^{-rt}K$ as the lower bound); 4)options with best bid equal to zero.

The estimation results for OTM options containing only one EA in their lifetime are presented in Appendix 6.1. In addition to those results, we also estimate our model for OTM options which contain more than one EA over their life span (results in Appendix

Figure 8: Sample Descriptive Statistics for INTC (figures in dollars)

Moneyness		T(days) < 30	[30; 60]	> 60	Subtotal N
< 0.97	price	0.69	1.10	1.20	
	spread	0.07	0.08	0.08	
	N	(613)	(1746)	(74)	(2433)
[0.97; 1.03]	price	2.17	3.18	3.50	
	spread	0.10	0.13	0.14	
	N	(307)	(696)	(36)	(1039)
> 1.03	price	17.59	15.03	16.75	
	spread	0.25	0.23	0.23	
	N	(3215)	(7733)	(405)	(11353)
Subtotal N		(4135)	(10175)	(515)	(14825)

6.2). The comparison of the latter kind of options with the former is needed to see if the simple version of our model (with one jump) is still applicable to options which actually have more than one jump. Intuitively, the model with one jump may fit if every trading day the market is concerned mostly with the closest EA, while the impact of all subsequent EA-s is quite distant in time from the closest one, so they can be neglected. For each company the number of OTM options with one EA is almost two times less than the amount of OTM options with more than one EA. Thus, to make the sample fit for both type of options comparable, we break the sample of the latter options by the median time to maturity. As a result, we report two tables for each company in Appendix 6.2. The section 'Estimation Results', in addition to outlining the main estimaiton results, explains how our estimates differ for the two kinds of options discussed.

3.2 Estimation Method

We estimate and compare four models: Black-Scholes (BS), Merton's jump-diffusion (JD) as in equation (4), our model without hedge premium (we denote it as SG) and our model with premium (SG+). We assume that market prices options correctly with no arbitrage possibilities, therefore we estimate model parameters from market option prices. Our assumption is plausible, since we deleted all contracts, which showed evident arbitrage.

The estimate of a vector of parameters Θ is obtained as

$$\hat{\Theta} = \underset{\Theta}{\operatorname{argmin}} \left(\frac{1}{N} \sum_{i=1}^{i=N} \left(M_i - V_i\left(\Theta\right)\right)^2 \right) \tag{19}$$

$$s.t.\ A \leq \Theta \leq B \tag{20}$$

where M is the market option price (average of best bid and best offer), N is the number of observed contracts, and $V(\Theta)$ is an option value prescribed by a certain model. Vectors A and B are lower an upper constraints on Θ. We specify

$$\Theta = (\sigma\ a\ \lambda\ k\ \delta\ \rho)' \tag{21}$$

$$A = (0\ 0\ 3\ \ -1\ 0\ 0)'\ ^{3} \tag{22}$$

$$B = (1\ 1\ 600\ 1\ 1\ 1)' \tag{23}$$

There are many variants for the error function to be minimised. Dubinsky and Johannes (2006) use relative error of the form[4]

$$\sum_{i=1}^{i=N} \left(\frac{M_i - V_i}{S_i} \right)^2 \tag{24}$$

where S_i is the current stock price of i-th company. Boes and Drost (2007) also use relative error, but with respect to market price:

$$\sum_{i=1}^{i=N} \left(\frac{M_i - V_i}{M_i} \right)^2 \tag{25}$$

Broadie, Chernov and Johannes (2007) use a more complex error function based on implied volatilities:

$$\sum_{i=1}^{i=N} \left(IV_i^{BS} - IV_i^{\text{model}}\left(\Theta\right) \right)^2 \tag{26}$$

where IV_i^{BS} is the Black-Scholes volatility implied from observed market prices and $IV_i^{\text{model}}(\Theta)$ is the volatility given by the model under consideration.

In this paper we choose to minimize standard MSE as in (19), first, because it has a simple interpretation: pricing error in dollars squared. Second, it automatically allows us to compare nested models using F-test and to calculate Akaike and Schwarz criteria.

[3]Note that the only constraint, which appeared binding is $3 \leq \lambda \leq 600$. We find negligible difference in fit if this constraint is omitted.

[4]this is simplified error function of that used by Dubinsky and Johannes. They use additional parameters in their function, which are not relevant to the current paragraph

Finally, as we estimate models on OTM options ($0 \leq \frac{S}{K} \leq 1$), we need to account for the fact that deep OTM options are traded less frequently that near OTM and they deserve lower weight in error function. MSE automatically weights deep OTM options, since they are the cheapest ones and dollar error in those options is small relative to those that are near OTM. Larger price makes near OTM options have more weight than deep OTM in MSE minimization. In contrast, relative error minimization does not produce such effect. The minimization algorithm is implemented using 'fmincon' function in Matlab with iterative starting value search for global optimum. The boundary vectors A and B are chosen to obtain coherent parameter estimates, which do not contradict to existing literature. Thus, σ was never observed to be greater than 1 in scheduled event pricing models (see Dubinsky and Johannes (2006)). The constraint on a and ρ follows from our model specification: a must be less than 1 to preserve positive stock price. Parameters k and δ exceeding 1 by magnitude would mean extreme events, for which we need some sort of special model different from Merton's one. We have chosen to impose the upper bound on λ at the level of 600 jumps per year and the lower at 3 jumps in face of results achieved by other researchers. For instance, Honore (Honore) estimates the lowest λ in his sample to be 5.17 and the highest to be 259. We slightly round up Honore's frames from below, but extend them from above, since we work not only with ATM options, but with OTM as well, which in turn may behave unpredictably in estimation. Furthermore, we set the lower bound on λ equal to 3 on purpose - we allow Merton's model to capture all four earning announcements during a year. The estimate of λ around 4 would mean a strong support for our model, since there are no other significant jumps to capture apart from the scheduled EA-s.

There were no ambiguities in constraint minimization for any model except Merton's JD. As this model has many parameters, the relative error in (19) showed a number of local minima, which complicates the global minimum search. Furthermore, in some cases large values of λ contrubited to σ estimates being close to zero. This may be handled by additional constraints on σ, but such constraints often appear (an in our paper too) to be binding. This means that a researcher imposing such constraints is effectively choosing an estimate himself, neglecting any data. In face of these issues, we did not impose additional constraints for σ to keep results in their rough form and to stress problems with the estimation of Merton's model.

One more empirical problem in Merton's model, noticed by us is the length of the sequence of Black-Scholes values, which approximates Merton's price. For small values of λ, 5-10 terms in a sequence suffice, however for large values of λ (for instance, more about 100) we need a sequence of size 50-80 for OTM options. Such a long sequence is inevitable, but computationally expensive. We restricted ourselves with a sequence of size 150 in our estimation, which was precise enough for OTM options we used.

After estimating parameters we run a number of formal tests. First, as BS is almost a linear restriction of SG (in the limit), F-tests for comparison of these models will be approximately correct. F-tests are quite common in option pricing literature to compare nested models (see Bakshi (1997)). They are used along with maximum likelihood tests, but since we estimated our parameters through MSE, we find F-test more appropriate than maximum likelihood in our paper. We run F-tests for three types of H_A. The first one is to test if SG better fits the sample than BS, in which $H_A = \{MSE(SG) < MSE(BS)\}$. The second test is to check if SG+ is better than BS is sample fitting, for this $H_A = \{MSE(SG+) < MSE(BS)\}$. The third is to see if hedge premium parameter ρ is significant, in this case hypothesis is $H_A = \{MSE(SG+) < MSE(SG)\}$. F-test is inaplicable to compare SG and AJ, since they are not nested. To compare them we run Diebold-Mariano test for the pair difference is MSE between these models. In this test $\text{Error}_i = SE_i(SG+) - SE_i(JD)$, where SE_i is squared error for a particilar observation The hypothesys is $H_A = \{E\left(SE(SG+) > SE(JD)\right)\}$. Since autocorrelation may be present in SE, then we use Newey-West variance estimator for this test. P-values of all the test above are reported in the table with estimation results (Appendix 6.1).

3.3 Estimation Results

The results of our estimates are presented in Appendix 6.1 and 6.2 as well as in the tables below. All our results are broken down by moneyness, by which we mean that for each company and each year we provide 2 tables: one with the results for OTM options ($\frac{S}{K} < 0.97$) and the other for ATM options ($0.97 \leq \frac{S}{K} \leq 1.03$) They clearly show that the introduction of scheduled-time jump is a reasonable model specification. Particularly, the jump amplitude a is significant at 5 percent for 38 subsamples out of 50 among OTM opitons and for 35 out of 50 subsamples among ATM options. Most importantly, the Diebold-Mariano test of SG relative to JD shows that JD is never significantly better than SG. Furthermore, JD model, being overparametrized, looses to SG in terms of AIC and SBC in about 75 percent of our subsamples. Those subsamples where JD is better than SG in terms of these criteria mostly contain insignificant a. However, the most striking feature is that there are many subsamples where SG model (or SG+), having only 2 free parameters (and 3 parameters for SG+), shows better sample fit that JD, which has four parameters. It happens in 22 (out of 50) subsamples among OTM options and 28 (out of 50) subsamples among ATM options, which is theoretically possible, as SG and JD are not nested. For instance, the $\sqrt{MSE}$ in fitting AMD options (out-of-the-money) in 2000 was 0.5814 for JD and 0.4422 for SG. This striking result shows that in most cases scheduled jumps is a better specification than unscheduled ones. Of course, a curious reader may think that the reason for JD fitting these samples worse than SG is that the constraint $5 \leq \lambda \leq 300$ is often binding, which stops MSE of JD from reaching global minimum. To check this idea, we estimated JD model without $3 \leq \lambda \leq 600$ constraint on every subsample under question. In all of these samples JD still showed a worse fit than SG, because the improvement from omitting $5 \leq \lambda \leq 600$ restriction was negligible.

Figure 9: OTM subsamples where SG model showed the best performance

			$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho} \times \mathbf{100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{MSE}$
AMD	2000	BS	0.84						0.3888	0.1545	0.5003
	(173)	SG	0.68	0.31					0.3014	0.1345	0.3915
		SG+	0.68	0.31	0.09				0.3003	0.1333	0.3913
		JD	0.00			39	0.13	0.00	0.3692	0.1529	0.4740
INTC	2008	BS	0.37						0.0801	0.3672	0.1162
	(198)	SG	0.26	0.15					0.0589	0.3028	0.0859
		SG+	0.26	0.15	0.27				0.0593	0.2919	0.0855
		JD	0.00			20.19	0.08	0.00	0.0735	0.3348	0.1069
MSFT	2002	BS	0.38						0.3166	0.5223	0.4022
	(222)	SG	0.21	0.20					0.3081	0.5123	0.3801
		SG+	0.21	0.20	0.18				0.3060	0.5077	0.3796
		JD	0.27			453	-0.01	0.01	0.3171	0.5208	0.4022
AAPL	2006	BS	0.41						0.1540	0.0852	0.1958
	(131)	SG	0.32	0.16					0.1274	0.0679	0.1565
		SG+	0.32	0.16	0.01				0.1275	0.0679	0.1565
		JD	0.00			143.58	0.03	0.00	0.1509	0.0830	0.1910

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3}; 7 \times 10^{-3})$

Figure 10: ATM subsamples where SG model showed the best performance

			$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho} \times \mathbf{100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{MSE}$
AMD	2000	BS	0.85						0.5013	0.0845	0.5856
	(47)	SG	0.67	0.32					0.3478	0.0611	0.4422
		SG+	0.67	0.32	0.00				0.3478	0.0611	0.4422
		JD	0.00			105	0.08	0.00	0.4975	0.0836	0.5814
CSCO	1999	BS	0.47						0.4215	0.0760	0.5178
	(182)	SG	0.43	0.11					0.3819	0.0718	0.4781
		SG+	0.43	0.11	0.00				0.3820	0.0718	0.4781
		JD	0.00			470.43	-0.02	0.00	0.4203	0.0758	0.5163
INTC	2008	BS	0.39						0.1277	0.1332	0.1645
	(101)	SG	0.29	0.13					0.0738	0.0778	0.1085
		SG+	0.29	0.13	0.01				0.0738	0.0778	0.1085
		JD	0.00			105.64	0.04	0.00	0.1277	0.1331	0.1645
AAPL	2006	BS	0.41						0.1476	0.0340	0.1817
	(97)	SG	0.34	0.14					0.1119	0.0258	0.1484
		SG+	0.34	0.14	0.00				0.1120	0.0258	0.1484
		JD	0.00			166.48	-0.03	0.00	0.1454	0.0336	0.1793

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3}; 7 \times 10^{-3})$

Figure 11: Estimation results averaged by company for OTM options with one EA in their life span

		$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho} \times \mathbf{100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{MSE}$
AMD	BS	0.61						0.1358	0.2270	0.1705
	SG	0.56	0.14					0.1243	0.2189	0.1563
	SG+	0.56	0.14	1.79				0.1240	0.2169	0.1560
	JD	0.06			125.14	0.08	0.05	0.1284	0.2102	0.1626
CSCO	BS	0.42						0.1966	0.2853	0.2428
	SG	0.38	0.10					0.1832	0.2586	0.2293
	SG+	0.38	0.10	0.42				0.1840	0.2606	0.2289
	JD	0.04			53.84	0.10	0.05	0.1803	0.2652	0.2267
INTC	BS	0.38						0.1627	0.3009	0.2053
	SG	0.34	0.08					0.1543	0.2759	0.1962
	SG+	0.34	0.08	0.37				0.1539	0.2719	0.1958
	JD	0.02			41.48	0.06	0.01	0.1526	0.2683	0.1967
AAPL	BS	0.54						0.5218	0.2821	0.6349
	SG	0.48	0.11					0.5082	0.2795	0.6128
	SG+	0.48	0.11	2.34				0.5103	0.2784	0.6117
	JD	0.09			26.03	0.02	0.15	0.4952	0.2873	0.6038
MSFT	BS	0.32						0.2178	0.3372	0.2741
	SG	0.28	0.06					0.2160	0.3191	0.2699
	SG+	0.28	0.06	1.64				0.2158	0.3112	0.2696
	JD	0.13			92.22	0.01	0.06	0.2109	0.3156	0.2676

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3}; 7 \times 10^{-3})$

Figure 12: Estimation results averaged by company for ATM options with one EA in their life span

		$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho \times 100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{MSE}$
AMD	BS	0.61						0.1631	0.0896	0.1915
	SG	0.51	0.19					0.1392	0.0821	0.1699
	SG+	0.51	0.19	0.01				0.1391	0.0822	0.1699
	JD	0.18			150.78	-0.04	0.01	0.1561	0.0854	0.1844
CSCO	BS	0.43						0.2087	0.1053	0.2676
	SG	0.39	0.10					0.1942	0.0960	0.2527
	SG+	0.39	0.10	0.02				0.1943	0.0961	0.2527
	JD	0.00			120.48	0.01	0.00	0.2070	0.1040	0.2655
INTC	BS	0.39						0.2078	0.1081	0.2585
	SG	0.35	0.08					0.1980	0.1002	0.2446
	SG+	0.35	0.08	0.02				0.1980	0.1002	0.2446
	JD	0.09			126.65	-0.02	0.00	0.2059	0.1064	0.2559
AAPL	BS	0.54						0.5782	0.1113	0.6981
	SG	0.46	0.14					0.5548	0.1059	0.6646
	SG+	0.46	0.14	0.05				0.5549	0.1059	0.6646
	JD	0.02			45.33	-0.02	0.06	0.5652	0.1081	0.6885
MSFT	BS	0.32						0.3172	0.1362	0.3759
	SG	0.28	0.06					0.3130	0.1325	0.3711
	SG+	0.28	0.06	0.04				0.3130	0.1326	0.3711
	JD	0.05			27.73	-0.06	0.06	0.3083	0.1272	0.3677

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3}; 7 \times 10^{-3})$

In addition to the criticism of JD model above, there are cases when JD estimates are troublesome to interpret. Those are mostly cases when σ is very close to zero (marked with * in the tables). Technically, the reason of such incoherent estimates is that $\left|\frac{\partial MSE}{\partial \sigma}\right| \leq \left|\frac{\partial MSE}{\partial \lambda}\right|$ for most paths of $(\sigma, \lambda, k, \delta)$. Thus, λ took most weight on itself in MSE minimization. In fact, large values of λ indicate that the most uncertainty lies in jump process, but not in diffusion. Theoretically it is possible, but a very large difference between σ estimated from JD and σ from BS is still confusing. In general, such troublesome outcomes are the result of overparametrization of JD model, which make us minimize MSE with respect to 4 parameters. Estimation is a lot easier when there's only one parameter (BS) or even two (SG). In contrast to JD model, SG did not yield unreasonable estimates in our subsamples.

There are other features of estimation results to be discussed. The hedging premium parameter ρ is significant at 10 percent for 11 subsamples out of 50. This is no way against our model - it just tells that investors are conservative about the option replicating portfolio only in a few cases. The insignificant premium is interpreted as the ability to hedge out the jump risk (as was also assumed by Merton (1976)). Also, relative errors reported in the table, which are of the form $\left|\frac{M-V(\Theta)}{V(\Theta)}\right|$ also support SG model. Again, in the majority of subsamples, relative error is less for SG model than for JD model. The size of relative error for BS model varies from 0.05 to 0.52. Large errors mostly occur for OTM options. It is well known that OTM options are the most difficult ones to fit, for example, Macbeth (1979) shows largest error on OTM options of the size 0.68. That value of 0.68 is an extreme case, but if you average Macbeth's estimates, they will be in line with ours. Just to remind the reader, the OTM options were used, first, because on these options our model is mostly different from Black Scholes. Second, any extension of Black-Scholes is not needed for ITM options, since on ITM options the BS model's relative error is very small (about 5 percent), which is an acceptable level of accuracy.

