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EQUAL WEIGHTS COAUTHORSHIP SHARING AND SHAPLEY VALUE ARE EQUIVALENT

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The publication credit allocation problem is one of the fundamental problems in bibliometrics. The solution of this problem provides the basis of further research. There are two solutions which do not require any additional information: equal weights measure and Shapley value. Until recently Shapley value is not used because of hardness of computing. The paper justifies the equal weights measure by showing equivalence with the Shapley value approach for sharing coauthors performance in specific games.

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1. Introduction

The noticeable trend towards increased coauthorships sets a challenge to modern bibliometrics (Hudson 1996; Bird 1997, and others). Researchers are faced with the problem of publication credit allocation. Frequently there is no any information about true coauthor input. There are several solutions which do not use any additional information. The first approach is assignment of full credit to each coauthor (full credit solution). It is a way of ignoring the problem, but this approach is very useful, e.g. the h-index is based on this assumption (Hirsch 2005). Another common approach is equal or egalitarian weights solution (Schreiber 2008). It is reasonable a priori solution.

Recent studies (Tol 2012; Papapetrou et al. 2011) propose to utilize the Shapley value solution (Shapley 1953). Shapley value can be considered as “fair allocation”. The main difficulty of the Shapley value solution is the hardness of computing. The problem of computing the Shapley value is an NP-complete problem (Castro et al. 2009).

There are three ways of avoiding this difficulty. The first approach is to simplify the Shapley value solution and calculate a modified measure (e.g. in (Tol 2012; Papapetrou et al. 2011) in which approximation which consists in considering not all possible coauthors' coalitions but only existing teams of coauthors is used. The second approach is to find algorithms to calculate the Shapley value precisely for particular classes of games, e.g. for the problem of sharing delay costs (Castro et al. 2008). The third approach is to find a measure which is equivalent to Shapley value but is simpler for computation. The third approach is quite popular. It is applied for some problems in supply chain management (Chen and Yin 2010) and for phylogenetic biodiversity measures (Hartmann 2012), but there is no such result for coauthorship sharing problem. This paper fills the gap in the literature.

There are some other approaches dealing with the coauthorship sharing problem which use some additional information: positionally weighted assignment (Lukovits and Vinkler 1995; Sekercioglu 2008; Zhang 2009; Abbas 2011 and others), rank-based (Assimakis and Adam 2011), Tailor based (Galam 2011) Pareto weights (Tol 2011), full credit to the senior coauthor (Hirsch 2010) and some mixed measures (Liu and Fang 2012).

There are several reasons to equal sharing. Articles with different number of authors are not equal. More coauthorship is associated with higher quality and greater length of publications (Holis 2001). Collaborative research tends to have higher number of citations (Katz and Hicks 1997; Levitt and Thelwall 2010). Individual's return from a coauthored paper with n authors is approximately l/n times that of a single-authored paper (Sauer 1988). There is a statistically significant increasing trend in yearly prevalence of equally credited authors in medicine journals (Akhaube and Lautenbach 2010, Tao et al. 2012).

The paper is organized as follows. The mathematical model including definition of games and main theorems belongs to Section 2. Section 3 shows clarification example. Section 4 concludes.

2. Problem setting

The mathematical model of the coauthorship sharing problem is described by a set of authors $\mathrm{N} = \{1,...,n\}$ and a set of publications $\mathrm{P} = \{1,...,m\}$. Each paper is associated with a set of coauthors $S_j \in 2^{\mathrm{N}}$ and a quality measured by a real number $q_j \in R$. It is possible to use the number of citations as the quality measure or to equate the quality of all papers to one. The first case is associated with the problem of citations sharing. In the latter case only the number of publications should be divided between authors. Mathematically these problems are the same. The aim is to find the performance measure attributed to each author ($y_i \in R$) subject to one constrain. The sum of the authors' performance measures should be equal to the sum of the publications quality

$$\sum_{i=1}^{n} y_i = \sum_{j=1}^{m} q_j \ . \tag{1}$$

There are two main solutions of this problem: equal weights coauthorship sharing and Shapley value.

Definition 1. Equal weights measure of performance. The quality of paper is divided equally between coauthors

$$y_i^{EW} = \sum_{j=1}^{m} \frac{\left|S_j \cap \{i\}\right|}{\left|S_j\right|} q_j \ , \tag{2}$$

where $\left|S_j\right|$ is the cardinality of set S_j, number of coauthors.

Before defining the Shapley value the description of appropriate cooperative games should be given. There are three reasonable approaches: Full obligation game, Full credit game and Equal weights game.

