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PROBABILISTIC EXTENTION OF THE CUMULATIVE PROSPECT THEORY

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PROBABILISTIC EXTENTION OF THE CUMULATIVE PROSPECT THEORY

A number of experiments indicate probabilistic preferences in cases where no one alternative is absolutely optimal. The task of predicting the choice of one of the alternatives among multiple alternatives is then practically important and not trivial. It can occur in situations of choice under risk when no one lottery stochastically dominates others.

For risky lotteries there are several complicated models of probabilistic binary preference. For the first time, we herein propose the probabilistic extension of the cumulative prospect theory (CPT). The presented visual graphic justification of this model is intuitively clear and does not use sophisticated cumulative summing or a Choquet integral.

Here we propose a model of selecting from a set of alternatives by continuous Markov random walks. It makes predicting the results of a choice easy because it fully uses dates received by probabilistic extension of CPT.

The proposed methods are quite simple and do not require a large amount of data for practical use.

JEL Classification: D81, D83, C44.

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1. Introduction

Practical experiments often show that in similar situations a rational individual can make a different choice from a pair of alternatives. This means that binary preferences cannot be determined uniquely and that we can only indicate the probability of choosing each alternative.

When discussing the axiomatization of stochastic models for choices over a set of alternatives, as seen in Dagsvik [2008], the nature of probabilistic choice are originally given. But in this article, we are interested in the causes of probability preferences.

Probabilistic binary preferences can occur if no single alternative is absolutely optimal. The first alternative is more preferable by some parameters (for example, price and weight) and the second is more preferable by other parameters (quality and ergonomics) [Swaita and Marleya 2013]. Probabilistic binary preferences can happen in situations of choice under risk, if no one lottery stochastically dominates another. Axiomatic models of probabilistic binary choice under risk in this situation have been studied by Blavatskyy [2012].

Here, for the first time, we introduce probabilistic binary preferences as an extension of the cumulative prospectuses theory of Tversky and Kahneman [1992]. We made it by using a visual graphical representation of prospects that are casually used by Wakker [2010, pp.160, 197] to explain the concept of rank-dependent utility. We also define the probabilistic binary preferences by graphic form, which is intuitively clear and does not require sophisticated cumulative summing or Choquet integrals in a continuous case.

The idea of probabilistic extends back to Fishburn [1978] and Kiruta et al [1980]. In comparing the alternatives x and y, an individual assesses why (or by how much) the first alternative is better than another and why (or by how much) the second alternative is better than the first. These values are referred to as the comparative utility[2] $\psi(\cdot,\cdot)$. If no one alternative absolutely dominates another, $\psi(x,y)>0$ and $\psi(y,x)>0$. The ratio of these values $\psi(x,y)/\psi(y,x)$ determines the ratio of probabilities for choosing each alternative.

The CPT defines specific utility $u(x)$ for each prospect x. The paradigm of rational choice assumes that the individual will choose the most useful alternative and the selection result will be univalent. However in this paper we determine the comparative utility function $\psi(\cdot,\cdot)$ for a pair of lotteries, so we use it to construct the probabilistic extension of CPT.

There are several quiet serious criticisms of CPT. Birnbaum et al [1999] presented examples of the violation of the first-order stochastic dominance principal. Wu and Markle

[2] Fishburn [1978] calls it "incremental expected utility advantage", but thanks to Kiruta et al [1980] we will here call it shorter as "comparative utility".

[2007] presented examples of the violation sine-depends principle. The applicability of CPT for mixed prospects seen by us is limited. That is why the probabilistic extension of CPT introduced here is only for non-negative prospects.

We then examine the following question in this paper. An individual has probabilistic binary preferences on a set of lotteries and must choose one alternative. How can one predict the choice result? There are several widely known approaches, such as Bradley-Terry-Luce and quit new (see Cattelan M. [2012], Blavatskyy [2009]). But in our case, the probability binary preferences are generated by comparative utility. So we propose to use a model of selection by continuous Markov random walk on a set of alternatives introduced in Zutler [2011]. This model can use values of comparative utility and allows us to easily receive the required prediction. It studies the course of the selection process in time. That is, it studies how an individual will go through and compare alternatives, and where it leads. The essential final question is formulated as follows: "Which alternative will an individual choose?" rather than "What is the most useful alternative to an individual?" Therefore, the received prediction coincides with our intuitive idea of a choice's result.