The parameter estimates are also worth attention. Values of a vary from 0.02 to 0.33, where the former value means that at earning announcement the stock price at its extreme can jump 2 percent up or 2 percent down, and the latter value means 33 percent up or 33 percent down. These estimates fit into real stock dynamics very well. You would not expect the stock price to change more than 20-30 percent after earning announcements in efficient markets. Furthermore, these values of a mean that the standard deviation of uniform jump, calculated as $\frac{a}{\sqrt{3}}$ varies approximately from 0.03 to 0.17. The average estimate of a across all companies and years is around 0.1, which means that the average volatility of jumps is roughly 8 percent. This result perfectly fits into those of Dubinsky and Johannes (2006), who get the average volatility of their jumps at the level of 7.93 percent. Concerning Merton's model estimates, they are mostly characterised with large λ and low k with δ. It means that we mostly capture frequent small shocks for the stock price. For instance, a randomly times newsflow which happens once per trading day would characterise our estimates very well. This is in line with what other researchers estimate. For instance, Honore (1998) obtains high estimates for λ (130 on average) and indicates that Merton's model does not capture extreme events (though, in his opinion, it was intended directly for rare events). A very interesting feature (an reasonable as well) is that $\sigma^{JD} \leq \sigma^{SG} \leq \sigma BS$. BS model yields the highest σ estimate, because variation both in jump and diffusion is contained in σ^{BS}, though it was intended only for diffusion by Black and Scholes. When jumps are introduced in SG and JD, they absorb part of the variation in stock price, which leads to decrease in diffusion variation σ.

As you can see from the historical stock prices in Appendix 6.3, all companies used in our estimations, were part of dotcom bubble in 2000-2001. This is captured by higher σ estimates for 2000-2001 years compared to other periods. For instance, σ^{BS} was about 0.8 in 2000 for AMD stock, while it fell to 0.6 in 2003 and 0.49 in 2004. Also, one can see that our model does not work well for every stock. Particularly, it works very well on CSCO (SG is significant in all years), but it works not so good for MSFT (SG significant in 7 years out of 10). In general, MSFT is a problematic stock to price with Black-Scholes style models. In the paper by Dubinsky and Johannes the stock of MSFT had largest absolute pricing errors compared to other technology stocks.

The discussion above was held only for OTM options with one EA in their life span. We also provide estimates for options with more than one EA (let's call them 'long options' in contrast to 'short' options, which have only one EA). A surprising result is that Merton's jump-diffuison is never significantly better than SG for long options in terms of sample fit. One could think that long options, which are characterised both by longer life span, contain more randomly-timed jumps than short options. This makes unscheduled jumps influence volatility stronger than scheduled EA-s for long options. As a result, Merton's model should be more appropriate for long options than SG model. Our estimations do not support this idea. Our result that Merton's model is never significantly better than SG for long options is corroborated by the fact that in some cases SG model actually provides lower MSE than jump-diffusion. Of course, the number of such cases is less than that for short options, but is still noticeable. For instance, AAPL long options with $T < 0.9342$ showed that SG provides smaller MSE than JD for five years out of ten.

The results for long options show than our model works well for them. Nevertheless, it is worth to extend our model for the general case of N earning announcements in order to explain long options better. This idea comes from the fact that the estimates of a (EA jump amplitude) are systematically higher for long options than for short ones. This fact indicates that by applying simple SG model with one jump to long options, we capture the impact of consecutive EA-s just in one jump. However, introducing N jumps will make the model more complex computationally. Therefore, in our further research we aim to compare the benefits of extending our model with the computational costs.

To conclude this section and to stress our main idea, SG model is better than JD model, because, first, it provides a better fit for the majority of subsamples. This result is corroborated by the fact that SG provides better fit for many subsamples of long options. Second, it is parsimonious: only 2 parameters without premium, compared to four parameters in JD. Third, the SG solution is a finite sequence, while Merton's solution is an infinite sequence, which means that SG is simpler and faster to calculate. Fourth, SG explicitly describes which jumps it uses for option pricing: we can list the times and names of each scheduled event, which should cause the jump according to our model. In contrast, Merton's model does not explicitly locate the jump event. Fifth, the parameter estimation in SG is much simpler than in JD (at least because JD shows a number of local minima in MSE). Having said all that, we claim that replacing unschedueld jumps with scheduled ones will not only increase the model's explanatory power and pricing accuracy in the majority of cases, but also will make it more parsimonious and simpler to estimate.

4 Conclusion

To sum up, this paper makes a meaningful contribution to option pricing literature by introducing SG pricing model based on scheduled uniform jumps. Our paper is distinguished from the others in a way that, first, we suggest uniform distribution in scheduled jumps, which greatly simplifies the resulting formula for option value and parameter estimation. Second, we justify our choice of martingale measure and explicitly show its analytical form by using Radon-Nikodym derivative. Many papers related to scheduled jumps neglect the martingale measure construction. Third, we compare our model with Merton's jump-diffusion and show that in the majority of samples our model provides a better fit that Merton's one. In face of this result, parsimony and easy estimation procedure of our model we suggest to use it instead of Merton's one.

We show that scheduled jumps is a better specification for stock price than unscheduled jumps. Thus, we suggest to replace the latter with the former in option pricing models.

This replacement will make a model to fit option prices better and strongly simplify parameter estimation. We believe that our paper can be beneficial for option pricing theorists, and even more beneficial for practitioners, who need simple and coherent models.

We see a number of directions for further research. First of all, we are going to extend the model for N scheduled jumps, which will allow us to price options covering any number of scheduled events during their life. Second, it is reasonable to extend our model for time-varying stochastic volatility both in diffusion and in jump. Although the solution for option price will become more complicated, the time-varying volatility may strongly improve market prices fit.

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5 Appendix

5.1 Estimation Results for options containing one EA in their life span

The ten tables below contain estimation results for five companies for the time period from 1999 to 2008. Estimation results for each company are broken down into 2 tables: the first for OTM options ($\frac{S}{K} < 0.97$) and the second for ATM options ($0.97 \leq \frac{S}{K} \leq 1.03$). The companies are: AMD, AAPL, CSCO, MSFT, INTC. In each table the number of observations is placed in brackets below the date. In the tables, MAE is Mean Absolute Error of the form $|M - V(\Theta)|$; $\sqrt{MSE}$ is the square root of Mean Square Error as in (19), measured in \$; RE means relative absolute error of the form $\left|\frac{M-V(\Theta)}{V(\Theta)}\right|$; AIC and BIC mean Akaike and Schwarz criteria respectively; the entry in p-val corresponding to SG model is the p-value of $H_A\{SG \text{ better than } BS\}$; entry in p-val corresponding to SG+ model is the p-value of $H_A\{SG+ \text{ better than } BS\}$; entry in p-val corresponding to JD model is the p-value of $H_A\{JD \text{ better than } SG\}$; entry in p-val' is the p-value of the test for significance of ρ

Table 1: Estimation Results for AMD (OTM options).

date		σ	a	$\rho \times 100$	λ	k	δ	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.76						0.2022	0.2271	0.2421	272	275		
(132)	SG	0.76	0.00					0.2017	0.2258	0.2418	274	280	0.56	
	SG+	0.76	0.00	14.19				0.2020	0.2264	0.2418	276	284	0.82	0.83
	JD	0.00			15	-0.06	0.21	0.1950	0.2126	0.2367	272	284	0.45	
2000	BS	0.84						0.3888	0.1545	0.5003	654	657		
(173)	SG	0.68	0.31					0.3014	0.1345	0.3915	571	577	0.00	
	SG+	0.68	0.31	0.09				0.3003	0.1333	0.3913	573	582	0.00	0.64
	JD	0.00			39	0.13	0.00	0.3692	0.1529	0.4740	641	654	0.62	
2001	BS	0.70						0.1892	0.2479	0.2466	591	594		
(225)	SG	0.68	0.13					0.1904	0.2541	0.2451	590	597	0.09	
	SG+	0.68	0.13	0.01				0.1904	0.2542	0.2451	592	602	0.24	1.00
	JD	0.07			559	-0.03	0.01	0.1880	0.2474	0.2462	596	609	0.51	
2002	BS	0.65						0.0946	0.2456	0.1149	1	4		
(75)	SG	0.63	0.11					0.0944	0.2460	0.1147	3	8	0.68	
	SG+	0.63	0.11	0.02				0.0944	0.2461	0.1147	5	12	0.92	1.00
	JD	0.49			524	-0.02	0.01	0.0947	0.2455	0.1148	7	16	0.51	
2003	BS	0.60						0.1002	0.2375	0.1177	54	57		
(114)	SG	0.48	0.24					0.0999	0.2403	0.1145	50	55	0.01	
	SG+	0.48	0.24	0.23				0.0991	0.2374	0.1143	51	60	0.04	0.57
	JD	0.00			37	0.09	0.00	0.0980	0.2274	0.1154	56	66	0.51	
2004	BS	0.48						0.0718	0.2063	0.0861	200	203		
(276)	SG	0.46	0.09					0.0712	0.2059	0.0852	196	203	0.02	
	SG+	0.46	0.09	0.70				0.0707	0.2044	0.0846	194	205	0.01	0.05
	JD	0.01			18	0.03	0.10	0.0636	0.1783	0.0788	157	171	0.43	
2005	BS	0.42						0.0977	0.2739	0.1271	226	229		
(195)	SG	0.36	0.14					0.0911	0.2558	0.1176	197	204	0.00	
	SG+	0.36	0.14	0.64				0.0890	0.2425	0.1161	194	204	0.00	0.03
	JD	0.00			3	0.20	0.04	0.0825	0.2158	0.1087	171	184	0.42	
2006	BS	0.49						0.0944	0.1423	0.1119	241	245		
(228)	SG	0.45	0.13					0.0777	0.1055	0.0964	175	182	0.00	
	SG+	0.45	0.13	0.15				0.0774	0.1035	0.0962	176	186	0.00	0.31
	JD	0.03			34	0.08	0.00	0.0843	0.1164	0.1025	207	221	0.54	
2007	BS	0.44						0.0574	0.1684	0.0858	-24	-22		
(106)	SG	0.40	0.11					0.0537	0.1572	0.0840	-27	-22	0.04	
	SG+	0.40	0.11	0.43				0.0550	0.1561	0.0837	-26	-18	0.08	0.40
	JD	0.01			17	0.08	0.05	0.0504	0.1541	0.0803	-32	-22	0.42	
2008	BS	0.74						0.0617	0.3664	0.0727	-66	-63		
(87)	SG	0.72	0.10					0.0614	0.3641	0.0726	-64	-59	0.61	
	SG+	0.72	0.10	1.46				0.0613	0.3646	0.0721	-63	-56	0.49	0.29
	JD	0.01			5	0.26	0.10	0.0585	0.3519	0.0685	-70	-60	0.37	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 2: Estimation Results for AMD (ATM options).

date		$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho \times 100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.68						0.2018	0.1212	0.2409	4	5		
(19)	SG	0.65	0.12					0.1948	0.1147	0.2396	6	8	0.67	
	SG+	0.65	0.12	0.00				0.1948	0.1147	0.2396	8	10	0.92	1.00
	JD	0.00			9	-0.24	0.00	0.1626	0.0969	0.1987	3	6	0.36	
2000	BS	0.85						0.5013	0.0845	0.5856	133	135		
(47)	SG	0.67	0.32					0.3478	0.0611	0.4422	108	112	0.00	
	SG+	0.67	0.32	0.00				0.3478	0.0611	0.4422	110	116	0.00	1.00
	JD	0.00			105	0.08	0.00	0.4975	0.0836	0.5814	138	145	0.69	
2001	BS	0.73						0.2754	0.0961	0.3228	54	56		
(38)	SG	0.67	0.18					0.2653	0.0950	0.3150	54	58	0.18	
	SG+	0.67	0.18	0.04				0.2650	0.0949	0.3149	56	61	0.42	0.97
	JD	0.00			7	-0.30	0.04	0.2657	0.0923	0.3136	58	65	0.49	
2002	BS	0.63						0.0674	0.0539	0.0757	-25	-24		
(9)	SG	0.53	0.21					0.0503	0.0436	0.0656	-25	-25	0.17	
	SG+	0.53	0.21	0.01				0.0502	0.0436	0.0656	-23	-23	0.42	1.00
	JD	0.01			15	-0.17	0.00	0.0583	0.0481	0.0689	-20	-20	0.56	
2003	BS	0.59						0.1360	0.1199	0.1514	-14	-13		
(18)	SG	0.25	0.33					0.1284	0.1131	0.1421	-14	-12	0.16	
	SG+	0.25	0.33	0.01				0.1284	0.1131	0.1421	-12	-10	0.38	1.00
	JD	0.58			411	0.00	0.01	0.1360	0.1199	0.1514	-8	-4	0.60	
2004	BS	0.49						0.0752	0.0718	0.0896	-22	-20		
(97)	SG	0.48	0.07					0.0750	0.0720	0.0891	-21	-16	0.29	
	SG+	0.48	0.07	0.02				0.0750	0.0721	0.0891	-19	-12	0.58	1.00
	JD	0.00			13	0.13	0.01	0.0699	0.0667	0.0843	-28	-18	0.42	
2005	BS	0.43						0.1235	0.1132	0.1609	57	59		
(78)	SG	0.37	0.13					0.1153	0.1130	0.1450	43	47	0.00	
	SG+	0.37	0.13	0.02				0.1153	0.1131	0.1450	45	52	0.00	0.99
	JD	0.00			77	0.05	0.00	0.1221	0.1119	0.1588	61	70	0.58	
2006	BS	0.50						0.1153	0.0550	0.1320	30	32		
(81)	SG	0.45	0.13					0.0894	0.0405	0.1100	2	7	0.00	
	SG+	0.45	0.13	0.01				0.0895	0.0405	0.1100	4	12	0.00	1.00
	JD	0.00			148	0.04	0.00	0.1136	0.0542	0.1312	35	45	0.64	
2007	BS	0.45						0.0544	0.0545	0.0636	-78	-76		
(50)	SG	0.38	0.14					0.0488	0.0491	0.0593	-83	-79	0.01	
	SG+	0.38	0.14	0.01				0.0488	0.0492	0.0593	-81	-75	0.04	1.00
	JD	0.44			357	0.00	0.00	0.0544	0.0545	0.0636	-72	-64	0.59	
2008	BS	0.76						0.0807	0.1259	0.0926	-29	-28		
(15)	SG	0.64	0.24					0.0766	0.1193	0.0916	-27	-26	0.60	
	SG+	0.64	0.24	0.01				0.0766	0.1193	0.0916	-25	-23	0.88	1.00
	JD	0.72			366	0.01	0.01	0.0807	0.1259	0.0926	-23	-20	0.52	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 3: Estimation Results for CSCO (OTM options).

date		σ	a	$\rho \times 100$	λ	k	δ	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.46						0.3875	0.1481	0.4503	303	306		
(100)	SG	0.41	0.12					0.3325	0.1342	0.4015	282	287	0.00	
	SG+	0.41	0.12	0.05				0.3324	0.1343	0.4014	284	292	0.00	0.87
	JD	0.00			19.14	0.10	0.01	0.3473	0.1278	0.4145	292	303	0.53	
2000	BS	0.57						0.7375	0.3072	0.9078	807	810		
(164)	SG	0.56	0.07					0.7358	0.3070	0.9047	808	814	0.30	
	SG+	0.56	0.07	0.93				0.7388	0.3090	0.9037	809	818	0.49	0.55
	JD	0.37			3.00	0.09	0.22	0.7019	0.3106	0.8790	802	814	0.44	
2001	BS	0.70						0.1608	0.2903	0.2048	579	583		
(247)	SG	0.69	0.08					0.1607	0.2898	0.2043	580	587	0.29	
	SG+	0.69	0.08	0.69				0.1605	0.2896	0.2041	582	592	0.43	0.46
	JD	0.02			48.06	0.01	0.10	0.1567	0.2834	0.2030	581	595	0.48	
2002	BS	0.56						0.1072	0.3039	0.1301	264	267		
(208)	SG	0.50	0.15					0.1051	0.3008	0.1259	252	259	0.00	
	SG+	0.50	0.15	0.09				0.1053	0.3018	0.1259	254	264	0.00	0.82
	JD	0.00			446.71	0.03	0.00	0.1066	0.3038	0.1299	269	283	0.55	
2003	BS	0.36						0.1133	0.3527	0.1428	161	164		
(146)	SG	0.31	0.12					0.1102	0.3449	0.1384	154	160	0.00	
	SG+	0.31	0.12	0.47				0.1107	0.3469	0.1381	155	164	0.01	0.41
	JD	0.00			3.00	0.16	0.06	0.1077	0.3428	0.1358	153	165	0.46	
2004	BS	0.33						0.0726	0.2330	0.0978	127	130		
(197)	SG	0.25	0.13					0.0580	0.1908	0.0809	54	60	0.00	
	SG+	0.25	0.13	0.23				0.0577	0.1887	0.0806	54	64	0.00	0.25
	JD	0.01			3.00	0.15	0.02	0.0626	0.2151	0.0837	72	85	0.53	
2005	BS	0.27						0.0598	0.2663	0.0709	-71	-69		
(66)	SG	0.23	0.09					0.0540	0.2458	0.0647	-81	-76	0.00	
	SG+	0.23	0.09	0.49				0.0531	0.2409	0.0642	-80	-73	0.00	0.30
	JD	0.00			3.00	0.12	0.04	0.0503	0.2225	0.0601	-87	-78	0.43	
2006	BS	0.27						0.0545	0.2343	0.0638	-88	-85		
(100)	SG	0.22	0.09					0.0380	0.1604	0.0439	-161	-156	0.00	
	SG+	0.22	0.09	0.35				0.0376	0.1555	0.0433	-161	-153	0.00	0.13
	JD	0.00			3.80	0.12	0.02	0.0346	0.1407	0.0404	-173	-163	0.41	
2007	BS	0.30						0.0879	0.2319	0.1028	-17	-15		
(73)	SG	0.23	0.10					0.0561	0.1341	0.0742	-62	-58	0.00	
	SG+	0.23	0.10	0.18				0.0570	0.1357	0.0741	-61	-54	0.00	0.56
	JD	0.00			4.24	0.13	0.00	0.0517	0.1322	0.0699	-67	-58	0.46	
2008	BS	0.39						0.1851	0.4850	0.2572	340	343		
(148)	SG	0.35	0.10					0.1812	0.4780	0.2544	338	344	0.07	
	SG+	0.35	0.10	0.75				0.1868	0.5038	0.2539	340	349	0.15	0.44
	JD	0.00			4.44	0.15	0.05	0.1832	0.5727	0.2507	338	350	0.46	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 4: Estimation Results for CSCO (ATM options).