Definition 2. Full obligation game. Each coauthor is crucial to publication. The absence of one coauthor leads to the absence of publication. The game is defined by characteristic function for all $S \in 2^{\mathrm{N}}$

$$v_1(S) = \sum_{j=1}^{m} 1_{S_j \subseteq S} q_j \ , \tag{3}$$

where $1_{S_j \subseteq S} = 1$ if $S_j \subseteq S$ and $1_{S_j \subseteq S} = 0$ otherwise.

Definition 3. Full credit game. Each coauthor takes full quality of the publication.

$$v_2(S) = \sum_{j=1}^{m} 1_{S_j \cap S \neq \varnothing} q_j \ . \tag{4}$$

Definition 4. Equal weights game. Each coauthor takes equal part of the quality of the publication.

$$v_3(S) = \sum_{i \in S} \sum_{j=1}^{m} \frac{\left|S_j \cap \{i\}\right|}{\left|S_j\right|} q_j \ . \tag{5}$$

The Shapley value is the average marginal contribution which author i adds to the coalition. One of the possible definitions of the Shapley value is the average marginal contribution obtained by player at random arrival. Let σ be the permutation of players, $\sigma(i)$ - the place of the player i in permutation σ. The marginal contribution of the player i is $v\left(S^{\sigma}_{\sigma(i)}\right) - v\left(S^{\sigma}_{\sigma(i)-1}\right)$, where $S^{\sigma}_{\sigma(i)-1}$ is the first $\sigma(i)-1$ players in permutation σ (the set of players in N which precede player i in the order σ). Taking into account the number of possible permutations the formula for calculating the Shapley value is derived

$$\phi_i(v) = \frac{1}{n!} \sum_{\sigma} \left[v\left(S^{\sigma}_{\sigma(i)}\right) - v\left(S^{\sigma}_{\sigma(i)-1}\right) \right]. \tag{6}$$

There are four different methods of measuring author's performance but they lead to the same result.

Theorem 1. (Equivalence theorem). For all coauthorship sharing problems

$$\phi_i(v_1) = \phi_i(v_2) = \phi_i(v_3) = y_i^{EW} \ . \tag{7}$$

Proof.

Taking into account definitions of the games we rewrite Shapley values

$$\phi_i(v_1)=\frac{1}{n!}\sum_{\sigma}\left[\sum_{j=1}^{m}1_{S_j\subseteq S^{\sigma}_{\sigma(i)}}q_j-\sum_{j=1}^{m}1_{S_j\subseteq S^{\sigma}_{\sigma(i)-1}}q_j\right]=\frac{1}{n!}\sum_{\sigma}\left[\sum_{j=1}^{m}\left(1_{S_j\subseteq S^{\sigma}_{\sigma(i)}}-1_{S_j\subseteq S^{\sigma}_{\sigma(i)-1}}\right)q_j\right];$$

$$\varphi_i(v_2)=\frac{1}{n!}\sum_{\sigma}\left[\sum_{j=1}^{m}1_{S_j\cap S^{\sigma}_{\sigma(i)}\neq\varnothing}q_j-\sum_{j=1}^{m}1_{S_j\cap S^{\sigma}_{\sigma(i)-1}\neq\varnothing}q_j\right]=\frac{1}{n!}\sum_{\sigma}\left[\sum_{j=1}^{m}\left(1_{S_j\cap S^{\sigma}_{\sigma(i)}\neq\varnothing}-1_{S_j\cap S^{\sigma}_{\sigma(i)-1\neq\varnothing}}\right)q_j\right]=$$

$$\frac{1}{n!}\sum_{\sigma}\left[\sum_{j=1}^{m}1_{S_j\cap S^{\sigma}_{\sigma(i)}=\{i\}}q_j\right];$$

$$\phi_i(v_3)=\frac{1}{n!}\sum_{\sigma}\left[\sum_{i\in S^{\sigma}_{\sigma(i)}}\sum_{j=1}^{m}\frac{|S_j\cap\{i\}|}{|S_j|}q_j-\sum_{i\in S^{\sigma}_{\sigma(i)-1}}\sum_{j=1}^{m}\frac{|S_j\cap\{i\}|}{|S_j|}q_j\right]=\sum_{j=1}^{m}\frac{|S_j\cap\{i\}|}{|S_j|}q_j\,. \tag{8}$$

$1_{S_j\in S^{\sigma}_{\sigma(i)}}-1_{S_j\in S^{\sigma}_{\sigma(i)-1}}=1$ is true for $i\in S_j$ and author i is the last author among coauthors S_j in ordering σ.