The proposed model has the following prerequisites. Firstly, how can individual actually choose? The simplest model of the selection process is a sequential scan that occurs in the mind of the individual. An individual "sets" all the alternatives in a row and then "takes" the left-most alternative and begins to compare it in order to option on the right. If an individual finds a better alternative, it "takes" it and "leaves" the old alternative. When an individual comes to the right-hand end of row, he finally chooses the alternative that remains "in hand".

Given the shortage of time, the sorting process can be more erratic. The individual "takes" some alternative in the hope that it is the best. He randomly inspects the remaining alternatives. Some of alternatives that catch the individual's eye will be worse than the alternative "in hand". The superiority of the other alternatives might not be notice and missed. But if the individual notices an alternative that is better, he or she will "grab" it and "throw" the old choice back into the pile. If the individual preferences are transitive, then he or she will eventually find a better alternative, and the process of brute force stops. For non-transitive preferences this formal stochastic process cannot ever stop. We then assume that the result of selection will be probabilistic and proportional to the time within which an individual considers a particular alternative as the best.

We assume that the selection process is implemented by a continuous homogeneous Markov random walk on a set of alternatives. That is, the intensity of the transition from the "taken" alternative to any other alternative depends only on which alternative he or she "holds" now. This process was called a Continuous Markov Chain Choice (CMCC). The system of differential equations for the CMCC model is obtained by standard methods. Regarding the ergodicity of the process, there will be a system of linear equations in a steady state.

The continuous Markov model has an advantage over a discrete model. If we have any numerical representations of binary relation on the set of alternatives, for the continuous model it does not require that we normalize the transition probabilities. These numerical values of advantage can be used explicitly as transitions intensities. In contrast, for a discrete Markov process it should be made to normalize the transition/no transition in each step. In case of a probabilistic extension of CPT, we will take the intensity of transition equal to comparative utility.

The paper is organized as follows: Section 2 introduces the probabilistic extension of the CPT and Section 3 presents a selection model by a continuous Markov random walk for this case.

2. Probabilistic extension of the cumulative prospect theory

2.1 Cumulative prospect theory and Prospect diagram

Suppose that we have a prospect (lottery[3]) x with positive and negative outcomes (gains and losses), where the probability of outcome x_i is p_i. To calculate the utility of a prospect, Tversky and Kahneman [1992][4] proposed a cumulative model, which is as follows. Let the outcomes be numbered in ascending order of their values, meaning that $i > j$ if the utility is $x_i > x_j$. For positive outcomes, positive indices are used, for negative ones, negative indices are used, for the zero outcomes (status quo), zero indices are used, x^+ and x^- denote the positive and negative parts of a prospect.

The weight function $W(p)$ reflects the subjective assessment of probability by an individual. The value function $V(x)$ reflects the desirability by an individual to receive x over nothing. The utility of the lottery is calculated as follows:

$$u(x) = V(x^+) + V(x^-),$$

$$V(x^+) = \sum_{i=1}^{n} \pi_i^+ V(x_i), \quad V(x^-) = \sum_{i=-m}^{0} \pi_i^- V(x_i),$$

$$\pi_n^+ = W^+(p_n),\ \pi_i^+ = W^+(p_i + p_{i+1} + \ldots + p_n) - W^+(p_{i+1} + \ldots + p_n),$$

$$\pi_m^- = W^-(p_m),\ \pi_i^- = W^-(p_{-m} + p_{-m+1} + \ldots + p_i) - W^-(p_{-m} + p_{-m+1} \ldots + p_{i-1}).$$

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[3] The fundamentals of the CPT are usually presented in a situation of uncertainty – the implementation of the possible states of nature. Here we made the presentation adopted in a situation of risk – a lottery.

[4] Here we present only a brief introduction to the CPT. The most detailed description of CPT is available in Wakker [2010]. For axiomatic foundations see Wakker and Tversky [1993].

Here π_i^+ means the difference between the values of the weight function for the outcome "not worse than x_i", and the weighting function of the outcomes "strictly better than x_i". For a continuous set of alternatives, this formula is transformed into a Choquet integral.

Let us present diagrams of the prospects. For positive prospects (lottery), the diagram of the prospects is the graph of reliably obtained values – a decreasing function $V(p)$ on the probability interval [0, 1]. For example, for the prospect x = ($ 200, 0.3; $ 100, 0.2; $ 50, 0.5) a diagram is equals to V($ 200) on the interval from 0 to 0.3, V($ 100) on the interval from 0.3 to 0.5, and V($ 50) on the interval from 0.5 to 1 (see Fig. 1).