date		σ	a	$\rho \times 100$	λ	k	δ	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.47						0.4215	0.0760	0.5178	203	206		
(182)	SG	0.43	0.11					0.3819	0.0718	0.4781	194	199	0.00	
	SG+	0.43	0.11	0.00				0.3820	0.0718	0.4781	196	203	0.01	1.00
	JD	0.00			470.43	-0.02	0.00	0.4203	0.0758	0.5163	209	218	0.57	
2000	BS	0.57						0.7574	0.1389	0.9742	463	465		
(101)	SG	0.51	0.13					0.7303	0.1325	0.9530	460	466	0.04	
	SG+	0.51	0.13	0.01				0.7306	0.1326	0.9530	462	470	0.12	0.99
	JD	0.00			83.43	-0.06	0.00	0.7523	0.1377	0.9696	468	478	0.55	
2001	BS	0.76						0.1982	0.1106	0.2367	40	42		
(43)	SG	0.73	0.12					0.1955	0.1102	0.2347	41	45	0.41	
	SG+	0.73	0.12	0.04				0.1955	0.1103	0.2347	43	48	0.71	0.99
	JD	0.00			89.45	0.08	0.00	0.1980	0.1106	0.2365	46	53	0.53	
2002	BS	0.58						0.1290	0.1222	0.1601	-9	-7		
(26)	SG	0.54	0.11					0.1271	0.1215	0.1584	-7	-5	0.48	
	SG+	0.54	0.11	0.04				0.1273	0.1216	0.1584	-5	-1	0.78	0.99
	JD	0.00			137.03	0.05	0.00	0.1300	0.1231	0.1596	-3	2	0.53	
2003	BS	0.38						0.0937	0.1048	0.1195	-22	-20		
(37)	SG	0.36	0.08					0.0874	0.0967	0.1170	-21	-18	0.23	
	SG+	0.36	0.08	0.03				0.0875	0.0969	0.1170	-19	-14	0.49	0.99
	JD	0.00			25.03	0.08	0.00	0.0927	0.1031	0.1183	-16	-10	0.53	
2004	BS	0.33						0.0814	0.0813	0.1081	-21	-19		
(57)	SG	0.29	0.10					0.0760	0.0789	0.0983	-30	-26	0.00	
	SG+	0.29	0.10	0.02				0.0761	0.0790	0.0983	-28	-22	0.01	0.99
	JD	0.00			16.90	-0.08	0.00	0.0748	0.0752	0.1008	-23	-15	0.52	
2005	BS	0.25						0.0568	0.0971	0.0755	-44	-41		
(120)	SG	0.21	0.08					0.0503	0.0849	0.0642	-81	-75	0.00	
	SG+	0.21	0.08	0.04				0.0503	0.0851	0.0642	-79	-70	0.00	0.93
	JD	0.00			181.33	0.02	0.00	0.0562	0.0952	0.0752	-38	-27	0.60	
2006	BS	0.29						0.0757	0.0931	0.0928	-36	-34		
(65)	SG	0.22	0.11					0.0527	0.0669	0.0679	-74	-70	0.00	
	SG+	0.22	0.11	0.01				0.0527	0.0669	0.0679	-72	-66	0.00	1.00
	JD	0.00			46.57	-0.04	0.00	0.0746	0.0935	0.0905	-33	-24	0.68	
2007	BS	0.30						0.0932	0.0906	0.1164	-4	-1		
(68)	SG	0.24	0.10					0.0623	0.0569	0.0847	-45	-40	0.00	
	SG+	0.24	0.10	0.01				0.0623	0.0569	0.0847	-43	-36	0.00	1.00
	JD	0.00			72.56	0.03	0.00	0.0900	0.0867	0.1129	-2	7	0.64	
2008	BS	0.38						0.1807	0.1385	0.2750	140	143		
(78)	SG	0.34	0.10					0.1783	0.1401	0.2704	140	144	0.11	
	SG+	0.34	0.10	0.03				0.1786	0.1405	0.2704	142	149	0.28	0.99
	JD	0.00			82.10	0.04	0.00	0.1810	0.1392	0.2748	146	156	0.55	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 5: Estimation Results for INTC (OTM opitons)

date		$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho} \times \mathbf{100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.46						0.2392	0.1545	0.3195	1360	1364		
(373)	SG	0.44	0.07					0.2246	0.1440	0.3038	1324	1332	0.00	
	SG+	0.44	0.07	0.00				0.2246	0.1440	0.3038	1326	1338	0.00	1.00
	JD	0.00			91.90	0.05	0.00	0.2276	0.1405	0.3099	1343	1359	0.54	
2000	BS	0.50						0.6007	0.4373	0.7432	2091	2095		
(389)	SG	0.47	0.08					0.5856	0.4196	0.7320	2081	2089	0.00	
	SG+	0.47	0.08	0.72				0.5822	0.4030	0.7307	2082	2094	0.00	0.24
	JD	0.00			16.77	0.11	0.00	0.5606	0.3773	0.7129	2065	2080	0.45	
2001	BS	0.51						0.2138	0.4192	0.2530	1471	1475		
(440)	SG	0.46	0.13					0.2064	0.4071	0.2466	1450	1458	0.00	
	SG+	0.46	0.13	0.23				0.2068	0.4100	0.2464	1452	1464	0.00	0.47
	JD	0.19			151.11	0.04	0.00	0.2124	0.4195	0.2525	1475	1491	0.55	
2002	BS	0.46						0.1845	0.4511	0.2224	1212	1216		
(404)	SG	0.42	0.10					0.1837	0.4514	0.2196	1204	1212	0.00	
	SG+	0.42	0.10	0.46				0.1836	0.4516	0.2194	1205	1217	0.00	0.32
	JD	0.00			38.72	0.07	0.00	0.1809	0.4427	0.2198	1208	1224	0.50	
2003	BS	0.36						0.1090	0.3132	0.1445	355	359		
(227)	SG	0.31	0.11					0.1025	0.2917	0.1374	334	341	0.00	
	SG+	0.31	0.11	0.27				0.1027	0.2926	0.1372	336	346	0.00	0.45
	JD	0.00			19.75	0.08	0.00	0.1044	0.2995	0.1403	348	361	0.53	
2004	BS	0.30						0.0683	0.2560	0.0869	134	137		
(233)	SG	0.29	0.06					0.0656	0.2419	0.0855	128	135	0.01	
	SG+	0.29	0.06	0.56				0.0639	0.2318	0.0851	128	138	0.01	0.15
	JD	0.00			6.77	0.10	0.01	0.0580	0.1990	0.0821	113	127	0.46	
2005	BS	0.24						0.0564	0.2725	0.0723	-68	-66		
(64)	SG	0.22	0.06					0.0503	0.2368	0.0677	-74	-70	0.00	
	SG+	0.22	0.06	0.79				0.0499	0.2279	0.0667	-74	-68	0.01	0.18
	JD	0.00			13.56	0.06	0.00	0.0436	0.2035	0.0590	-88	-79	0.34	
2006	BS	0.27						0.0349	0.1747	0.0452	-120	-118		
(57)	SG	0.26	0.04					0.0304	0.1435	0.0377	-139	-135	0.00	
	SG+	0.26	0.04	0.33				0.0304	0.1444	0.0376	-137	-131	0.00	0.61
	JD	0.00			28.08	0.05	0.00	0.0310	0.1547	0.0389	-132	-123	0.54	
2007	BS	0.30						0.0401	0.1638	0.0503	-97	-95		
(46)	SG	0.29	0.04					0.0356	0.1207	0.0453	-105	-101	0.00	
	SG+	0.29	0.04	0.08				0.0358	0.1218	0.0453	-103	-97	0.01	0.93
	JD	0.01			27.97	0.01	0.05	0.0347	0.1117	0.0445	-102	-95	0.47	
2008	BS	0.37						0.0801	0.3672	0.1162	197	200		
(198)	SG	0.26	0.15					0.0589	0.3028	0.0859	79	86	0.00	
	SG+	0.26	0.15	0.27				0.0593	0.2919	0.0855	79	89	0.00	0.17
	JD	0.00			20.19	0.08	0.00	0.0735	0.3348	0.1069	170	183	0.61	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 6: Estimation Results for INTC (ATM opitons)

date		$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho} \times \mathbf{100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val’
1999	BS	0.46						0.3174	0.0665	0.4048	727	730		
(206)	SG	0.44	0.08					0.2954	0.0650	0.3722	694	701	0.00	
	SG+	0.44	0.08	0.01				0.2954	0.0650	0.3722	696	706	0.00	1.00
	JD	0.41			151.54	-0.02	0.00	0.3173	0.0666	0.4048	733	746	0.57	
2000	BS	0.49						0.7129	0.1489	0.8876	882	885		
(178)	SG	0.47	0.07					0.7188	0.1499	0.8823	882	888	0.15	
	SG+	0.47	0.07	0.00				0.7188	0.1499	0.8823	884	893	0.35	1.00
	JD	0.01			151.38	-0.04	0.00	0.7136	0.1488	0.8863	887	900	0.51	
2001	BS	0.53						0.2878	0.1492	0.3327	312	315		
(120)	SG	0.49	0.12					0.2732	0.1400	0.3231	307	313	0.01	
	SG+	0.49	0.12	0.02				0.2732	0.1400	0.3231	309	318	0.03	1.00
	JD	0.00			15.35	-0.14	0.00	0.2733	0.1403	0.3184	308	319	0.48	
2002	BS	0.47						0.2329	0.1545	0.2774	209	212		
(101)	SG	0.44	0.08					0.2332	0.1546	0.2733	208	213	0.08	
	SG+	0.44	0.08	0.04				0.2334	0.1546	0.2733	210	218	0.23	0.98
	JD	0.47			326.04	0.00	0.00	0.2329	0.1545	0.2774	215	226	0.56	
2003	BS	0.39						0.1373	0.1366	0.1721	79	81		
(84)	SG	0.36	0.08					0.1325	0.1303	0.1639	72	77	0.00	
	SG+	0.36	0.08	0.07				0.1325	0.1303	0.1639	74	82	0.02	0.95
	JD	0.00			151.09	0.03	0.00	0.1384	0.1371	0.1716	84	94	0.59	
2004	BS	0.31						0.0895	0.0869	0.1083	22	24		
(103)	SG	0.30	0.05					0.0882	0.0851	0.1066	20	25	0.07	
	SG+	0.30	0.05	0.02				0.0882	0.0851	0.1066	22	30	0.20	1.00
	JD	0.00			166.44	0.02	0.00	0.0892	0.0866	0.1076	26	37	0.52	
2005	BS	0.25						0.0739	0.0845	0.1112	-14	-12		
(62)	SG	0.20	0.08					0.0728	0.0917	0.0978	-28	-24	0.00	
	SG+	0.20	0.08	0.03				0.0729	0.0919	0.0978	-26	-20	0.00	0.98
	JD	0.00			36.10	-0.04	0.00	0.0739	0.0842	0.1085	-12	-3	0.57	
2006	BS	0.26						0.0334	0.0466	0.0421	-99	-97		
(37)	SG	0.24	0.06					0.0287	0.0391	0.0382	-104	-101	0.01	
	SG+	0.24	0.06	0.01				0.0287	0.0391	0.0382	-102	-97	0.04	1.00
	JD	0.00			77.89	0.03	0.00	0.0302	0.0408	0.0395	-98	-91	0.52	
2007	BS	0.32						0.0654	0.0738	0.0840	-49	-47		
(43)	SG	0.30	0.06					0.0635	0.0682	0.0806	-51	-47	0.07	
	SG+	0.30	0.06	0.01				0.0635	0.0682	0.0806	-49	-44	0.19	1.00
	JD	0.00			84.98	-0.03	0.00	0.0627	0.0719	0.0806	-47	-40	0.50	
2008	BS	0.39						0.1277	0.1332	0.1645	104	106		
(101)	SG	0.29	0.13					0.0738	0.0778	0.1085	22	27	0.00	
	SG+	0.29	0.13	0.01				0.0738	0.0778	0.1085	24	31	0.00	1.00
	JD	0.00			105.64	0.04	0.00	0.1277	0.1331	0.1645	110	120	0.71	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 7: Estimation Results for AAPL (OTM options)

date		σ	a	$\rho \times 100$	λ	k	δ	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.70						1.0306	0.3565	1.0880	422	425		
(90)	SG	0.70	0.00					1.0299	0.3562	1.0875	424	429	0.76	
	SG+	0.70	0.00	0.17				1.0299	0.3562	1.0875	426	434	0.96	1.00
	JD	0.14			3.00	-0.37	0.63	0.9732	0.3440	1.0505	422	432	0.44	
2000	BS	0.77						0.4688	0.1934	0.7549	2026	2030		
(377)	SG	0.69	0.21					0.4993	0.3007	0.7175	1990	1998	0.00	
	SG+	0.69	0.21	0.19				0.5059	0.3081	0.7169	1992	2003	0.00	0.42
	JD	0.00			10.24	0.22	0.03	0.4600	0.2382	0.6963	1972	1987	0.46	
2001	BS	0.66						0.1749	0.3490	0.2231	948	952		
(336)	SG	0.66	0.00					0.1746	0.3479	0.2228	949	957	0.34	
	SG+	0.66	0.00	16.77				0.1762	0.3555	0.2225	951	962	0.43	0.39
	JD	0.52			3.00	-0.07	0.27	0.1729	0.3658	0.2211	948	964	0.47	
2002	BS	0.48						0.1094	0.4351	0.1292	333	336		
(239)	SG	0.31	0.23					0.0905	0.3707	0.1063	241	248	0.00	
	SG+	0.31	0.23	0.30				0.0884	0.3505	0.1054	239	250	0.00	0.05
	JD	0.00			52.95	0.06	0.00	0.1050	0.4118	0.1274	332	346	0.62	
2003	BS	0.42						0.0354	0.1329	0.0460	-162	-159		
(118)	SG	0.42	0.01					0.0354	0.1329	0.0460	-160	-154	0.92	
	SG+	0.42	0.01	2.95				0.0349	0.1288	0.0459	-158	-150	0.74	0.44
	JD	0.00			16.12	-0.04	0.11	0.0322	0.1190	0.0430	-172	-161	0.43	
2004	BS	0.49						0.2284	0.4178	0.2728	820	824		
(272)	SG	0.40	0.18					0.2093	0.3753	0.2530	781	788	0.00	
	SG+	0.40	0.18	0.35				0.2094	0.3953	0.2513	779	790	0.00	0.06
	JD	0.00			17.12	0.11	0.01	0.2197	0.4784	0.2597	799	814	0.52	
2005	BS	0.43						0.1753	0.2007	0.2659	666	670		
(236)	SG	0.38	0.13					0.1695	0.1955	0.2473	634	641	0.00	
	SG+	0.38	0.13	0.46				0.1687	0.1847	0.2454	632	643	0.00	0.06
	JD	0.00			6.96	0.14	0.02	0.1731	0.2155	0.2374	619	633	0.46	
2006	BS	0.41						0.1540	0.0852	0.1958	213	216		
(131)	SG	0.32	0.16					0.1274	0.0679	0.1565	157	162	0.00	
	SG+	0.32	0.16	0.01				0.1275	0.0679	0.1565	159	167	0.00	0.97
	JD	0.00			143.58	0.03	0.00	0.1509	0.0830	0.1910	213	224	0.62	
2007	BS	0.46						0.9379	0.2407	1.1124	484	486		
(100)	SG	0.35	0.18					0.8475	0.2388	1.0323	471	476	0.00	
	SG+	0.35	0.18	0.16				0.8574	0.2435	1.0316	473	481	0.00	0.72
	JD	0.00			4.34	0.19	0.03	0.8375	0.2327	1.0114	471	481	0.47	
2008	BS	0.57						1.9031	0.4095	2.2609	1758	1761		
(246)	SG	0.56	0.04					1.8988	0.4087	2.2583	1759	1766	0.46	
	SG+	0.56	0.04	2.08				1.9046	0.3931	2.2543	1760	1771	0.49	0.35
	JD	0.27			3.00	-0.08	0.37	1.8272	0.3843	2.2000	1750	1764	0.41	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 8: Estimation Results for AAPL (ATM options)