$1_{S_j\cap S^{\sigma}_{\sigma(i)}=\{i\}}=1$ is true for $i\in S_j$ and author i is the first author among coauthors S_j in ordering σ.

If $i\in S_j$ then has equal chance to be first, second or last among coauthors S_j in the ordering σ. The probability of being last is $\frac{|S_j\cap\{i\}|}{|S_j|}$. The same can be derived by counting all permutations which satisfy respective condition. The number of such permutations is the number of ways of placing the unordered set of coauthors of the paper j in the ordering from N multiplying on the number of permutations of coauthors of the paper j except the author i and the number of permutation of all authors except coauthors of the paper j

$$\phi_i(v_1)=\sum_{j=1}^{m}\frac{\binom{n}{|S_j|}\cdot(n-|S_j|)!\cdot(|S_j|-1)!}{n!}\cdot 1_{i\in S_j}\cdot q_j=\sum_{j=1}^{m}\frac{|S_j\cap\{i\}|}{|S_j|}\cdot q_j\,. \tag{9}$$

The same is true for the probability of being first, then

$$\phi_i(v_1)=\phi_i(v_2)=y_i^{EW}=\sum_{j=1}^{m}\frac{|S_j\cap\{i\}|}{|S_j|}\cdot q_j\,.\ \blacksquare \tag{10}$$

For all games which do not distinguish all authors and all publications by names or some other characteristics we obtain the equal weights solution as equivalent of the Shapley value solution.

There are several justifications of the Shapley value. The Shapley value is unique solution satisfying the efficiency, symmetry, dummy, and additivity axioms (Shapley 1953). Hart and Mas-Colell (1989) describes the Shapley value as unique solution satisfying symmetry, efficiency and consistency. Chun (1989) describes the Shapley value as unique solution satisfying efficiency, triviality, coalitional strategic equivalence, and fair ranking. All these properties are reasonable for our problem but fair ranking (the ranking of players' payoffs within a coalition depends only on the collaboration opportunities, possible marginal contributions of their members, but not on the worth of the coalition) is particularly important. The equal weights solution possesses all properties listed above and there is no such another solution.

Another basic solution of cooperative games is the core. In core solution there is no coalition which can increase own payoff. Each coalition obtains at least as the value of the coalition in the game

$$C(\mathrm{N},v)=\left\{ y\in R^n \middle| \sum_{i\in N} y_i = v(\mathrm{N}), \forall S\subset \mathrm{N}\ \sum_{i\in S} y_i \geq v(S) \right\}. \tag{11}$$

The dual solution is anticore (Monderer et al. 1992). Each coalition obtains at most as the value of the coalition in the game

$$AC(\mathrm{N},v)=\left\{ y\in R^n \middle| \sum_{i\in N} y_i = v(\mathrm{N}), \forall S\subset \mathrm{N}\ \sum_{i\in S} y_i \leq v(S) \right\}. \tag{12}$$

There is a close relation between core and anticore. The anticore coincides with the core of dual game. (This fact is proved in theorem 3). The dual game of v is the game v^d with the following characteristic function

$$v^d(S)=v(\mathrm{N})-v(\mathrm{N}/S),\ \ \forall S\subseteq \mathrm{N}. \tag{13}$$

The relation between the core, the anticore and equal weights solution is given in theorems 2 and 3.

Theorem 2. $C(\mathrm{N},v_3)$ is non empty single valued solution and $y_i^{EW}=C(\mathrm{N},v_3)$.

Proof.

Game is convex if its characteristic function is supermodular:

$$\forall A,B\subseteq \mathrm{N}\ \ v(A)+v(B)\leq v(A\cap B)+v(A\cup B). \tag{14}$$

The supermodularity condition for v_3

$$\sum_{i \in A}\sum_{j=1}^{m}\frac{\left|S_j \cap \{i\}\right|}{\left|S_j\right|}q_j + \sum_{i \in B}\sum_{j=1}^{m}\frac{\left|S_j \cap \{i\}\right|}{\left|S_j\right|}q_j \le \sum_{i \in A\cap B}\sum_{j=1}^{m}\frac{\left|S_j \cap \{i\}\right|}{\left|S_j\right|}q_j + \sum_{i \in A\cup B}\sum_{j=1}^{m}\frac{\left|S_j \cap \{i\}\right|}{\left|S_j\right|}q_j \ . \quad (15)$$

Because of $\sum_{i \in A}\sum_{j=1}^{m}\frac{\left|S_j \cap \{i\}\right|}{\left|S_j\right|}q_j + \sum_{i \in B}\sum_{j=1}^{m}\frac{\left|S_j \cap \{i\}\right|}{\left|S_j\right|}q_j \le \sum_{i \in A\cup B}\sum_{j=1}^{m}\frac{\left|S_j \cap \{i\}\right|}{\left|S_j\right|}q_j$ the supermodularity condition holds.