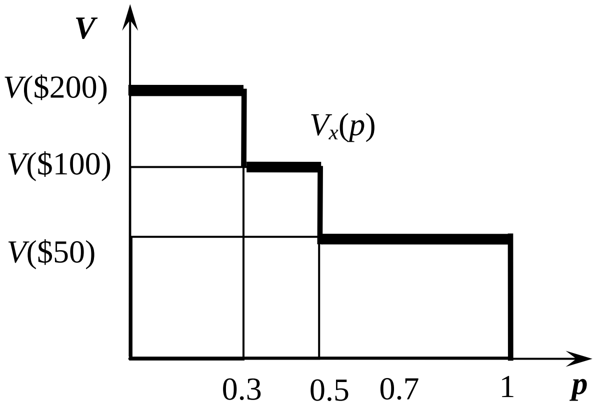


Fig. 1

A prospect diagram allows us to calculate cumulative utility easier. For a positive prospect:

$$u(x) = \int_0^1 V(p)\omega(p)\, dp \qquad (1)$$

where

$$\omega(p) = \frac{dW(p)}{dp}$$

meaning that $\omega(p)$ is the density of the weight function $W(p)$.

In the general case for a mixed prospect, we can construct diagrams as follow. For the positive part of the prospects it is the graph of reliably obtained values (on the interval [0, 1]). For the negative part of the prospects it is the graph of reliable loss values (on the interval [-1, 0]). For example, the prospect for z = (-50$, 0.2; -10$, 0.1; 100$, 0.4; 50$, 0.3) is shown on Fig. 2.

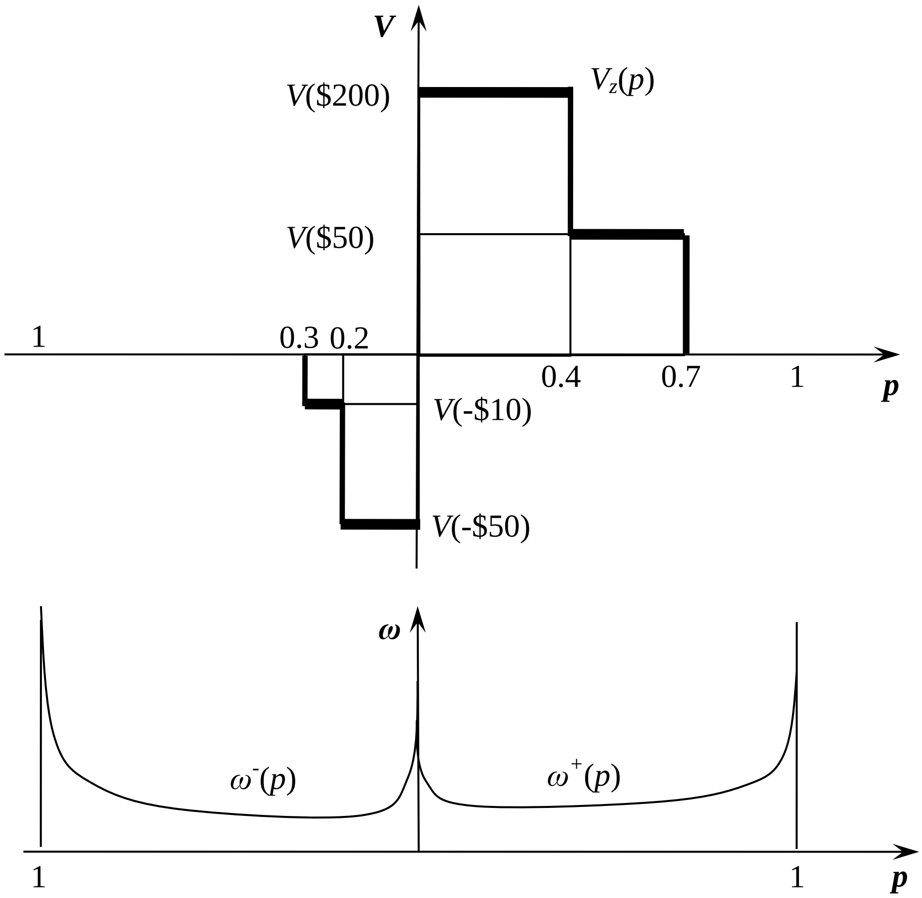


Fig. 2

For the mixed prospect:

$$u(x) = \int_{-1}^{0} V^-(p)\omega^-(p)\,dp + \int_{0}^{1} V^+(p)\omega^+(p)\,dp$$

where $V^-(p)$ and $V^+(p)$ – a graphs for its positive and negative parts,

$$\omega^+(p) = \frac{dW^+(p)}{dp}, \quad \omega^-(p) = -\frac{dW^-(-p)}{dp}$$

is the density of the weight function for gains and losses.