date		σ	a	$\rho \times 100$	λ	k	δ	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.69						1.0605	0.1995	1.1549	186	188		
(45)	SG	0.68	0.00					1.0604	0.1995	1.1548	188	192	0.94	
	SG+	0.68	0.00	0.05				1.0604	0.1995	1.1548	190	196	1.00	1.00
	JD	0.09			3.00	-0.23	0.62	0.9892	0.1845	1.1115	189	196	0.44	
2000	BS	0.73						0.6948	0.0799	0.9502	436	438		
(97)	SG	0.62	0.24					0.6814	0.0938	0.8777	422	428	0.00	
	SG+	0.62	0.24	0.02				0.6812	0.0939	0.8777	424	432	0.00	0.96
	JD	0.01			19.05	0.16	0.01	0.6850	0.0784	0.9418	440	450	0.59	
2001	BS	0.70						0.2867	0.1348	0.3273	73	75		
(45)	SG	0.70	0.00					0.2867	0.1348	0.3273	75	78	0.90	
	SG+	0.70	0.00	0.40				0.2867	0.1348	0.3273	77	82	0.99	1.00
	JD	0.08			11.69	-0.21	0.00	0.2766	0.1296	0.3207	77	84	0.43	
2002	BS	0.52						0.1718	0.1263	0.1975	35	37		
(50)	SG	0.38	0.20					0.1323	0.0950	0.1536	12	16	0.00	
	SG+	0.38	0.20	0.02				0.1323	0.0949	0.1536	14	20	0.00	0.98
	JD	0.00			62.50	-0.07	0.00	0.1714	0.1265	0.1960	41	48	0.69	
2003	BS	0.42						0.0687	0.0665	0.0837	-39	-38		
(22)	SG	0.29	0.19					0.0558	0.0557	0.0665	-47	-45	0.00	
	SG+	0.29	0.19	0.01				0.0558	0.0557	0.0665	-45	-42	0.01	1.00
	JD	0.00			40.54	0.06	0.00	0.0673	0.0654	0.0815	-34	-30	0.68	
2004	BS	0.48						0.2830	0.1040	0.3295	150	152		
(72)	SG	0.40	0.16					0.2542	0.0938	0.2880	133	137	0.00	
	SG+	0.40	0.16	0.01				0.2543	0.0939	0.2880	135	141	0.00	0.99
	JD	0.00			15.75	0.12	0.00	0.2633	0.0975	0.3195	152	161	0.60	
2005	BS	0.43						0.2575	0.0778	0.3760	274	277		
(102)	SG	0.37	0.13					0.2461	0.0767	0.3356	253	258	0.00	
	SG+	0.37	0.13	0.00				0.2461	0.0767	0.3356	255	263	0.00	1.00
	JD	0.00			11.53	0.12	0.00	0.2576	0.0780	0.3677	276	286	0.57	
2006	BS	0.41						0.1476	0.0340	0.1817	115	117		
(97)	SG	0.34	0.14					0.1119	0.0258	0.1484	78	83	0.00	
	SG+	0.34	0.14	0.00				0.1120	0.0258	0.1484	80	87	0.00	1.00
	JD	0.00			166.48	-0.03	0.00	0.1454	0.0336	0.1793	118	129	0.62	
2007	BS	0.45						1.0124	0.1424	1.1866	569	572		
(112)	SG	0.33	0.18					0.9160	0.1343	1.1179	557	563	0.00	
	SG+	0.33	0.18	0.00				0.9162	0.1343	1.1179	559	568	0.00	1.00
	JD	0.00			17.88	-0.11	0.00	1.0045	0.1408	1.1788	573	584	0.56	
2008	BS	0.56						1.7988	0.1481	2.1932	641	644		
(103)	SG	0.48	0.16					1.8028	0.1495	2.1759	642	647	0.21	
	SG+	0.48	0.16	0.01				1.8035	0.1496	2.1759	644	651	0.45	1.00
	JD	0.00			104.89	-0.05	0.00	1.7920	0.1472	2.1881	647	657	0.52	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 9: Estimation Results for MSFT (OTM options)

date		σ	a	$\rho \times 100$	λ	k	δ	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.43						0.4968	0.1624	0.5884	619	622		
(155)	SG	0.43	0.00					0.4956	0.1614	0.5877	621	627	0.54	
	SG+	0.43	0.00	8.18				0.4963	0.1617	0.5875	623	632	0.80	0.79
	JD	0.31			3	-0.01	0.19	0.4792	0.1494	0.5741	618	630	0.45	
2000	BS	0.46						0.52634	0.3079	0.6507	755	759		
(175)	SG	0.46	0.00					0.5301	0.3166	0.6513	758	764	1.00	
	SG+	0.46	0.00	0.79				0.5302	0.3168	0.6513	760	769	1.00	1.00
	JD	0.24			3	-0.10	0.29	0.5000	0.3472	0.6239	747	759	0.42	
2001	BS	0.41						0.4054	0.4170	0.4953	260	263		
(85)	SG	0.41	0.00					0.4043	0.4156	0.4941	262	267	0.53	
	SG+	0.41	0.00	5.28				0.4037	0.4152	0.4941	264	271	0.82	0.92
	JD	0.00			43	0.06	0.00	0.3971	0.4322	0.4902	264	274	0.46	
2002	BS	0.38						0.3166	0.5223	0.4022	431	434		
(138)	SG	0.21	0.20					0.3081	0.5123	0.3801	417	423	0.00	
	SG+	0.21	0.20	0.18				0.3059	0.5077	0.3796	419	427	0.00	0.56
	JD	0.27			453	-0.01	0.01	0.3171	0.5208	0.4022	437	448	0.55	
2003	BS	0.33						0.0967	0.4317	0.1229	141	144		
(159)	SG	0.29	0.10					0.0912	0.4077	0.1183	131	137	0.00	
	SG+	0.29	0.10	0.31				0.0889	0.3896	0.1179	132	141	0.00	0.33
	JD	0.00			68	0.04	0.00	0.0930	0.3969	0.1215	144	156	0.54	
2004	BS	0.23						0.0530	0.3365	0.0666	-81	-79		
(83)	SG	0.19	0.08					0.0514	0.3254	0.0646	-84	-79	0.03	
	SG+	0.19	0.08	0.37				0.0515	0.3223	0.0641	-83	-76	0.05	0.26
	JD	0.00			23	0.04	0.00	0.0482	0.3080	0.0624	-86	-76	0.44	
2005	BS	0.18						0.0357	0.3154	0.0437	-99	-98		
(39)	SG	0.16	0.05					0.0305	0.2506	0.0364	-112	-108	0.00	
	SG+	0.16	0.05	0.76				0.0297	0.2405	0.0351	-112	-107	0.00	0.12
	JD	0.00			10	0.05	0.00	0.0293	0.2633	0.0347	-111	-105	0.45	
2006	BS	0.19						0.0135	0.1083	0.0182	-55	-55		
(119)	SG	0.19	0.00					0.0128	0.1045	0.0158	-56	-55	0.14	
	SG+	0.19	0.00	0.02				0.0128	0.1045	0.0158	-54	-53	0.37	1.00
	JD	0.13			16	0.01	0.03	0.0119	0.1058	0.0161	-52	-50	0.53	
2007	BS	0.22						0.0655	0.3838	0.0889	74	77		
(186)	SG	0.21	0.03					0.0643	0.3267	0.0884	73	80	0.13	
	SG+	0.21	0.03	0.21				0.0643	0.3267	0.0884	75	85	0.31	0.75
	JD	0.00			300	0.01	0.00	0.0639	0.3019	0.0870	655	667	0.71	
2008	BS	0.36						0.1685	0.3864	0.2646	1300	1304		
(192)	SG	0.30	0.12					0.1716	0.3701	0.2620	1295	1303	0.01	
	SG+	0.30	0.12	0.31				0.1745	0.3274	0.2618	1296	1308	0.02	0.42
	JD	0.31			5	0.00	0.08	0.1686	0.3306	0.2644	2048	2064	0.74	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 10: Estimation Results for MSFT (ATM options)

date		σ	a	$\rho\times 100$	λ	k	δ	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.43						0.5807	0.0876	0.6641	429	432		
(110)	SG	0.43	0.00					0.5801	0.0875	0.6637	431	436	0.70	
	SG+	0.43	0.00	0.08				0.5801	0.0875	0.6637	433	441	0.93	1.00
	JD	0.00			43	-0.01	0.07	0.5774	0.0867	0.6625	434	445	0.49	
2000	BS	0.44						0.5602	0.1214	0.6771	351	354		
(93)	SG	0.44	0.00					0.5602	0.1216	0.6749	352	357	0.44	
	SG+	0.44	0.00	0.05				0.5602	0.1216	0.6749	354	362	0.75	1.00
	JD	0.34			3	-0.12	0.15	0.5583	0.1195	0.6701	355	365	0.47	
2001	BS	0.45						0.5625	0.1309	0.6196	111	113		
(40)	SG	0.45	0.00					0.5624	0.1309	0.6197	113	117	1.00	
	SG+	0.45	0.00	0.09				0.5624	0.1309	0.6197	115	120	1.00	1.00
	JD	0.00			8	-0.03	0.20	0.5531	0.1288	0.6056	115	122	0.46	
2002	BS	0.40						0.6245	0.2063	0.6749	134	136		
(44)	SG	0.30	0.16					0.6154	0.2027	0.6669	135	138	0.32	
	SG+	0.30	0.16	0.03				0.6153	0.2028	0.6669	137	142	0.61	0.98
	JD	0.02			12	-0.12	0.00	0.6258	0.2066	0.6704	139	146	0.51	
2003	BS	0.35						0.1875	0.1753	0.2314	92	94		
(69)	SG	0.30	0.10					0.1766	0.1608	0.2203	87	92	0.01	
	SG+	0.30	0.10	0.04				0.1764	0.1608	0.2203	89	96	0.04	0.97
	JD	0.00			21	-0.08	0.00	0.1835	0.1719	0.2280	96	105	0.57	
2004	BS	0.21						0.0985	0.1285	0.1259	-5	-3		
(56)	SG	0.09	0.11					0.0905	0.1125	0.1102	-18	-14	0.00	
	SG+	0.09	0.11	0.01				0.0905	0.1126	0.1102	-16	-10	0.00	0.99
	JD	0.00			14	-0.06	0.00	0.0917	0.1174	0.1166	-7	1	0.56	
2005	BS	0.17						0.0740	0.1233	0.0898	-43	-41		
(52)	SG	0.11	0.07					0.0741	0.1263	0.0845	-48	-44	0.01	
	SG+	0.11	0.07	0.04				0.0742	0.1267	0.0845	-46	-40	0.05	0.96
	JD	0.09			151	0.01	0.00	0.0741	0.1227	0.0897	-37	-29	0.57	
2006	BS	0.18						0.0788	0.1242	0.0934	-40	-38		
(44)	SG	0.18	0.02					0.0838	0.1327	0.0981	-34	-30	1.00	
	SG+	0.18	0.02	0.03				0.0838	0.1328	0.0981	-32	-26	1.00	1.00
	JD	0.00			9	-0.06	0.00	0.0674	0.1055	0.0830	-45	-37	0.30	
2007	BS	0.24						0.1523	0.1185	0.1605	-9	-9		
(46)	SG	0.24	0.00					0.1474	0.1143	0.1572	-7	-8	0.70	
	SG+	0.24	0.00	0.05				0.1474	0.1143	0.1572	-5	-6	0.94	1.00
	JD	0.02			3	0.00	0.20	0.1000	0.0675	0.1317	-6	-6	0.30	
2008	BS	0.36						0.2529	0.1458	0.4218	60	62		
(33)	SG	0.24	0.15					0.2393	0.1355	0.4155	61	64	0.34	
	SG+	0.24	0.15	0.02				0.2400	0.1360	0.4155	63	68	0.64	0.99
	JD	0.00			14	-0.10	0.00	0.2521	0.1454	0.4192	66	72	0.54	

*The entries 0.00 are not identically zero, but within $(2\times 10^{-3} 7\times 10^{-3})$

5.2 Estimation Results for options containing more than one EA in their life span

The five tables below contain estimation results for five companies for the time period from 1999 to 2008. The companies are: AMD, AAPL, CSCO, MSFT, INTC. The data for each company is divided in two parts by the median of time to maturity. Thus, two tables for each company are reported: the first shows estimation results for options with time to maturity less than the median and the second depicts results for those with time to maturity higher than the median. In each table the number of observations is placed in brackets below the date. In the tables, MAE is Mean Absolute Error of the form $|M - V(\Theta)|$; $\sqrt{MSE}$ is the square root of Mean Square Error as in (19), measured in \$; RE means relative absolute error of the form $\left|\frac{M-V(\Theta)}{V(\Theta)}\right|$; AIC and BIC mean Akaike and Schwarz criteria respectively; the entry in p-val corresponding to SG model is the p-value of $H_A\{SG \text{ better than } BS\}$; entry in p-val corresponding to SG+ model is the p-value of $H_A\{SG+ \text{ better than } BS\}$; entry in p-val corresponding to JD model is the p-value of $H_A\{JD \text{ better than } SG\}$; entry in p-val' is the p-value of the test for significance of ρ

Table 11: Estimation Results for AMD for $T < 0.9342$

date		$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho \times 100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.42						0.2021	0.1293	0.4224	2715	2720		
(561)	SG	0.36	0.25					0.1953	0.1255	0.3986	2650	2658	0.00	
	SG+	0.36	0.25	0.00				0.1963	0.1265	0.3986	2652	2665	0.00	1.00
	JD	0.33			155.05	-0.01	0.01	0.2024	0.1306	0.4221	6092	6110	0.81	
2000	BS	0.46						0.3970	0.1081	0.8630	2388	2392		
(646)	SG	0.39	0.30					0.3490	0.1105	0.8543	2382	2390	0.00	
	SG+	0.39	0.30	0.07				0.3472	0.1091	0.8541	2384	2396	0.01	0.67
	JD	0.15			5.00	-0.03	0.21	0.3734	0.0816	0.8572	2389	2405	0.51	
2001	BS	0.50						0.1478	0.1357	0.3044	3362	3367		
(667)	SG	0.50	0.00					0.1467	0.1406	0.3044	3364	3373	1.00	
	SG+	0.50	0.00	0.00				0.1467	0.1406	0.3044	3366	3380	1.00	1.00
	JD	0.10			5.23	-0.11	0.22	0.1228	0.1014	0.2766	3219	3238	0.42	
2002	BS	0.40						0.1384	0.2767	0.3250	2835	2839		
(508)	SG	0.30	0.34					0.1284	0.2471	0.3139	2790	2799	0.00	
	SG+	0.30	0.34	0.07				0.1285	0.2472	0.3138	2792	2806	0.00	0.63
	JD	0.04			33.09	-0.04	0.06	0.1304	0.2598	0.3221	4615	4633	0.72	
2003	BS	0.34						0.1411	0.2539	0.2482	1506	1510		
(303)	SG	0.34	0.00					0.1411	0.2539	0.2482	1508	1516	1.00	
	SG+	0.34	0.00	3.62				0.1400	0.2453	0.2482	1510	1522	1.00	0.96
	JD	0.07			5.00	-0.06	0.16	0.1144	0.1792	0.2413	1487	1503	0.43	
2004	BS	0.29						0.0966	0.1467	0.1005	599	603		
(319)	SG	0.29	0.00					0.0966	0.1467	0.1005	602	610	1.00	
	SG+	0.29	0.00	0.01				0.0959	0.1458	0.1005	604	616	1.00	1.00
	JD	0.12			5.01	-0.05	0.12	0.0956	0.1437	0.0944	553	570	0.44	
2005	BS	0.22						0.0873	0.1332	0.0871	326	330		
(181)	SG	0.22	0.00					0.0873	0.1332	0.0871	328	336	1.00	
	SG+	0.22	0.00	11.51				0.0853	0.1275	0.0871	330	341	0.95	0.67
	JD	0.10			5.01	-0.02	0.09	0.0807	0.1144	0.0858	323	338	0.45	
2006	BS	0.24						0.0753	0.1827	0.0880	289	293		
(161)	SG	0.21	0.14					0.0611	0.1809	0.0844	264	271	0.00	
	SG+	0.21	0.14	0.35				0.0605	0.1767	0.0838	261	273	0.00	0.04
	JD	0.00			14.18	0.06	0.00	0.0736	0.1760	0.0837	1452	1467	0.71	
2007	BS	0.25						0.0996	0.2642	0.1363	1273	1278		
(318)	SG	0.17	0.24					0.0997	0.2634	0.1044	983	992	0.00	
	SG+	0.17	0.24	0.37				0.1003	0.2443	0.1020	960	973	0.00	0.00
	JD	0.00			13.28	0.07	0.00	0.0986	0.2645	0.1319	2981	2999	0.73	
2008	BS	0.32						0.0795	0.2677	0.2993	2483	2487		
(759)	SG	0.24	0.26					0.0795	0.2676	0.2910	2450	2459	0.00	
	SG+	0.24	0.26	0.44				0.0795	0.2679	0.2901	2448	2461	0.00	0.04
	JD	0.00			18.42	0.07	0.00	0.0789	0.2675	0.2978	3941	3959	0.78	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3}; 7 \times 10^{-3})$