Because of convexity the core of this game exists (Shapley 1971). For any $y \in C(\mathrm{N}, v_3)$ we have $y_i \ge v_3(\{i\})$.

Taking into account the definition of the game we obtain

$$y_i \ge v_3(\{i\}) = \sum_{j=1}^{m}\frac{\left|S_j \cap \{i\}\right|}{\left|S_j\right|}q_j = y_i^{EW} \ . \quad (16)$$

Because of $\sum_{i \in \mathrm{N}} y_i = \sum_{j=1}^{m} q_j = \sum_{i \in \mathrm{N}} y_i^{EW}$ the core coincides with equal weights solution $y_i = y_i^{EW}$.■

Theorem 2 shows that equal weights game does not contribute any additional solutions. The core and the Shapley value solution coincides with equal weights solution. For the other games the core or the anticore is wider and includes the Shapley value solution.

Theorem 3. $y^{EW} \in C(\mathrm{N}, v_1)$ and $y^{EW} \in AC(\mathrm{N}, v_2)$.

Proof.

Let define the dual game for v_1

$$v_1^d(S) = v_1(\mathrm{N}) - v_1(\mathrm{N}/S) = \sum_{j=1}^{m} q_j - \sum_{j=1}^{m} 1_{S_j \subseteq \mathrm{N}/S}\, q_j = \sum_{j=1}^{m} 1_{S_j \cap S \ne \varnothing}\, q_j = v_2(S) \ . \quad (17)$$

The game v_2 is the dual game for v_1. It is easy to show, that the core of v_1 is the anticore of its dual game

$$AC(\mathrm{N}, v_2) = \left\{ y \in R^n \,\middle|\, \sum_{i \in N} y_i = v_2(\mathrm{N}), \forall S \subset \mathrm{N} \ \sum_{i \in S} y_i \le v_2(S) \right\} =$$

$$\left\{ y \in R^n \,\middle|\, \sum_{i \in N} y_i = v_1(\mathrm{N}), \forall S \subset \mathrm{N} \ \sum_{i \in \mathrm{N}/S} y_i \ge v_1(\mathrm{N}/S) \right\} = C(\mathrm{N}, v_1) \ . \quad (18)$$

The supermodularity condition for v_1

$$\sum_{j=1}^{m} 1_{S_j \subseteq A} q_j + \sum_{j=1}^{m} 1_{S_j \subseteq B} q_j \leq \sum_{j=1}^{m} 1_{S_j \subseteq A \cap B} q_j + \sum_{j=1}^{m} 1_{S_j \subseteq A \cup B} q_j \ . \tag{19}$$

Because of $1_{S_j \subseteq A} q_j + 1_{S_j \subseteq B} q_j \leq 1_{S_j \subseteq A \cup B} q_j$ the supermodularity condition holds.

Because of convexity of the game the core of v_1 exists (Shapley 1971). If the core is nonempty then $\phi(v_1) \in C(\mathrm{N}, v_1)$ and $\phi(v_3) \in AC(\mathrm{N}, v_2)$ (Shapley 1971). By theorem 1 $y^{EW} \in C(\mathrm{N}, v_1)$ and $y^{EW} \in AC(\mathrm{N}, v_2)$.■

The core and the anticore are extended to the undominated set (Derks et al. 2012). By theorem 3 equal weights solution belongs to the undominated set which means that it cannot be improved by sidepayments changing the allocation or the game (domination concept).

Four possible solutions of coauthorship sharing problem which does not require any additional information are equivalent. For all games considered in the text the Shapley value solution coincides with the equal weights solution. The Shapley value justifies equal weights method. The equal weights solution also belongs to the core (natural compromise division) of full obligation game, belongs to the anticore of full credit game and coincides with the core of equal weights game.

3. Example

This section provides an example illustrating the equivalence theorem. There are 3 authors $\mathrm{N} = \{1,2,3\}$. There are six publications produced by these authors (Table 1). Publication 1 has one author, but publication 6 – three coauthors. There is no information about the contribution of coauthors in each publication.