Example

Let us consider violations of the Independence Axiom of the Allais-type paradox in an example from Kahneman and Tversky [1979]. Of the two alternatives x_1 = ($ 4000, 0.8; $ 0, 0.2) and y_1 = ($ 3,000, 1.0), about 80% of the respondents choose the alternative y_1. That is to say that 80% of the respondents prefer to receive $3000 for sure, rather than to participate in a lottery where the gain is $4000 with a probability of 0.8. If the probability of gain in both lotteries is

reduced by 4 times, then from the obtained alternatives x_2 = (\$4000, 0.2; \$0, 0.8) and y_2 = (\$3000, 0.25), about 65% of the respondents choose the alternative x_2.
To explain the paradox, we consider the prospect diagrams given in Fig. 3 and formula (1).

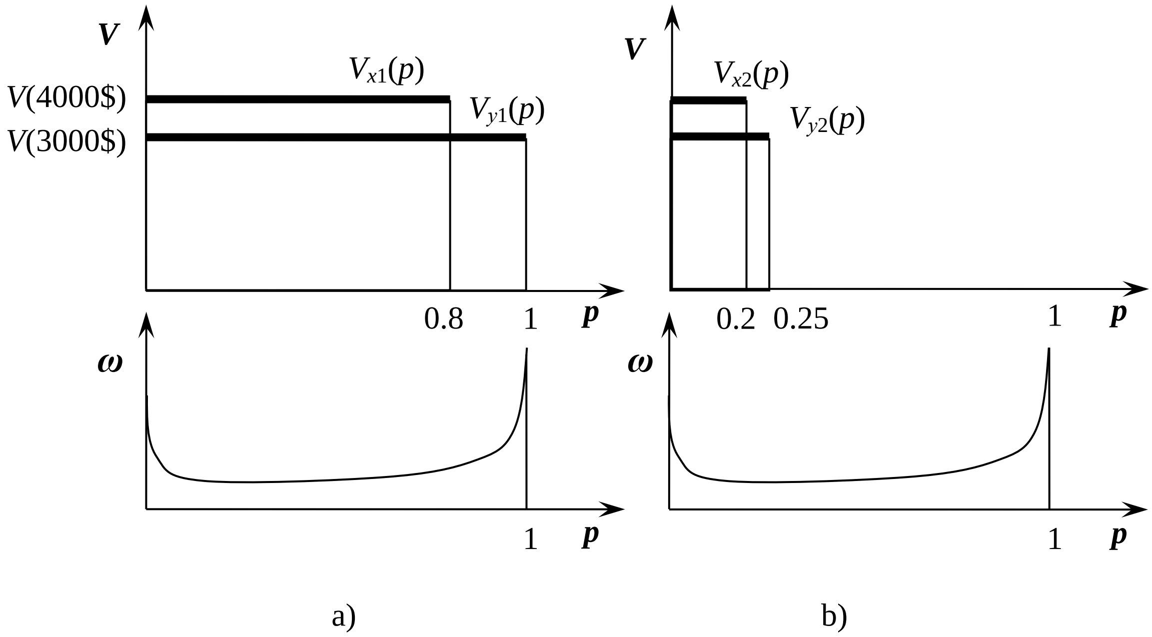


Fig. 3

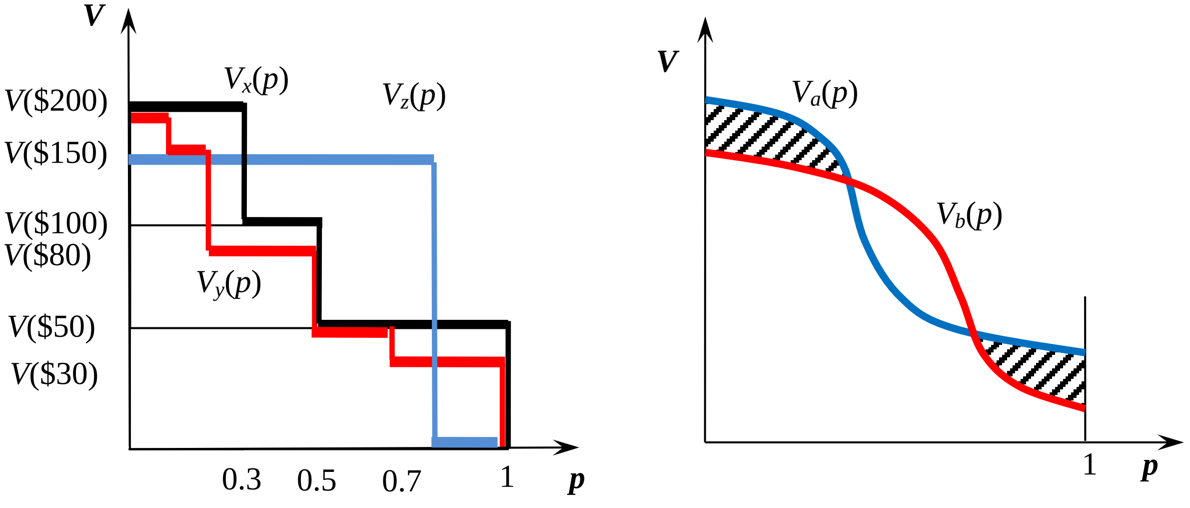
Since the density of the weighting function significantly increases in the vicinity of p = 1 (the respondents overvalued the lack of risk), then:

$$\int_0^{0.8} V(4000\$)\omega(p)dp < \int_0^{1} V(3000\$)\omega(p)dp \qquad \text{but} \qquad \int_0^{0.2} V(4000\$)\omega(p)dp > \int_0^{0.25} V(3000\$)\omega(p)dp$$

which explains this paradox.

2.2 Stochastic dominance and Probabilistic choice

Of the two alternative prospects x = (\$ 200, 0.3; \$ 100, 0.2; \$ 50, 0.5) and x' = (\$ 190, 0.3; \$ 90, 0.2; \$ 40, 0.5), an individual's choice is clearly in favor of the first one. Considering the same chances, it ensures greater gain. This means stochastic dominance. For a comparison of the prospects in less obvious situations, such as x and y = (\$ 200, 0.1; \$ 150, 0.1; \$ 80, 0.3; \$ 50, 0.2; \$ 30, 0.3), we can construct auxiliary diagrams of the prospects (see Fig. 4a). The diagram of prospect x is strictly higher than y, a choice of the individual will be in its favor. As $V_x(p) \geq V_y(p)$ for all p, a prospect x stochastically dominates y, too.