Table 12: Estimation Results for AMD for $T > 0.9342$

date		$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho \times 100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.64						0.2992	0.0755	0.3781	2095	2099		
(492)	SG	0.62	0.32					0.2985	0.0748	0.3739	2086	2094	0.00	
	SG+	0.62	0.32	0.03				0.2986	0.0748	0.3739	2088	2100	0.00	0.90
	JD	0.28			43.00	0.04	0.07	0.2884	0.0717	0.3692	4636	4652	0.89	
2000	BS	0.75						0.3166	0.0440	0.4975	1069	1073		
(257)	SG	0.75	0.00					0.3164	0.0440	0.4975	1071	1078	1.00	
	SG+	0.75	0.00	0.00				0.3164	0.0440	0.4975	1073	1084	1.00	1.00
	JD	0.00			5.00	-0.05	0.34	0.3087	0.0420	0.4677	2113	2127	0.58	
2001	BS	0.67						0.2639	0.0904	0.3212	2663	2667		
(636)	SG	0.57	0.73					0.2041	0.0778	0.2701	2445	2454	0.00	
	SG+	0.57	0.73	0.00				0.2041	0.0779	0.2701	2447	2460	0.00	1.00
	JD	0.43			47.04	-0.04	0.06	0.2613	0.0850	0.3195	6088	6106	0.79	
2002	BS	0.64						0.3306	0.2664	0.4113	3734	3739		
(767)	SG	0.64	0.00					0.3306	0.2664	0.4113	3736	3745	0.98	
	SG+	0.64	0.00	0.01				0.3306	0.2664	0.4113	3738	3752	1.00	1.00
	JD	0.00			5.00	-0.13	0.29	0.3272	0.2576	0.4061	4740	4759	0.68	
2003	BS	0.56						0.3014	0.2720	0.3748	2646	2650		
(597)	SG	0.56	0.01					0.3014	0.2720	0.3747	2648	2657	0.93	
	SG+	0.56	0.01	24.77				0.2997	0.2655	0.3743	2649	2662	0.46	0.22
	JD	0.00			5.00	0.13	0.19	0.2848	0.2468	0.3587	4275	4292	0.71	
2004	BS	0.47						0.2517	0.1347	0.3130	2245	2249		
(560)	SG	0.45	0.33					0.2410	0.1303	0.3083	2230	2239	0.00	
	SG+	0.45	0.33	0.05				0.2410	0.1303	0.3083	2232	2245	0.00	0.86
	JD	0.22			43.66	0.03	0.05	0.2487	0.1339	0.3111	4601	4618	0.86	
2005	BS	0.41						0.2592	0.1161	0.3206	2429	2434		
(591)	SG	0.37	0.38					0.2456	0.1088	0.3065	2378	2387	0.00	
	SG+	0.37	0.38	0.05				0.2445	0.1082	0.3065	2380	2393	0.00	0.79
	JD	0.25			44.31	0.02	0.05	0.2558	0.1116	0.3187	5120	5137	0.83	
2006	BS	0.44						0.2033	0.0532	0.2554	1584	1588		
(464)	SG	0.43	0.06					0.2034	0.0533	0.2555	1587	1595	1.00	
	SG+	0.43	0.06	0.02				0.2033	0.0533	0.2555	1589	1601	1.00	1.00
	JD	0.28			45.88	-0.01	0.05	0.2032	0.0531	0.2555	4316	4333	0.85	
2007	BS	0.45						0.2428	0.1327	0.3122	2357	2362		
(583)	SG	0.45	0.00					0.2429	0.1328	0.3122	2359	2368	1.00	
	SG+	0.45	0.00	0.06				0.2429	0.1328	0.3122	2361	2375	1.00	1.00
	JD	0.00			5.00	-0.06	0.21	0.2395	0.1284	0.3088	3735	3752	0.71	
2008	BS	0.68						0.1556	0.2679	0.1993	1859	1863		
(589)	SG	0.52	1.00					0.1395	0.2546	0.1852	1774	1783	0.00	
	SG+	0.52	1.00	0.00				0.1395	0.2546	0.1852	1776	1790	0.00	1.00
	JD	0.00			5.00	-0.26	0.26	0.1444	0.2718	0.1780	1823	1841	0.54	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3}; 7 \times 10^{-3})$

Table 13: Estimation Results for CISCO for $T < 0.7332$

date		σ	a	$\rho \times 100$	λ	k	δ	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.46						0.8831	0.1254	1.0865	5475	5479		
(799)	SG	0.46	0.06					0.8839	0.1256	1.0866	5477	5486	1.00	
	SG+	0.46	0.06	0.04				0.8841	0.1257	1.0866	5479	5493	1.00	0.97
	JD	0.15			168.16	-0.03	0.02	0.8802	0.1243	1.0854	5479	5498	0.49	
2000	BS	0.50						0.9784	0.1666	1.1236	5727	5731		
(824)	SG	0.50	0.00					0.9785	0.1666	1.1238	5729	5738	1.00	
	SG+	0.50	0.00	0.02				0.9785	0.1666	1.1238	5731	5745	1.00	1.00
	JD	0.00			5.00	-0.06	0.24	0.9162	0.1550	1.0684	5649	5668	0.41	
2001	BS	0.59						0.1823	0.1963	0.2311	4085	4089		
(1021)	SG	0.59	0.04					0.1823	0.1963	0.2311	4086	4096	0.70	
	SG+	0.59	0.04	0.00				0.1823	0.1963	0.2311	4088	4103	0.93	1.00
	JD	0.06			8.30	-0.10	0.21	0.1711	0.1730	0.2188	3979	3999	0.45	
2002	BS	0.51						0.0785	0.1493	0.1016	1285	1290		
(666)	SG	0.48	0.19					0.0776	0.1483	0.1005	1273	1282	0.00	
	SG+	0.48	0.19	0.00				0.0776	0.1483	0.1005	1275	1288	0.00	1.00
	JD	0.20			16.76	-0.10	0.07	0.0656	0.1169	0.0888	1112	1130	0.40	
2003	BS	0.34						0.1396	0.2793	0.2071	1330	1334		
(449)	SG	0.33	0.05					0.1397	0.2797	0.2070	1332	1340	0.83	
	SG+	0.33	0.05	0.60				0.1414	0.2856	0.2070	1334	1346	0.85	0.59
	JD	0.15			5.01	-0.05	0.14	0.1369	0.2719	0.2062	1332	1349	0.48	
2004	BS	0.32						0.0918	0.1951	0.1102	1087	1091		
(564)	SG	0.32	0.00					0.0918	0.1951	0.1102	1089	1098	1.00	
	SG+	0.32	0.00	1.29				0.0916	0.1949	0.1102	1091	1104	1.00	0.84
	JD	0.15			7.76	-0.03	0.10	0.0911	0.1906	0.1095	1086	1104	0.48	
2005	BS	0.23						0.0532	0.1554	0.0616	57	61		
(314)	SG	0.23	0.03					0.0532	0.1549	0.0615	58	66	0.44	
	SG+	0.23	0.03	1.19				0.0526	0.1510	0.0613	57	69	0.19	0.10
	JD	0.11			47.34	0.01	0.03	0.0513	0.1457	0.0599	45	60	0.44	
2006	BS	0.24						0.0653	0.1296	0.0841	466	470		
(423)	SG	0.24	0.00					0.0653	0.1292	0.0841	468	476	0.80	
	SG+	0.24	0.00	7.72				0.0653	0.1291	0.0841	470	482	0.89	0.69
	JD	0.08			5.01	-0.03	0.11	0.0630	0.1232	0.0819	449	465	0.46	
2007	BS	0.30						0.2286	0.3562	0.2696	2421	2425		
(632)	SG	0.30	0.02					0.2286	0.3565	0.2697	2423	2432	1.00	
	SG+	0.30	0.02	1.93				0.2284	0.3517	0.2694	2424	2437	0.65	0.27
	JD	0.12			5.02	-0.03	0.13	0.2244	0.3355	0.2681	2420	2437	0.47	
2008	BS	0.31						0.0908	0.2155	0.1170	1490	1494		
(671)	SG	0.31	0.00					0.0908	0.2156	0.1170	1492	1501	1.00	
	SG+	0.31	0.00	0.02				0.0908	0.2156	0.1170	1494	1508	1.00	1.00
	JD	0.12			24.91	-0.04	0.05	0.0802	0.1915	0.1065	1370	1388	0.38	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 14: Estimation Results for CISCO for $T > 0.7332$

date		$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho} \times \mathbf{100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.50						1.0238	0.0573	1.3228	1051	1054		
(182)	SG	0.46	0.34					1.0292	0.0577	1.2998	1047	1053	0.01	
	SG+	0.46	0.34	0.00				1.0292	0.0577	1.2998	1049	1058	0.04	1.00
	JD	0.00			5.00	0.15	0.14	1.0127	0.0569	1.2892	1048	1060	0.48	
2000	BS	0.53						0.8281	0.0588	0.9894	1267	1271		
(233)	SG	0.53	0.00					0.8281	0.0588	0.9895	1269	1276	1.00	
	SG+	0.53	0.00	0.02				0.8281	0.0588	0.9895	1271	1282	1.00	1.00
	JD	0.31			45.94	0.01	0.06	0.8190	0.0579	0.9839	1271	1284	0.29	
2001	BS	0.58						0.3159	0.1198	0.3719	3416	3420		
(738)	SG	0.55	0.33					0.3061	0.1164	0.3636	3384	3394	0.00	
	SG+	0.55	0.33	0.00				0.3061	0.1164	0.3636	3387	3400	0.00	1.00
	JD	0.00			16.34	-0.15	0.00	0.3095	0.1131	0.3644	3391	3410	0.50	
2002	BS	0.50						0.1249	0.0979	0.1577	2327	2331		
(783)	SG	0.49	0.22					0.1249	0.1038	0.1563	2315	2324	0.00	
	SG+	0.49	0.22	0.00				0.1249	0.1038	0.1563	2317	2331	0.00	1.00
	JD	0.00			6.80	-0.20	0.05	0.1069	0.0945	0.1322	2057	2075	0.37	
2003	BS	0.35						0.3266	0.3239	0.4204	3386	3391		
(702)	SG	0.34	0.21					0.3264	0.3297	0.4192	3384	3393	0.05	
	SG+	0.34	0.21	0.23				0.3283	0.3351	0.4191	3386	3399	0.12	0.58
	JD	0.08			34.47	0.05	0.02	0.3286	0.3441	0.4196	3390	3408	0.51	
2004	BS	0.31						0.1503	0.1370	0.1802	2363	2368		
(742)	SG	0.30	0.17					0.1454	0.1303	0.1779	2346	2356	0.00	
	SG+	0.30	0.17	0.00				0.1454	0.1303	0.1779	2348	2362	0.00	1.00
	JD	0.11			25.45	-0.02	0.05	0.1514	0.1449	0.1783	2353	2371	0.50	
2005	BS	0.24						0.0728	0.0840	0.0909	547	552		
(430)	SG	0.24	0.00					0.0728	0.0840	0.0910	551	559	1.00	
	SG+	0.24	0.00	4.91				0.0726	0.0834	0.0910	552	565	1.00	0.85
	JD	0.18			43.43	-0.01	0.02	0.0727	0.0869	0.0903	548	564	0.44	
2006	BS	0.24						0.1152	0.0705	0.1404	646	650		
(339)	SG	0.24	0.00					0.1154	0.0706	0.1406	649	657	1.00	
	SG+	0.24	0.00	0.01				0.1154	0.0706	0.1406	651	663	1.00	1.00
	JD	0.08			13.79	-0.04	0.05	0.1046	0.0578	0.1298	599	614	0.33	
2007	BS	0.32						0.5043	0.1851	0.5557	1511	1515		
(327)	SG	0.32	0.00					0.5043	0.1851	0.5558	1513	1521	1.00	
	SG+	0.32	0.00	0.04				0.5043	0.1851	0.5558	1515	1527	1.00	1.00
	JD	0.20			42.70	0.01	0.04	0.5019	0.1846	0.5543	1515	1531	0.42	
2008	BS	0.32						0.1904	0.1290	0.2396	1149	1153		
(374)	SG	0.32	0.00					0.1904	0.1290	0.2396	1151	1159	1.00	
	SG+	0.32	0.00	0.06				0.1904	0.1290	0.2396	1153	1165	1.00	1.00
	JD	0.00			5.00	-0.06	0.14	0.1859	0.1401	0.2271	1115	1131	0.43	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 15: Estimation Results for INTC for $T < 0.7240$

date		σ	a	$\rho \times 100$	λ	k	δ	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.42						0.3189	0.0574	0.4224	2715	2720		
(584)	SG	0.36	0.25					0.3194	0.0593	0.3986	2650	2658	0.00	
	SG+	0.36	0.25	0.00				0.3194	0.0593	0.3986	2652	2665	0.00	1.00
	JD	0.33			155.05	-0.01	0.01	0.3190	0.0575	0.4221	2721	2738	0.57	
2000	BS	0.46						0.7501	0.1526	0.8630	2388	2392		
(416)	SG	0.39	0.30					0.7496	0.1557	0.8543	2382	2390	0.00	
	SG+	0.39	0.30	0.07				0.7541	0.1560	0.8541	2384	2396	0.01	0.67
	JD	0.15			5.00	-0.03	0.21	0.7379	0.1446	0.8572	2389	2405	0.51	
2001	BS	0.50						0.2358	0.2512	0.3044	3362	3367		
(784)	SG	0.50	0.00					0.2358	0.2512	0.3044	3364	3373	1.00	
	SG+	0.50	0.00	0.00				0.2358	0.2512	0.3044	3366	3380	1.00	1.00
	JD	0.10			5.23	-0.11	0.22	0.2167	0.2218	0.2766	3218	3237	0.41	
2002	BS	0.40						0.2532	0.3985	0.3250	2835	2839		
(666)	SG	0.30	0.34					0.2457	0.3680	0.3139	2790	2799	0.00	
	SG+	0.30	0.34	0.07				0.2467	0.3696	0.3138	2792	2806	0.00	0.63
	JD	0.04			33.09	-0.04	0.06	0.2462	0.3709	0.3221	2829	2847	0.55	
2003	BS	0.34						0.1886	0.2696	0.2482	1506	1510		
(452)	SG	0.34	0.00					0.1886	0.2696	0.2482	1508	1516	1.00	
	SG+	0.34	0.00	3.62				0.1886	0.2698	0.2482	1510	1522	1.00	0.96
	JD	0.07			5.00	-0.06	0.16	0.1800	0.2598	0.2413	1486	1503	0.43	
2004	BS	0.29						0.0782	0.1393	0.1005	599	603		
(416)	SG	0.29	0.00					0.0782	0.1393	0.1005	602	610	1.00	
	SG+	0.29	0.00	0.01				0.0782	0.1393	0.1005	604	616	1.00	1.00
	JD	0.12			5.01	-0.05	0.12	0.0741	0.1267	0.0944	553	569	0.44	
2005	BS	0.22						0.0606	0.1308	0.0871	326	330		
(341)	SG	0.22	0.00					0.0606	0.1306	0.0871	328	336	1.00	
	SG+	0.22	0.00	11.51				0.0606	0.1303	0.0871	330	341	0.95	0.67
	JD	0.10			5.01	-0.02	0.09	0.0592	0.1284	0.0858	322	338	0.45	
2006	BS	0.24						0.0691	0.1601	0.0880	289	293		
(318)	SG	0.21	0.14					0.0674	0.1590	0.0844	264	271	0.00	
	SG+	0.21	0.14	0.35				0.0670	0.1510	0.0838	261	273	0.00	0.04
	JD	0.00			14.18	0.06	0.00	0.0650	0.1479	0.0837	262	278	0.49	
2007	BS	0.25						0.1042	0.2442	0.1363	1273	1278		
(548)	SG	0.17	0.24					0.0827	0.2374	0.1044	983	992	0.00	
	SG+	0.17	0.24	0.37				0.0795	0.1838	0.1020	960	973	0.00	0.00
	JD	0.00			13.28	0.07	0.00	0.0980	0.2276	0.1319	1244	1261	0.61	
2008	BS	0.32						0.2117	0.4038	0.2993	2483	2487		
(618)	SG	0.24	0.26					0.2060	0.3789	0.2910	2450	2459	0.00	
	SG+	0.24	0.26	0.44				0.2103	0.3700	0.2901	2448	2461	0.00	0.04
	JD	0.00			18.42	0.07	0.00	0.2097	0.4149	0.2978	2482	2500	0.55	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 16: Estimation Results for INTC for $T > 0.7240$

date		σ	a	$\rho \times 100$	λ	k	δ	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.41						0.4320	0.0377	0.5704	254	257		
(78)	SG	0.39	0.20					0.4178	0.0341	0.5580	253	258	0.07	
	SG+	0.39	0.20	0.00				0.4178	0.0341	0.5580	255	262	0.19	1.00
	JD	0.01			5.02	0.15	0.08	0.3970	0.0328	0.5302	249	258	0.43	
2000	BS	0.52						0.3823	0.0362	0.5495	295	297		
(89)	SG	0.51	0.20					0.3780	0.0355	0.5479	296	301	0.47	
	SG+	0.51	0.20	0.00				0.3780	0.0355	0.5479	298	306	0.77	1.00
	JD	0.00			5.00	0.13	0.17	0.3685	0.0338	0.5169	290	300	0.42	
2001	BS	0.47						0.4383	0.1125	0.4986	2102	2106		
(446)	SG	0.44	0.36					0.3998	0.1072	0.4814	2073	2081	0.00	
	SG+	0.44	0.36	0.00				0.3998	0.1072	0.4814	2075	2087	0.00	1.00
	JD	0.00			12.43	-0.14	0.00	0.4242	0.0982	0.4893	2091	2108	0.52	
2002	BS	0.43						0.4683	0.2384	0.5492	3395	3400		
(644)	SG	0.43	0.00					0.4683	0.2384	0.5493	3397	3406	1.00	
	SG+	0.43	0.00	0.00				0.4683	0.2384	0.5493	3399	3413	1.00	1.00
	JD	0.00			5.00	-0.06	0.19	0.4625	0.2319	0.5399	3379	3397	0.44	
2003	BS	0.35						0.4392	0.3472	0.5447	4775	4779		
(861)	SG	0.33	0.21					0.4379	0.3488	0.5433	4772	4782	0.04	
	SG+	0.33	0.21	0.30				0.4402	0.3498	0.5432	4774	4788	0.08	0.44
	JD	0.20			44.35	0.01	0.04	0.4378	0.3496	0.5432	4776	4795	0.50	
2004	BS	0.29						0.1559	0.1189	0.1977	2280	2284		
(691)	SG	0.29	0.10					0.1553	0.1191	0.1972	2278	2287	0.06	
	SG+	0.29	0.10	0.00				0.1553	0.1191	0.1972	2280	2294	0.17	1.00
	JD	0.00			20.98	-0.07	0.00	0.1529	0.1230	0.1932	2254	2272	0.46	
2005	BS	0.22						0.1271	0.0966	0.1670	826	830		
(358)	SG	0.22	0.08					0.1272	0.0956	0.1668	827	835	0.33	
	SG+	0.22	0.08	0.22				0.1266	0.0946	0.1667	829	840	0.56	0.65
	JD	0.13			45.65	0.01	0.02	0.1258	0.0933	0.1655	825	841	0.44	
2006	BS	0.24						0.1241	0.0814	0.1531	478	481		
(262)	SG	0.24	0.00					0.1242	0.0814	0.1531	480	487	1.00	
	SG+	0.24	0.00	2.79				0.1242	0.0814	0.1531	482	492	1.00	0.94
	JD	0.15			51.42	0.01	0.02	0.1234	0.0804	0.1523	481	495	0.41	
2007	BS	0.25						0.2482	0.1228	0.3061	757	761		
(242)	SG	0.22	0.22					0.2507	0.1426	0.2998	749	756	0.00	
	SG+	0.22	0.22	0.06				0.2520	0.1439	0.2998	751	762	0.01	0.87
	JD	0.15			56.11	0.01	0.02	0.2478	0.1251	0.3031	759	773	0.53	
2008	BS	0.33						0.2670	0.2402	0.3660	1373	1377		
(355)	SG	0.31	0.19					0.2703	0.2487	0.3644	1372	1380	0.08	
	SG+	0.31	0.19	0.58				0.2708	0.2510	0.3638	1373	1384	0.12	0.27
	JD	0.12			17.75	0.04	0.06	0.2610	0.2502	0.3568	1361	1376	0.39	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 17: Estimation Results for AAPL for $T < 0.7315$