Table 1. Set of publications

1	2	3	4	5	6
$\{1\}$	$\{1\}$	$\{2\}$	$\{1,2\}$	$\{1,3\}$	$\{1,2,3\}$

The first author is the most distinguished, but the majority of his papers are written in collaboration. The problem is to share six publications between three authors. By obvious reasons the contribution of the first author is not less than two publications (single-authored) but not more than five publications (all written singly or in collaboration). Finding the exact number of publications of each author requests a model and some calculations.

Utilizing the model described above for sharing the number of publications problem we have $q_j = 1 \ \forall j \in \mathrm{P}$. The first approach is the equal weights measure of performance. Each publication is divided equally between its coauthors. Calculations are showed in the Table 2, where $\frac{|S_j \cap \{i\}|}{|S_j|} \cdot q_j$ is pointed in the each cell.

Table 2. Equal weights measure of performance

Authors	$\{1\}$	$\{1\}$	$\{2\}$	$\{1,2\}$	$\{1,3\}$	$\{1,2,3\}$	y_i^{EW}
1	1	1	0	1/2	1/2	1/3	3.33
2	0	0	1	1/2	0	1/3	1.83
3	0	0	0	0	1/2	1/3	0.83

The obtained solution $y^{EW} = (3.33,\ 1.83,\ 0.83)$ seems to be relevant.

On the surface the Shapley value approach looks differently. We can define various games and in each of them find own solution. The full obligation game is based on the minimal number of publications associated with each coalition. No doubt coalition S without counter-coalition $\mathrm{N} \setminus S$ can take such publications that $S_j \subseteq S$. The characteristic function of full obligation game is given in Table 3.

Table 3. Full obligation game

S	Ø	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v_1(S)$	0	2	1	0	4	3	1	6

By definition of the Shapley value we consider all possible permutations and calculate the average marginal contribution (Table 4).

Table 4. Full obligation game Shapley value

Permutations	$v_1(S) - v_1(S / \{1\})$	$v_1(S) - v_1(S / \{2\})$	$v_1(S) - v_1(S / \{3\})$
123	2	2	2
132	2	3	1
213	3	1	2
231	5	1	0
312	3	3	0
321	5	1	0
Shapley value	3.33	1.83	0.83

Let's explain the calculation of the figures in the Table 4 e.g. third line. Author 2 is the first in permutation. He takes marginal contribution from $v_1(\varnothing)$ to $v_1(\{2\})$ which is equal to $v_1(\{2\}) = 1$. Author 1 is the second in permutation and he gets $v_1(\{1,2\}) - v_1(\{2\}) = 3$. Author 3 gets residual 2 publications. In accordance with equivalence theorem the Shapley value coincides with the equal weights solution.

The full credit game is based on the maximal number of publications associated with each coalition. Coalition S without counter-coalition $\mathrm{N} \setminus S$ can take such publications that at least one coauthor of the set S_j belongs to the coalition S. The presence of only one author from coauthors' set S_j is sufficient to take publication credit. The characteristic function of full credit game is given in the Table 5.

Table 5. Full credit game

S	Ø	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v_2(S)$	0	5	3	2	6	5	4	6

Performing the same calculations as in the Table 4 we obtain the Shapley value. It is equal to quoted above. Moreover the Table 6 can be found by permutation of lines of the Table 4. It is not accidental coincidence. The Shapley values of dual games are always equal (Funaki 1998). By definition one can show the duality of the games v_1 and v_2 in this example.

Table 6. Full credit game Shapley value

Permutations	$v_2(S) - v_2(S/\{1\})$	$v_2(S) - v_2(S/\{2\})$	$v_2(S) - v_2(S/\{3\})$
123	5	1	0
132	5	1	0
213	3	3	0
231	2	3	1
312	3	1	2
321	2	2	2
Shapley value	3.33	1.83	0.83

This example shows the equivalence of the equal weights solution and the Shapley value for basic games. In general case the computational complexity of the Shapley value calculation is harder then equal weights solution calculation but in this small example the difference is inconsiderable.

4. Conclusion

This paper fills the gap in the literature showing equivalence of two existing solutions. The equivalence theorem from one hand provides a game-theoretical justification of the equal weights measure of performance and from another hand provides a simple method of computing Shapley value by substituting on equal weights measure. Equal weights measure can be used keeping all properties of Shapley value. Moreover it is unique such solution.

The core of full obligation game and the anticore of full credit game include equal weights solution. The two main solutions of cooperative game theory supports equal weight solution. This justifies Schreiber’s approach to author performance measure (Schreiber 2008) and many empirical studies which uses equal weights coauthorship sharing.

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