a) b)

Fig. 4

Finally, if an individual will compare prospects *x* and *z* = ($150, 0.8; $0, 0.2), then no one stochastically dominates the other. There is some reasons to choose prospects *x* and some reasons *y*. It means that the choice can be probabilistic.

If a prospect stochastically dominates over the second one, the rational choice of an individual should be unambiguous. However, if neither of the two prospects dominates, each prospect is in some way better than the other. The individual has incentives to select each of the prospects and it can be assumed that the choice will be probabilistic in nature.

We define the function of the comparative cumulative utility $\psi(\cdot,\cdot)$ on the pairs of positive prospects:

$$\psi(a,b)=\int_0^1 \max\left(V_a(p)-V_b(p),0\right)\omega(p)dp \qquad (2)$$

This function corresponds to an area between the prospects diagrams, where prospect *a* is higher than the prospect *b*, which is represented by the crosshatched area in Fig. 4b (*a* - blue, *b* - red). If the density of the weighting function were identically equal to unity, the value of the comparative utility would be equal to the square of this area. It is obvious that $u(x)-u(y)=\psi(x,y)-\psi(y,x)$.

2.3 Probabilistic extension of the cumulative prospect theory

We denote $x \succ_a y$ (there $a \in (0.5;1]$) of the two alternatives *x* and *y*, an individual chooses *x* with probability *a*, and chooses *y* with probability (1-*a*). Then notion $x \succ_1 y$ will mean that an individual prefers *x* over *y* for sure. We denote *x* ~ *y*, then of the two alternatives *x* and *y* an individual chooses *x* or *y* with equal probabilities (*a* = ½).