date		$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho} \times \mathbf{100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.62						1.3719	0.1972	1.9867	1561	1564		
(229)	SG	0.38	0.58					1.3072	0.1823	1.8974	1542	1549	0.00	
	SG+	0.38	0.58	0.06				1.3105	0.1827	1.8973	1544	1554	0.00	0.88
	JD	0.09			15.79	0.14	0.05	1.3581	0.1948	1.9833	2067	2081	0.76	
2000	BS	0.68						0.9416	0.2054	1.4249	1785	1789		
(281)	SG	0.40	0.65					0.8037	0.2431	1.0978	1641	1648	0.00	
	SG+	0.40	0.65	0.11				0.7949	0.2376	1.0968	1642	1653	0.00	0.49
	JD	0.00			12.77	0.18	0.00	0.9036	0.1779	1.4034	2592	2606	0.66	
2001	BS	0.72						0.6720	0.5333	0.7757	3078	3082		
(533)	SG	0.59	0.51					0.6718	0.5324	0.7564	3053	3061	0.00	
	SG+	0.59	0.51	0.50				0.6667	0.5437	0.7544	3052	3065	0.00	0.10
	JD	0.00			5.00	0.20	0.18	0.6393	0.5559	0.7516	3038	3055	0.47	
2002	BS	0.57						0.7157	0.6593	0.9025	3631	3635		
(588)	SG	0.43	0.47					0.7095	0.6384	0.8834	3608	3617	0.00	
	SG+	0.43	0.47	0.60				0.7130	0.6584	0.8814	3607	3620	0.00	0.10
	JD	0.00			5.00	0.16	0.16	0.6932	0.6827	0.8867	3611	3628	0.50	
2003	BS	0.53						0.6853	0.7619	0.8438	2484	2488		
(433)	SG	0.41	0.43					0.6776	0.7412	0.8288	2470	2478	0.00	
	SG+	0.41	0.43	0.66				0.6717	0.7573	0.8272	2470	2483	0.00	0.19
	JD	0.00			5.00	0.13	0.16	0.6632	0.7735	0.8298	2472	2488	0.50	
2004	BS	0.53						1.2514	0.6871	1.5341	3424	3429		
(486)	SG	0.24	0.58					1.2319	0.6337	1.4732	3387	3395	0.00	
	SG+	0.24	0.58	0.11				1.2326	0.6389	1.4731	3389	3402	0.00	0.76
	JD	0.00			40.43	-0.09	0.00	1.2483	0.6699	1.5324	4160	4177	0.68	
2005	BS	0.53						1.7246	0.5926	2.1646	5505	5510		
(682)	SG	0.05	0.65					1.5574	0.4369	1.9846	5389	5398	0.00	
	SG+	0.05	0.65	0.00				1.5575	0.4370	1.9846	5391	5405	0.00	1.00
	JD	0.00			5.00	-0.21	0.19	1.6497	0.4910	2.1067	5474	5493	0.55	
2006	BS	0.47						2.1146	0.3554	2.7260	3210	3214		
(401)	SG	0.00	0.56					1.8927	0.2916	2.4083	3112	3120	0.00	
	SG+	0.00	0.56	0.00				1.8927	0.2916	2.4083	3114	3126	0.00	1.00
	JD	0.04			5.00	-0.22	0.06	2.0981	0.3428	2.6961	3207	3223	0.61	
2007	BS	0.47						3.5508	0.3772	4.9220	3299	3303		
(363)	SG	0.19	0.52					3.2560	0.3457	4.5554	3245	3252	0.00	
	SG+	0.19	0.52	0.05				3.2632	0.3474	4.5553	3246	3258	0.00	0.88
	JD	0.00			5.00	0.19	0.02	3.5012	0.3779	4.8864	3292	3308	0.57	
2008	BS	0.46						3.4193	0.2858	4.6717	4271	4275		
(463)	SG	0.27	0.44					3.1267	0.2499	4.3154	4200	4208	0.00	
	SG+	0.27	0.44	0.23				3.1507	0.2536	4.3125	4201	4214	0.00	0.43
	JD	0.09			10.66	0.10	0.08	3.4357	0.3071	4.6296	4424	4441	0.64	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 18: Estimation Results for AAPL for $T > 0.7315$

date		σ	a	$\rho \times 100$	λ	k	δ	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.34						2.8112	0.3261	3.6757	2125	2128		
(260)	SG	0.00	0.70					2.4823	0.2774	3.2324	2060	2067	0.00	
	SG+	0.00	0.70	0.07				2.4867	0.2781	3.2323	2062	2073	0.00	0.90
	JD	0.22			45.06	0.01	0.04	2.8100	0.3262	3.6738	2130	2145	0.62	
2000	BS	0.40						2.2845	0.4153	3.1000	3564	3568		
(428)	SG	0.20	0.70					2.0055	0.3909	2.7594	3466	3474	0.00	
	SG+	0.20	0.70	0.24				1.9982	0.3813	2.7581	3468	3480	0.00	0.53
	JD	0.25			44.68	0.01	0.05	2.2816	0.4139	3.0985	3569	3586	0.60	
2001	BS	0.48						1.3466	0.8140	1.6785	6039	6044		
(784)	SG	0.23	0.95					1.2097	0.6938	1.5007	5865	5875	0.00	
	SG+	0.23	0.95	0.29				1.2087	0.7084	1.4996	5866	5880	0.00	0.28
	JD	0.31			44.72	0.00	0.06	1.3480	0.8213	1.6779	6044	6063	0.62	
2002	BS	0.33						1.0174	0.7824	1.3164	4900	4904		
(691)	SG	0.18	0.63					0.9845	0.7407	1.2266	4804	4813	0.00	
	SG+	0.18	0.63	0.83				0.9853	0.7631	1.2220	4801	4815	0.00	0.02
	JD	0.17			44.13	0.02	0.04	1.0136	0.7837	1.3122	4901	4920	0.59	
2003	BS	0.27						0.7795	0.8398	1.0017	2414	2418		
(402)	SG	0.14	0.51					0.7602	0.7903	0.9315	2358	2366	0.00	
	SG+	0.14	0.51	0.84				0.7570	0.8239	0.9280	2357	2369	0.00	0.08
	JD	0.19			45.00	0.00	0.03	0.7809	0.8489	1.0019	2420	2436	0.59	
2004	BS	0.26						1.3325	0.5876	1.7858	3497	3501		
(477)	SG	0.03	0.56					1.2536	0.5638	1.5659	3374	3382	0.00	
	SG+	0.03	0.56	0.24				1.2546	0.5687	1.5653	3375	3388	0.00	0.53
	JD	0.17			44.48	0.01	0.03	1.3312	0.5904	1.7835	3502	3519	0.60	
2005	BS	0.27						1.8621	0.6019	2.4999	5993	5997		
(713)	SG	0.00	0.57					1.5900	0.4697	2.0336	5700	5709	0.00	
	SG+	0.00	0.57	0.14				1.5887	0.4698	2.0332	5702	5716	0.00	0.60
	JD	0.16			44.15	0.02	0.03	1.8621	0.6062	2.4973	5997	6015	0.64	
2006	BS	0.23						2.5123	0.4102	3.3721	3230	3234		
(385)	SG	0.00	0.48					2.0118	0.3208	2.6475	3046	3054	0.00	
	SG+	0.00	0.48	0.05				2.0137	0.3216	2.6474	3048	3060	0.00	0.90
	JD	0.18			44.83	0.01	0.02	2.5120	0.4106	3.3714	3236	3252	0.68	
2007	BS	0.25						5.3336	0.4752	7.3679	3385	3389		
(344)	SG	0.00	0.50					4.3159	0.4023	5.9886	3245	3252	0.00	
	SG+	0.00	0.50	0.02				4.3198	0.4031	5.9886	3247	3258	0.00	0.96
	JD	0.17			44.80	0.00	0.03	5.3388	0.4772	7.3662	3391	3406	0.68	
2008	BS	0.24						5.3444	0.4244	7.5748	4295	4299		
(425)	SG	0.07	0.48					4.5257	0.3439	6.6631	4188	4196	0.00	
	SG+	0.07	0.48	0.41				4.5755	0.3459	6.6567	4189	4202	0.00	0.37
	JD	0.14			44.64	0.01	0.03	5.3274	0.4232	7.5577	4299	4316	0.63	

*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

Table 19: Estimation Results for MSFT for $T < 0.9342$

date		$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho} \times \mathbf{100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.67						0.2021	0.1293	0.2541	2066	2071		
(572)	SG	0.60	0.37					0.1953	0.1255	0.2470	2036	2045	0.00	
	SG+	0.60	0.37	0.11				0.1963	0.1265	0.2469	2037	2050	0.00	0.43
	JD	0.32			142	0.03	0.04	0.2024	0.1306	0.2532	2068	2086	0.55	
2000	BS	0.81						0.3970	0.1081	0.4862	3462	3467		
(681)	SG	0.70	0.49					0.3490	0.1105	0.4294	3295	3304	0.00	
	SG+	0.70	0.49	0.04				0.3472	0.1091	0.4293	3297	3310	0.00	0.56
	JD	0.00			48	0.11	0.00	0.3734	0.0816	0.4771	3443	3461	0.57	
2001	BS	0.71						0.1478	0.1357	0.1870	2099	2103		
(666)	SG	0.69	0.22					0.1467	0.1406	0.1853	2089	2098	0.00	
	SG+	0.69	0.22	0.00				0.1467	0.1406	0.1853	2091	2104	0.00	1.00
	JD	0.26			40	-0.07	0.09	0.1228	0.1014	0.1582	1882	1900	0.33	
2002	BS	0.64						0.1384	0.2767	0.1813	1356	1361		
(488)	SG	0.52	0.49					0.1284	0.2471	0.1665	1275	1283	0.00	
	SG+	0.52	0.49	0.00				0.1285	0.2472	0.1665	1277	1290	0.00	1.00
	JD	0.00			43	-0.10	0.00	0.1304	0.2598	0.1790	1350	1367	0.58	
2003	BS	0.58						0.1411	0.2539	0.1837	905	909		
(361)	SG	0.58	0.00					0.1411	0.2539	0.1837	907	914	1.00	
	SG+	0.58	0.00	63.71				0.1400	0.2453	0.1836	908	920	0.79	0.49
	JD	0.00			5	-0.06	0.28	0.1144	0.1792	0.1633	826	841	0.24	
2004	BS	0.48						0.0966	0.1467	0.1152	1284	1289		
(612)	SG	0.48	0.00					0.0966	0.1467	0.1152	1286	1295	1.00	
	SG+	0.48	0.00	6.57				0.0959	0.1458	0.1152	1287	1301	0.69	0.38
	JD	0.25			5	-0.03	0.19	0.0956	0.1437	0.1138	1274	1292	0.46	
2005	BS	0.43						0.0873	0.1332	0.1214	1290	1295		
(594)	SG	0.43	0.00					0.0873	0.1332	0.1214	1292	1301	0.75	
	SG+	0.43	0.00	43.16				0.0853	0.1275	0.1211	1292	1305	0.31	0.13
	JD	0.00			5	-0.06	0.20	0.0807	0.1144	0.1110	1190	1208	0.41	
2006	BS	0.44						0.1753	0.1827	0.2203	3183	3188		
(854)	SG	0.36	0.30					0.1611	0.1809	0.2005	3024	3033	0.00	
	SG+	0.36	0.30	0.14				0.1605	0.1767	0.2001	3022	3037	0.00	0.07
	JD	0.09			142	0.03	0.02	0.1736	0.1760	0.2181	3172	3191	0.58	
2007	BS	0.44						0.0996	0.2642	0.1400	2182	2186		
(794)	SG	0.36	0.30					0.0997	0.2634	0.1307	2074	2084	0.00	
	SG+	0.36	0.30	0.30				0.1003	0.2443	0.1301	2069	2083	0.00	0.01
	JD	0.00			83	0.05	0.00	0.0986	0.2645	0.1393	2180	2199	0.57	
2008	BS	0.67						0.0795	0.2677	0.1031	987	991		
(555)	SG	0.67	0.02					0.0795	0.2676	0.1031	989	997	1.00	
	SG+	0.67	0.02	0.40				0.0795	0.2679	0.1031	991	1004	1.00	0.95
	JD	0.28			59	-0.04	0.07	0.0789	0.2675	0.1020	981	998	0.46	

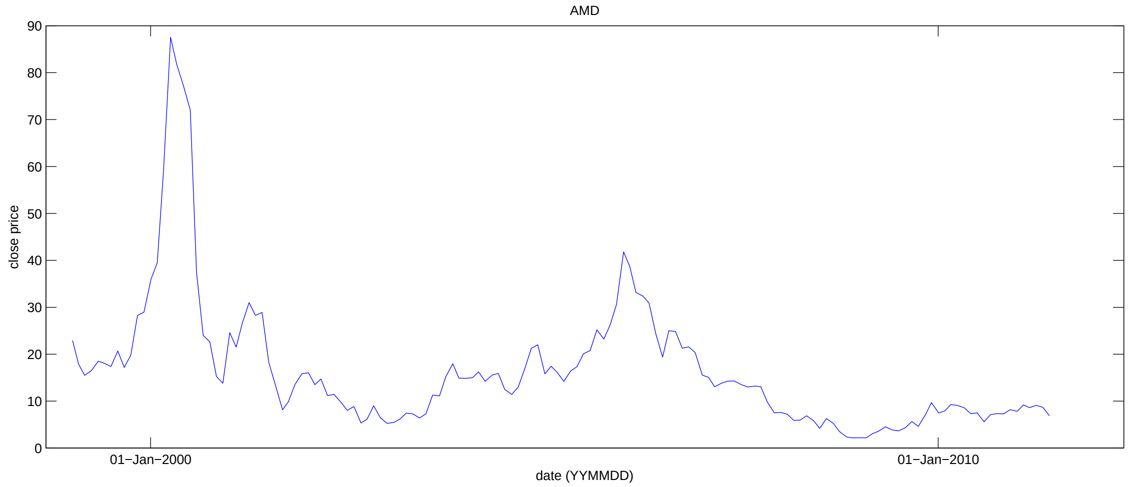
*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

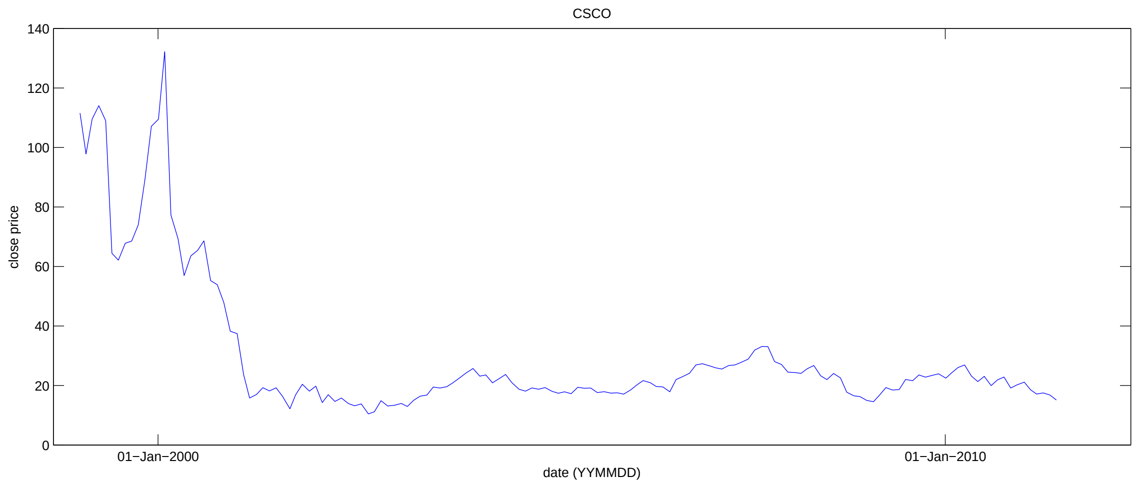
Table 20: Estimation Results for MSFT for $T > 0.9342$