Let us consider the following model of probabilistic preference relations. For any pair of alternatives x and y, there are a pair of non-negative values $\psi(x, y)$ и $\psi(y, x)$, which demonstrate the advantage alternatives x over y and advantage alternative y over alternative x. Both of these values can be positive simultaneously as if in some parameters where x is better than y, and in some parameters alternative y is better than x. The probability that an individual selects each of the alternatives is determined by the ratio $\psi(x, y)$ and $\psi(y, x)$.

We say that the probabilistic preference relation is represented by a positive function of comparative utility if there exists (unique up to positive multiplication) a binary function $\psi(\cdot, \cdot)$ such that:

$$x \succ_a y \Leftrightarrow \frac{\psi(x, y)}{\psi(y, x)} = \frac{a}{1-a}, \; a = \frac{\psi(x, y)}{\psi(y, x) + \psi(x, y)} \tag{3}$$

Formally:

$$\begin{cases} \psi(x, y) > \psi(y, x) > 0 & \Rightarrow \quad \dfrac{\psi(x, y)}{\psi(y, x)} = \dfrac{a}{1-a}, \quad a = \dfrac{\psi(x, y)}{\psi(y, x) + \psi(x, y)}, \quad x \succ_a y \\ \psi(x, y) > \psi(y, x) = 0 & \Rightarrow \quad a = \dfrac{\psi(x, y)}{\psi(y, x) + \psi(x, y)} = 1, \quad x \succ_1 y \\ \psi(x, y) = \psi(y, x) & \Rightarrow \quad a = \dfrac{\psi(x, y)}{\psi(y, x) + \psi(x, y)} = \dfrac{1}{2}, \quad x \sim y \end{cases}.$$

Based on the comparative utility function defined by (2), we can determine a probability binary relation on the set of prospects by formula (3).

For lotteries in Fig. 4a (for simplicity, we assume that the density of the weight function is equal to the unity and linear value function: $w(p)=1$ and $V(x)=x$):

$\psi(x, y) = 27, \quad \psi(y, x) = 0$ and $x \succ_1 y$.

$\psi(x, z) = 25, \quad \psi(z, x) = 40$ and $z \succ_{40/65} x$ or $z \succ_{0.62} x$.

Example

We return to the Allais paradox and ask why the preferences of the respondents were divided. Why did the choice of 20% of the respondents in the first poll and 35% in the second poll not coincide with the choice of the majority? Clearly, the respondents may have different value or weight functions. However, according to the probabilistic model constructed above, even with identical value and weight functions the responses could be divided. In fact, as seen in Fig. 3:

$$\psi(x_1, y_1) = \int_0^1 \max\left(V_{y1}(p) - V_{x1}(p), 0\right)\omega(p)dp = \int_{0.8}^1 V(3000\$)\omega(p)dp > 0 \quad \text{and}$$

$$\psi(y_1, x_1) = \int_0^1 \max\left(V_{x1}(p) - V_{y1}(p), 0\right)\omega(p)dp = \int_0^{0.8} \left(V(4000\$) - V(3000\$)\right)\omega(p)dp > 0 \, .$$

That is, with a non-zero probability, x_1 and y_1 can be selected, which can be calculated by formula (3). Similarly, for x_2= ($ 4000, 0.2, 0, $ 0.8) and y_2= ($3000, 0.25):

$$\psi(x_2, y_2) = \int_0^1 \max\left(V_{y2}(p) - V_{x2}(p), 0\right)\omega(p)dp = \int_{0.2}^{0.25} V(3000\$)\omega(p)dp > 0 \quad \text{and}$$

$$\psi(y_2, x_2) = \int_0^1 \max\left(V_{x2}(p) - V_{y2}(p), 0\right)\omega(p)dp = \int_0^{0.2} \left(V(4000\$) - V(3000\$)\right)\omega(p)dp > 0 \, ,$$

which explains why the responses were divided in the second poll.

3. Model of selecting from a set of alternatives by continuous Markov random walks on a set of alternatives

3.1 Continuous Markov Chain Choice

Choosing by means of a continuous homogeneous Markov random walk is performed as follows. A set of process states coincides with a set of alternatives and an individual may “take” any alternative as the best. At any given time he or she can probabilistically move to another state by “taking” another alternative. At the same time, the intensity of the transition depends only on the current state of the alternative that he or she “holds”. In the case of ergodicity of the walk process, the result of the probabilistic choice of probabilistic corresponds to the stationary distribution.