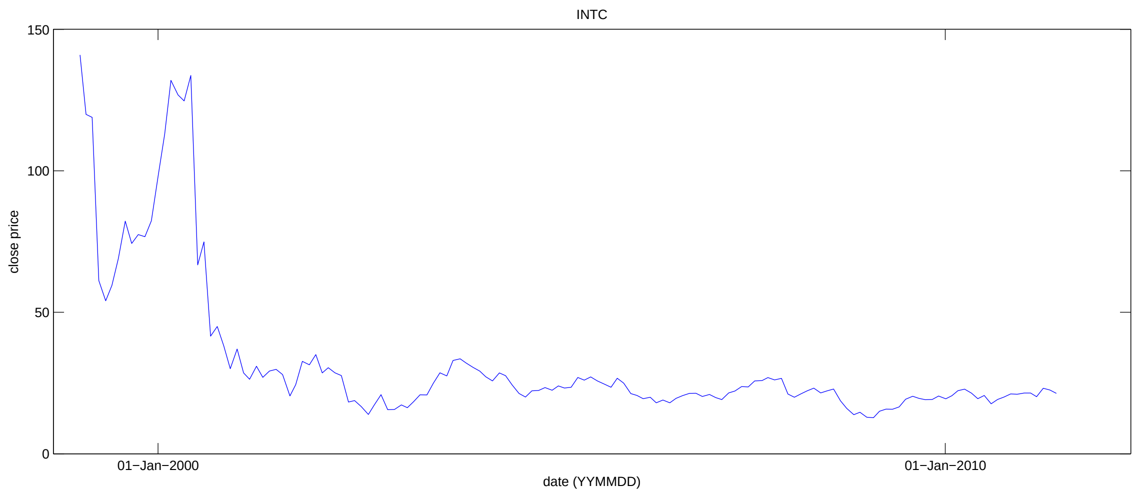
date		$\boldsymbol{\sigma}$	$\boldsymbol{a}$	$\boldsymbol{\rho \times 100}$	$\boldsymbol{\lambda}$	$\boldsymbol{k}$	$\boldsymbol{\delta}$	MAE	RE	$\sqrt{\text{MSE}}$	AIC	BIC	p-val	p-val'
1999	BS	0.64						0.2992	0.0755	0.3781	2095	2099		
(492)	SG	0.62	0.32					0.2985	0.0748	0.3739	2086	2094	0.00	
	SG+	0.62	0.32	0.03				0.2986	0.0748	0.3739	2088	2100	0.00	0.90
	JD	0.28			43	0.04	0.07	0.2884	0.0717	0.3692	2077	2094	0.46	
2000	BS	0.75						0.3166	0.0440	0.4975	1069	1073		
(257)	SG	0.75	0.00					0.3164	0.0440	0.4975	1071	1078	1.00	
	SG+	0.75	0.00	0.00				0.3164	0.0440	0.4975	1073	1084	1.00	1.00
	JD	0.00			5	-0.05	0.34	0.3087	0.0420	0.4677	1043	1058	0.38	
2001	BS	0.67						0.2639	0.0904	0.3212	2663	2667		
(636)	SG	0.57	0.73					0.2041	0.0778	0.2701	2445	2454	0.00	
	SG+	0.57	0.73	0.00				0.2041	0.0779	0.2701	2447	2460	0.00	1.00
	JD	0.43			47	-0.04	0.06	0.2613	0.0850	0.3195	2662	2680	0.60	
2002	BS	0.64						0.3306	0.2664	0.4113	3734	3739		
(767)	SG	0.64	0.00					0.3306	0.2664	0.4113	3736	3745	0.98	
	SG+	0.64	0.00	0.01				0.3306	0.2664	0.4113	3738	3752	1.00	1.00
	JD	0.00			5	-0.13	0.29	0.3272	0.2576	0.4061	3720	3739	0.45	
2003	BS	0.56						0.3014	0.2720	0.3748	2646	2650		
(594)	SG	0.56	0.01					0.3014	0.2720	0.3747	2648	2657	0.93	
	SG+	0.56	0.01	24.77				0.2997	0.2655	0.3743	2649	2662	0.46	0.22
	JD	0.00			5	0.13	0.19	0.2848	0.2468	0.3587	2600	2618	0.38	
2004	BS	0.47						0.2517	0.1347	0.3130	2245	2249		
(560)	SG	0.45	0.33					0.2410	0.1303	0.3083	2230	2239	0.00	
	SG+	0.45	0.33	0.05				0.2410	0.1303	0.3083	2232	2245	0.00	0.86
	JD	0.22			44	0.03	0.05	0.2487	0.1339	0.3111	2244	2261	0.52	
2005	BS	0.41						0.2592	0.1161	0.3206	2429	2434		
(591)	SG	0.37	0.38					0.2456	0.1088	0.3065	2378	2387	0.00	
	SG+	0.37	0.38	0.05				0.2445	0.1082	0.3065	2380	2393	0.00	0.79
	JD	0.25			44	0.02	0.05	0.2558	0.1116	0.3187	2428	2446	0.56	
2006	BS	0.44						0.2033	0.0532	0.2554	1584	1588		
(464)	SG	0.43	0.06					0.2034	0.0533	0.2555	1587	1595	1.00	
	SG+	0.43	0.06	0.02				0.2033	0.0533	0.2555	1589	1601	1.00	1.00
	JD	0.28			46	-0.01	0.05	0.2032	0.0531	0.2555	1591	1607	0.50	
2007	BS	0.45						0.2428	0.1327	0.3122	2357	2362		
(583)	SG	0.45	0.00					0.2429	0.1328	0.3122	2359	2368	1.00	
	SG+	0.45	0.00	0.06				0.2429	0.1328	0.3122	2361	2375	1.00	1.00
	JD	0.00			5	-0.06	0.21	0.2395	0.1284	0.3088	2351	2368	0.46	
2008	BS	0.68						0.1556	0.2679	0.1993	1859	1863		
(555)	SG	0.52	1.00					0.1395	0.2546	0.1852	1774	1783	0.00	
	SG+	0.52	1.00	0.00				0.1395	0.2546	0.1852	1776	1790	0.00	1.00
	JD	0.00			5	-0.26	0.26	0.1444	0.2718	0.1780	1732	1749	0.47	

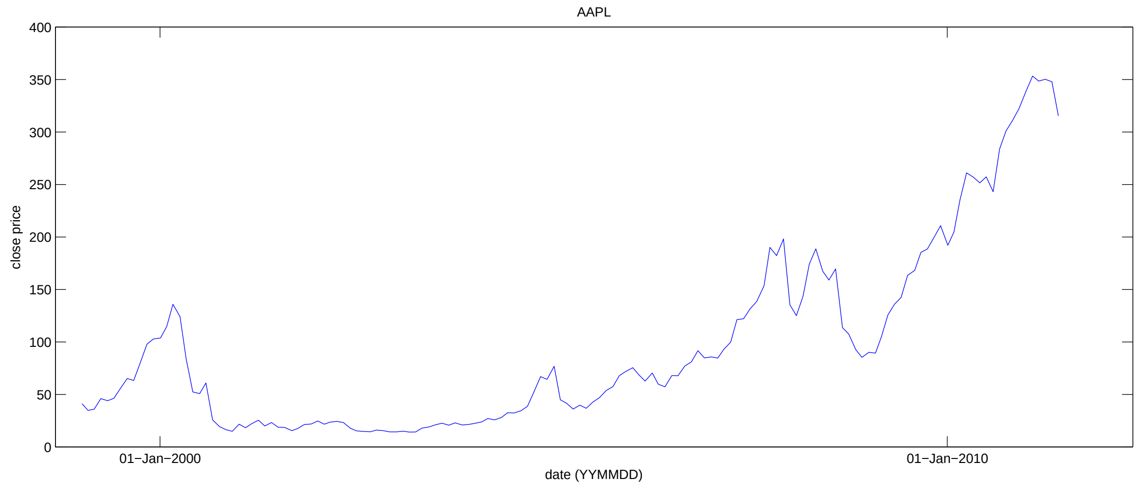
*The entries 0.00 are not identically zero, but within $(2 \times 10^{-3} 7 \times 10^{-3})$

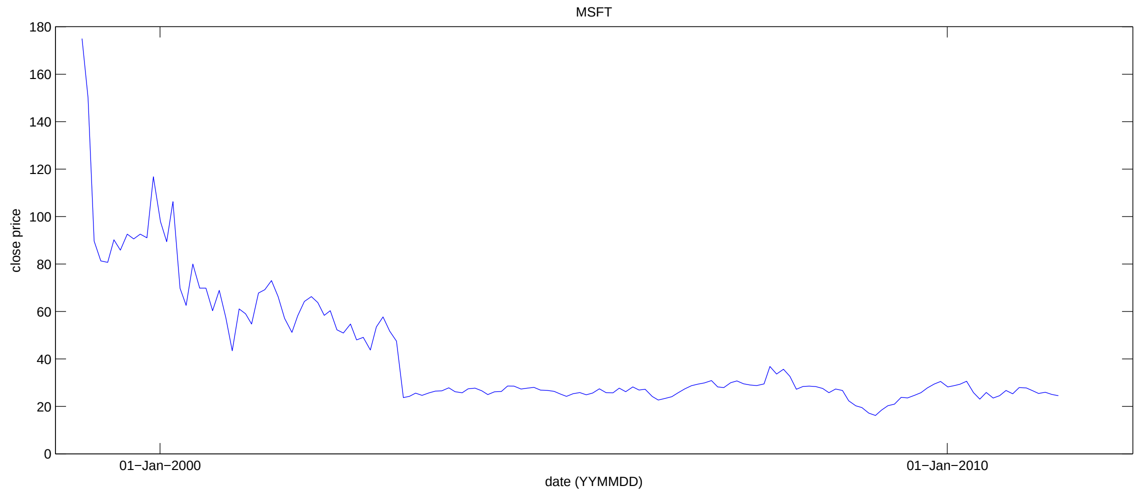
5.3 Historical Stock Prices For Five Companies











5.4 Solving equation for stock price movement

Below we proove that $S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right) \times \xi_t$ indeed solves the equaiton $dS_t = \mu S_t dt + \sigma S_t dW_t + S_{t-} d\xi_t^1$. For this consider S_t as a function $S(t, W_t, \xi_t)$ and obtain its differential using Ito formula for processes with jumps.

$$dS = S_t^{'} dt + S_W^{'} dW_t + \frac{1}{2} S_{WW}^{''} dt + \sum_{i=0}^{i=t} S(\xi_i) - S(\xi_{i-})$$

$$S_t^{'} = S\left(\mu - \frac{\sigma^2}{2}\right)$$

$$S_W^{'} = S\sigma$$

$$S_{WW}^{''} = S\sigma^2$$

And according to our definition of ξ_t, the difference $S(\xi_t) - S(\xi_{t-})$ is zero for any $t \neq t_1$. It is equal to $S(t-) \times d\xi_t$ for $t = t_1$ Thus, we obtain that

$$dS_t = S_t\left(\mu - \frac{\sigma^2}{2}\right)dt + S_t\sigma dW_t + \frac{1}{2}S_t\sigma^2 dt + S(t-) \times d\xi_t =$$

$$= \mu S_t dt + \sigma S_t dW_t + \frac{1}{2}S_t\sigma^2 dt - \frac{1}{2}S_t\sigma^2 dt + S(t-) \times d\xi_t = \mu S_t dt + \sigma S_t dW_t + S(t-) \times d\xi_t$$

This verifies the solution

5.5 Derivation of $\mathbf{Z}_1$

Z_1 is such Radon-Nikodym derivative, which switches from uniform distribution with one amplitude to uniform distribution with another amplitude, preserving the same mean equal to 1. To find Z_1, define $\Omega = [0; 2]$. Then for $0 \leq x \leq y \leq 2$

$$P[x,y] = \begin{cases} \int_x^y \frac{1}{2b}d\omega = \frac{1}{2b}(y-x), & \text{if } 1-b \leq x \leq 1+b \text{ and } 1-b \leq y \leq 1+b \\ \int_{1-b}^y \frac{1}{2b}d\omega = \frac{1}{2b}(y+b-1), & \text{if } x < 1-b \text{ and } 1-b \leq y \leq 1+b \\ \int_x^{1+b} \frac{1}{2b}d\omega = \frac{1}{2b}(1+b-x), & \text{if } 1-b \leq x \leq 1+b \text{ and } y > 1+b \\ \int_{1-b}^{1+b} \frac{1}{2b}d\omega = 1, & \text{if } x < 1-b \text{ and } y > 1+b \\ 0, & \text{otherwise} \end{cases} \tag{27}$$

$$Q[x,y] = \begin{cases} \int_x^y \frac{1}{2a}d\omega = \frac{1}{2a}(y-x), & \text{if } 1-a \leq x \leq 1+a \text{ and } 1-a \leq y \leq 1+a \\ \int_{1-a}^y \frac{1}{2a}d\omega = \frac{1}{2a}(y+a-1), & \text{if } x < 1-a \text{ and } 1-a \leq y \leq 1+a \\ \int_x^{1+a} \frac{1}{2a}d\omega = \frac{1}{2a}(1+a-x), & \text{if } 1-a \leq x \leq 1+a \text{ and } y > 1+a \\ \int_{1-a}^{1+a} \frac{1}{2a}d\omega = 1, & \text{if } x < 1-a \text{ and } y > 1+a \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

Now consider 2 cases: the first one when $a < b$ (the case $a > b$ is symmetric) and the second one $a = b$. In the first case, when $1 - b \leq x \leq y \leq 1 + b$ we see that $dP = \frac{1}{2b}d\omega$. We can substitute this into the system above, writing $2b \times dP$ instead of $d\omega$. Make that substitution and examine such events that $1 - b \leq x \leq y \leq 1 - a$. On these events $Q[x,y] = \int_x^y 0 \times dw = \int_x^y 0 \times 2b \times dP$. This implies that such random variable Z, which allows $Q[\omega] = \int_\omega Z \times dP$ is $Z = 0$. However, this contradicts the definition of Radon-Nikodym derivative, which is required to be positive almost surely. It means that in case 1, Radon-Nikodym derivative does not exist and equivalent measure cannot be constructed. Intuitively it happens because when $a \neq b$, there exist events which have zero probability in P, but non-zero probability in Q and vice versa. Nevertheless in case 2 when $a = b$, we have $dP = \frac{1}{2b} d\omega = \frac{1}{2a} d\omega$ and the following equality holds: $Q[\omega] = \int_\omega \frac{1}{2a} dw = \int_\omega \frac{2a}{2a} dP = \int_\omega 1 \times dP$. It means that Radon-Nikodym derivative is defined and equal to 1. The logic above shows that given our assumptions about jump distribution under Q, the proper value of a in our model specification is $a = b$, which implies $Z_1 = 1$.

5.6 Proof that $Q(A) = \int_A Z_0 Z_1 dP(A)$ is equivalent martingale measure

In case of no dividends, an equivalent martingale measure by definition is such measure under which discounted stock price is a martingale. Therefore we need to verify that $E^Q\left(e^{-rt} S_t \mid \mathcal{F}_s\right) = e^{-rs} S_s$ for $s \leq t$. Recalling equation (12), a similar property exists for conditional expectation. For this we define Radon-Nikodym derivative process $Z(t) = E\left(Z \mid \mathcal{F}_t\right)$, where $Z = \frac{dQ}{dP}$. The property we need is the following: if X is an $\mathcal{F}_t$ - measurable random variable and $0 \leq s \leq t \leq T$, then

$$E^Q\left(X \mid \mathcal{F}_s\right) = \frac{1}{Z(s)} E\left(X Z(t) \mid \mathcal{F}_s\right) \tag{29}$$

For Radon-Nikodym derivatives Z_0 and Z_1 assumed in this paper, the Radon-Nikodym Derivative processes will be

$$Z_0(t) = e^{-\frac{\mu - r}{\sigma} W_t - \frac{1}{2}\left(\frac{\mu - r}{\sigma}\right)^2 t} \tag{30}$$

$$Z_1(t) = 1 \tag{31}$$

Now, we proceed to verifying that $E^Q\left(e^{-rt} S_t \mid \mathcal{F}_s\right) = e^{-rs} S_s$ for $s \leq t$

$$\begin{aligned}
&E^Q\left(e^{-rt} S_t \mid \mathcal{F}_s\right) = \\
&= \frac{1}{Z_0(s) Z_1(s)} E\left(e^{-rt} Z_0(t) Z_1(t) S_t \mid \mathcal{F}_s\right) \\
&= \frac{1}{Z_0(s) Z_1(s)} E\left(e^{-rt} e^{-\frac{\mu - r}{\sigma} W_t - \frac{1}{2}\left(\frac{\mu - r}{\sigma}\right)^2 t} S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t} \xi_t \mid \mathcal{F}_s\right) \\
&= \frac{1}{Z_0(s) Z_1(s)} S_0 e^{-\frac{1}{2}\left(\frac{\mu - r}{\sigma}\right)^2 t + \left(\mu - \frac{\sigma^2}{2}\right) t - rt} E\left(e^{-\frac{\mu - r}{\sigma} W_t + \sigma W_t} \xi_t \mid \mathcal{F}_s\right) \\
&= \frac{1}{Z_0(s) Z_1(s)} S_0 e^{-\frac{1}{2}\left(\frac{\mu - r}{\sigma}\right)^2 t + \left(\mu - \frac{\sigma^2}{2}\right) t - rt} e^{\left(\sigma - \frac{\mu - r}{\sigma}\right) W_s} E\left(e^{\left(\sigma - \frac{\mu - r}{\sigma}\right)(W_t - W_s)} \xi_t \mid \mathcal{F}_s\right)
\end{aligned}$$

Now consider 2 cases: case 1 means $s < t_1 < t$ and case 2 means everything else. In case 1 we see some randomness in ξ_t, because it involves a jump with random size at t_1. However, ξ_t is independent of $\mathcal{F}_s$ in case 1. Together with the fact that the increment of brownian motion $W_t - W_s$ is also independent of $\mathcal{F}_s$, it is true that

$$
\begin{aligned}
E\left(e^{\left(\sigma-\frac{\mu-r}{\sigma}\right)(W_t-Ws)}\xi_t \mid \mathcal{F}_s\right) = & \\
&= E\left(e^{\left(\sigma-\frac{\mu-r}{\sigma}\right)(W_t-Ws)}\xi_t\right) \\
&= E\left(e^{\left(\sigma-\frac{\mu-r}{\sigma}\right)(W_t-Ws)}\right)E\left(\xi_t\right) \\
&= E\left(e^{\left(\sigma-\frac{\mu-r}{\sigma}\right)(W_t-Ws)}\right)\xi_s \qquad (32)
\end{aligned}
$$

simply because $E(\xi_t) = 1 = \xi_s$ in case 1. Case 2 is different only in a way that there's no randomness in ξ_t and it is just equal to its previous value ξ_s. In case 2 we get $E\left(e^{\left(\sigma-\frac{\mu-r}{\sigma}\right)(W_t-Ws)}\xi_t \mid \mathcal{F}_s\right) = E\left(e^{\left(\sigma-\frac{\mu-r}{\sigma}\right)(W_t-Ws)}\right)\xi_s$. The result for both cases is similar, as shown in this paragraph.

$$
\begin{aligned}
&E^Q\left(e^{-rt}S_t \mid \mathcal{F}_s\right) = \\
&= \frac{1}{Z_0(s)Z_1(s)} S_0 e^{-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 t+\left(\mu-\frac{\sigma^2}{2}\right)t-rt} e^{\left(\sigma-\frac{\mu-r}{\sigma}\right)W_s} e^{\frac{1}{2}\left(\sigma-\frac{\mu-r}{\sigma}\right)^2(t-s)}\xi_s \\
&= \frac{1}{Z_0(s)Z_1(s)} S_0\xi_s e^{\left(\sigma-\frac{\mu-r}{\sigma}\right)W_s} \times e^{\left(-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 t+\left(\mu-\frac{\sigma^2}{2}\right)t-rt+\frac{\sigma^2}{2}t-\mu t+rt+\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 t-\frac{\sigma^2}{2}s+\mu s-rs-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 s\right)} \\
&= \frac{1}{Z_0(s)Z_1(s)} S_0\xi_s e^{\sigma W_s-\frac{\mu-r}{\sigma}W_s} e^{\left(-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 s\right)} e^{\mu s-\frac{\sigma^2}{2}s} e^{-rs} \\
&= \frac{1}{Z_0(s)Z_1(s)} S_0 e^{-rs} e^{\mu s-\frac{\sigma^2}{2}s} e^{\sigma W_s} e^{-\frac{\mu-r}{\sigma}W_s-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 s} \\
&= \frac{1}{Z_0(s)Z_1(s)} e^{-rs} S_s Z_0(s) Z_1(s) \\
&= e^{-rs} S_s
\end{aligned}
$$

Which verifies that under Q, the process $e^{-rt}S_t$ is a martingale.

5.7 The derivation of V_0

$E^Q\left(1_{\{S_T>K\}}\right) = E\left(Z \times 1_{\{S_T>K\}}\right)$, where Z is the Radom-Nikodym derivative.

$$
\begin{aligned}
&E^Q\left(1_{\{S_T>K\}}\right) = \\
&= E\left(E\left(e^{-\frac{\mu-r}{\sigma}W_T-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 T} \times 1_{\left\{S_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)T+\sigma W_T} \times \xi_T > K\right\}} | W_T\right)\right) \\
&= E\left(e^{-\frac{\mu-r}{\sigma}W_T-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 T} \times P\left(\xi_T > \frac{K}{S_0} e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T} | W_T\right)\right) \\
&= E\left(e^{-\frac{\mu-r}{\sigma}W_T-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 T}\left[1 - F_{\xi_T}\left(\frac{K}{S_0} e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T}\right)\right]\right)
\end{aligned}
$$

Where F_{ξ_T} is the cumulative density function of ξ_T Just recall that $0 < t_1 < T$, which means that the uniformly distributed jump in ξ_t will occur during the life of the option. Since this jump is the only source of uncertainty in ξ_t, the variable ξ_T is viewed as uniformly distributed variable on the interval [1-a; 1+a] for the investor who prices the option at t=0. Therefore F_{ξ_T} will be just the standard cdf of uniform variable:

$$
F_{\xi_T}(x) = \begin{cases} 0, & \text{if } x < 1-a \\ \frac{a-1}{2a} + \frac{1}{2a}x, & \text{if } 1-a \le x \le 1+a \\ 1, & \text{if } x > 1+a \end{cases} \tag{33}
$$

Following that idea,

$$E^Q\left(1_{\{S_T>K\}}\right)=$$
$$=E\left(e^{-\frac{\mu-r}{\sigma}W_T-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2T}\left[1-\frac{a-1}{2a}-\frac{K}{2aS_0}\left(e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T}\right)\right]\times 1_{\left\{1-a\leq\frac{K}{S_0}e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T}\leq 1+a\right\}}\right)+$$
$$+E\left(1\times e^{-\frac{\mu-r}{\sigma}W_T-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2T}\times 1_{\left\{\frac{K}{S_0}e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T}\leq 1-a\right\}}\right)+$$
$$+E\left(0\times e^{-\frac{\mu-r}{\sigma}W_T-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2T}\times 1_{\left\{\frac{K}{S_0}e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T}>1+a\right\}}\right)=$$
$$=\frac{a+1}{2a}e^{-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2T}E\left(e^{-\frac{\mu-r}{\sigma}W_T}\times 1_{\left\{1-a\leq\frac{K}{S_0}e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T}\leq 1+a\right\}}\right)-$$
$$-\frac{K}{2aS_0}e^{-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2T-\mu T+\frac{\sigma^2}{2}T}E\left(e^{-\frac{\mu-r}{\sigma}W_T-\sigma W_T}\times 1_{\left\{1-a\leq\frac{K}{S_0}e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T}\leq 1+a\right\}}\right)+$$
$$+e^{-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2T}E\left(e^{-\frac{\mu-r}{\sigma}W_T}\times 1_{\left\{\frac{K}{S_0}e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T}\leq 1-a\right\}}\right)=$$
$$=\frac{a+1}{2a}e^{-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2T}E\left(e^{-\frac{\mu-r}{\sigma}\sqrt{T}N(0;1)}\times 1_{\{b_1\leq N(0;1)\leq b_2\}}\right)-$$
$$-\frac{K}{2aS_0}e^{-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2T-\mu T+\frac{\sigma^2}{2}T}E\left(e^{\left(-\frac{\mu-r}{\sigma}-\sigma\right)\sqrt{T}N(0;1)}\times 1_{\{b_1\leq N(0;1)\leq b_2\}}\right)+$$
$$+e^{-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2T}E\left(e^{-\frac{\mu-r}{\sigma}\sqrt{T}N(0;1)}\times 1_{\{N(0;1)>b_2\}}\right)$$

Where $b_1=\frac{ln\frac{(1+a)S_0}{K}+\mu T-\frac{\sigma^2}{2}T}{-\sigma\sqrt{T}}$ and $b_2=\frac{ln\frac{(1-a)S_0}{K}+\mu T-\frac{\sigma^2}{2}T}{-\sigma\sqrt{T}}$

$$E^Q\left(1_{\{S_T>K\}}\right)=$$
$$=\frac{a+1}{2a}\left(\Phi\left\{b_2+\frac{\mu-r}{\sigma}\sqrt{T}\right\}-\Phi\left\{b_1+\frac{\mu-r}{\sigma}\sqrt{T}\right\}\right)-$$
$$-\frac{K}{2aS_0}exp\left[\sigma^2T-rT\right]\left(\Phi\left\{b_2+\frac{\mu-r}{\sigma}\sqrt{T}+\sigma\sqrt{T}\right\}-\Phi\left\{b_1+\frac{\mu-r}{\sigma}\sqrt{T}+\sigma\sqrt{T}\right\}\right)+$$
$$+\Phi\left\{-\frac{\mu-r}{\sigma}\sqrt{T}-b_2\right\}$$

Denote $d_1=b_1+\frac{\mu-r}{\sigma}\sqrt{T}=\frac{ln\frac{K}{(1+a)S_0}+\frac{\sigma^2}{2}T-rT}{\sigma\sqrt{T}}$ and $d_2=b_2+\frac{\mu-r}{\sigma}\sqrt{T}=\frac{ln\frac{K}{(1-a)S_0}+\frac{\sigma^2}{2}T-rT}{\sigma\sqrt{T}}$ and finally get the formula

$$E^Q\left(1_{\{S_T>K\}}\right)=\frac{a+1}{2a}\left(\Phi\{d_2\}-\Phi\{d_1\}\right)-\frac{K}{2aS_0}exp\left[\sigma^2T-rT\right]\times$$
$$\times\left(\Phi\left\{d_2+\sigma\sqrt{T}\right\}-\Phi\left\{d_1+\sigma\sqrt{T}\right\}\right)+\Phi\{-d_2\}\quad(34)$$

After that we proceed to calculating calculate

$$E^Q\left(S_T\times 1_{\{S_T>K\}}\right)=E\left(E\left(Z\times S_T\times 1_{\{S_T>K\}}|W_T\right)\right),$$

where Z is the Radon-Nikodym derivative

$$E^Q\left(S_T \times 1_{(S_T>K)}\right) =$$
$$= E\left(E\left(S_0 e^{-\frac{\mu-r}{\sigma}W_T-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 T} e^{\mu T-\frac{\sigma^2}{2}T+\sigma W_T}\xi_T \times 1_{\left\{S_0 e^{\mu T-\frac{\sigma^2}{2}T+\sigma W_T}\xi_T>K\right\}}|W_T\right)\right)$$
$$= E\left(S_0 e^{-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 T+\mu T-\frac{\sigma^2}{2}T} e^{-\frac{\mu-r}{\sigma}W_T+\sigma W_T} E\left(\xi_T \times 1_{\left\{S_0 e^{\mu T-\frac{\sigma^2}{2}T+\sigma W_T \times}\xi_T>K\right\}}|W_T\right)\right)$$
$$= E\left(S_0 e^{-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 T+\mu T-\frac{\sigma^2}{2}T} e^{-\frac{\mu-r}{\sigma}W_T+\sigma W_T} E\left(\xi_T \times 1_{\left\{\xi_T>\frac{K}{S_0}e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T \times}\right\}}|W_T\right)\right)$$

The last term of this expression is an expectation of a function of a uniform variable. We calculate this expectation separately, recalling that the probability density function f_{ξ_T} is a piecewise defined function.

$$f_{\xi_T}(x) = \begin{cases} 0, & \text{if } x < 1-a \\ \frac{1}{2a}, & \text{if } 1-a \le x \le 1+a \\ 0, & \text{if } x > 1+a \end{cases} \tag{35}$$

$$E\left(\xi_T \times 1_{\left\{\xi_T>\frac{K}{S_0}exp\left(-\mu T+\frac{\sigma^2}{2}T-\sigma W_T \times\right)\right\}}|W_T\right) =$$
$$= E\left(\xi_T\right) \times 1_{\left\{\frac{K}{S_0}e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T \times}<1-a\right\}} + \left(\int\limits_{\frac{K}{S_0}e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T}}^{(1+a)} \xi\frac{1}{2a}d\xi\right) \times 1_{\left\{1-a\le\frac{K}{S_0}e^{-\mu T+\frac{\sigma^2}{2}T-\sigma W_T \times}\le 1+a\right\}} +$$
$$+ 0 \times 1_{\left\{\frac{K}{S_0}exp\left(-\mu T+\frac{\sigma^2}{2}T-\sigma W_T \times\right)>1+a\right\}} =$$
$$= 1_{\left\{W_T>\frac{ln\frac{(1-a)S_0}{K}+\mu T-\frac{\sigma^2}{2}T}{-\sigma}\right\}} + \left(\frac{(1+a)^2}{4a} - \frac{K^2}{4aS_0^2}e^{-2\mu T+\sigma^2 T-2\sigma W_T}\right) \times$$
$$\times 1_{\left\{\frac{ln\frac{(1+a)S_0}{K}+\mu T-\frac{\sigma^2}{2}T}{-\sigma}\le W_T\le\frac{ln\frac{(1-a)S_0}{K}+\mu T-\frac{\sigma^2}{2}T}{-\sigma}\right\}}$$

After conducting intermediate calculations we return back to the expectation we were looking for:

$$\frac{E^Q\left(S_T \times 1_{(S_T>K)}\right)}{S_0 e^{-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 T+\mu T-\frac{\sigma^2}{2}T}} =$$

$$= E\left(e^{-\frac{\mu-r}{\sigma}W_T+\sigma W_T} \times 1_{\left\{W_T>\frac{ln\frac{(1-a)S_0}{K}+\mu T-\frac{\sigma^2}{2}T}{-\sigma}\right\}}\right) +$$

$$+ E\left(e^{-\frac{\mu-r}{\sigma}W_T+\sigma W_T}\left(\frac{(1+a)^2}{4a} - \frac{K^2}{4aS_0^2}e^{-2\mu T+\sigma^2 T-2\sigma W_T}\right) \times\right.$$

$$\left.\times 1_{\left\{\frac{ln\frac{(1+a)S_0}{K}+\mu T-\frac{\sigma^2}{2}T}{-\sigma} \le W_T \le \frac{ln\frac{(1-a)S_0}{K}+\mu T-\frac{\sigma^2}{2}T}{-\sigma}\right\}}\right) =$$

$$= E\left(e^{\left(\sigma-\frac{\mu-r}{\sigma}\right)\sqrt{T}N(0;1)} \times 1_{\{N(0;1)>b_2\}}\right) + \frac{(1+a)^2}{4a}E\left(e^{\left(\sigma-\frac{\mu-r}{\sigma}\right)\sqrt{T}N(0;1)} \times 1_{\{b_1\le N(0;1)\le b_2\}}\right) -$$

$$- \frac{K^2}{4aS_0^2}e^{-2\mu T+\sigma^2 T}E\left(e^{\left(-\sigma-\frac{\mu-r}{\sigma}\right)\sqrt{T}N(0;1)} \times 1_{\{b_1\le N(0;1)\le b_2\}}\right) =$$

$$= e^{\frac{1}{2}\left(\sigma-\frac{\mu-r}{\sigma}\right)^2 T}\Phi\left\{\left(\sigma-\frac{\mu-r}{\sigma}\right)\sqrt{T}-b_2\right\} +$$

$$+ \frac{(1+a)^2}{4a}e^{\frac{1}{2}\left(\sigma-\frac{\mu-r}{\sigma}\right)^2 T}\left(\Phi\left\{b_2-\left(\sigma-\frac{\mu-r}{\sigma}\right)\sqrt{T}\right\} - \Phi\left\{b_1-\left(\sigma-\frac{\mu-r}{\sigma}\right)\sqrt{T}\right\}\right) -$$

$$- \frac{K^2}{4aS_0^2}e^{-2\mu T+\sigma^2 T}e^{\frac{1}{2}\left(-\sigma-\frac{\mu-r}{\sigma}\right)^2 T}\left(\Phi\left\{b_2-\left(-\sigma-\frac{\mu-r}{\sigma}\right)\sqrt{T}\right\} -\right.$$

$$\left.-\Phi\left\{b_1-\left(-\sigma-\frac{\mu-r}{\sigma}\right)\sqrt{T}\right\}\right) =$$

$$= e^{\frac{1}{2}\left(\sigma-\frac{\mu-r}{\sigma}\right)^2 T}\Phi\left\{-d_2+\sigma\sqrt{T}\right\} +$$

$$+ \frac{(1+a)^2}{4a}e^{\frac{1}{2}\left(\sigma-\frac{\mu-r}{\sigma}\right)^2 T}\left(\Phi\left\{d_2-\sigma\sqrt{T}\right\} - \Phi\left\{d_1-\sigma\sqrt{T}\right\}\right) -$$

$$- \frac{K^2}{4aS_0^2}e^{-2\mu T+\sigma^2 T}e^{\frac{1}{2}\left(-\sigma-\frac{\mu-r}{\sigma}\right)^2 T}\left(\Phi\left\{d_2+\sigma\sqrt{T}\right\} - \Phi\left\{d_1+\sigma\sqrt{T}\right\}\right)$$

After some algebraic simplifications, the answer becomes

$$E^Q\left(S_T \times 1_{(S_T>K)}\right) = S_0 e^{rT}\Phi\left\{-d_2+\sigma\sqrt{T}\right\} + S_0 e^{rT}\frac{(1+a)^2}{4a}\left(\Phi\left\{d_2-\sigma\sqrt{T}\right\} -\right.$$
$$\left.- \Phi\left\{d_1-\sigma\sqrt{T}\right\}\right) - \frac{K^2}{4aS_0}e^{\sigma^2 T-rT}\left(\Phi\left\{d_2+\sigma\sqrt{T}\right\} - \Phi\left\{d_1+\sigma\sqrt{T}\right\}\right) \quad (36)$$

Finally, recall that the option value is

$$V(0) = e^{-rT}E^Q\left(S_T \times 1_{(S_T>K)}\right) - e^{-rT}K \times E\left(1_{(S_T>K)}\right)$$

Thus, we combine equations (24) and (26) and after straightforward calculations, the answer is

$$V(0) = S_0\frac{(1+a)^2}{4a}\left(\Phi\left\{d_2-\sigma\sqrt{T}\right\} - \Phi\left\{d_1-\sigma\sqrt{T}\right\}\right) +$$
$$+ \frac{K^2}{4aS_0}e^{\sigma^2 T-2rT}\left(\Phi\left\{d_2+\sigma\sqrt{T}\right\} - \Phi\left\{d_1+\sigma\sqrt{T}\right\}\right) +$$
$$+ S_0\Phi\left\{-d_2+\sigma\sqrt{T}\right\} - Ke^{-rT}\frac{a+1}{2a}\left(\Phi\{d_2\} - \Phi\{d_1\}\right) - Ke^{-rT}\Phi\{-d_2\} \quad (37)$$