Let us recall that the Markov property states that the conditional probability distribution for the system at the future depends only on the current state of the system, and not additionally on the state of the system in the past.

Let $S = \{s_1, s_2, ..., s_m\}$ – a set of states and the probability of transition from s_i to s_j $i, j \in \overline{1..m}$, $i \neq j$ during time Δt is $\psi(s_j, s_i)\Delta t + o(\Delta t)$, where $\psi(s_j, s_i)$ is the non-negative constant, or the intensity of the transition. This process is called a continuous-time homogeneous Markov chain.

The Markov process is called ergodic if for any initial state there are marginal probabilities of the state process at $t \to \infty$ which do not depend on the initial state. In particular, this condition is satisfied if the vertices of the graph are connected and between any two vertices there is a directed path. That is the stationary probability distribution of the process.

In the case of ergodicity of the walk process, the result of the probabilistic choice corresponds to the stationary distribution.

3.2 Governing equations of CMCC

Let us determine the equation of the CMCC model. If at time t the process is in state s_j with probability $p_j(t)$. Then, at time $t+\Delta t$ it goes into s_i with probability $\psi(s_i, s_j)\Delta t + o(\Delta t)$. Thus, we have:

$$p_i(t+\Delta t) = \left(1 - \sum_{\substack{k=1 \\ k\neq i}}^{m} \psi(s_k, s_i)\Delta t\right) p_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{m} \psi(s_i, s_j)\Delta t \, p_j(t) + o(\Delta t) .$$

Transiting to the limit $\Delta t \to 0$, we obtain a system of differential equations:

$$\frac{d}{dt} p_i(t) = -p_i(t) \sum_{\substack{k=1 \\ k\neq i}}^{m} \psi(s_k, s_i) + \sum_{\substack{j=1 \\ j\neq i}}^{m} \psi(s_i, s_j) p_j(t) \quad \text{for all } i .$$

In the case that a stationary probability distribution exists, the derivatives in the right-hand side vanish when $t \to \infty$. Thus, we obtain a system of linear equations (actually, dependent, rank = m +1):

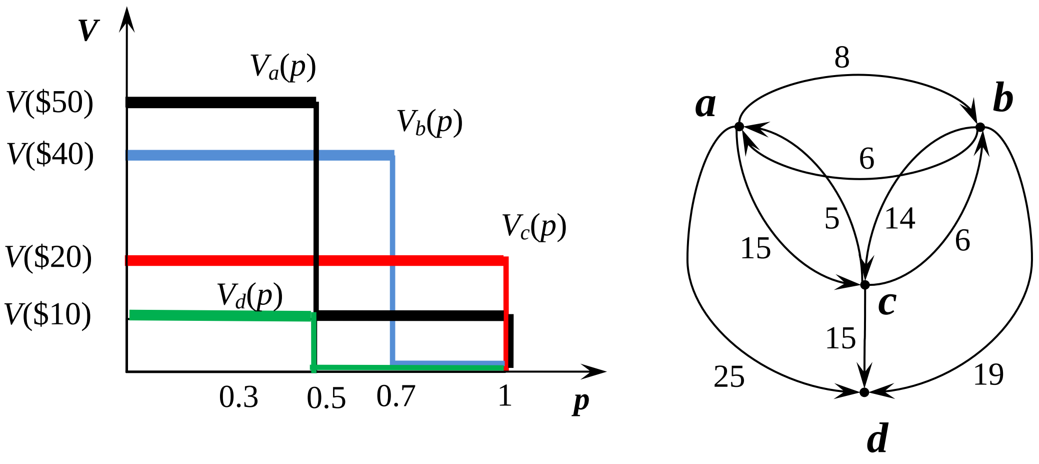
$$\begin{cases} \sum\limits_{\substack{j=1 \\ j\neq i}}^{m} \psi(s_i, s_j) p_j - p_i \sum\limits_{\substack{k=1 \\ k\neq i}}^{m} \psi(s_k, s_i) = 0 \quad \text{for all } i , \\ \sum\limits_{j=1}^{m} p_j = 1. \end{cases} \tag{4}$$

The resulting probability distribution $p = (p_1, p_2, ... p_n)$ is interpreted as the probability that an individual will select either of the alternatives of a set S.

In the case of probabilistic extension of CPT, we will take the intensity of transition for a CMCC model as equal to comparative utility. Note that in case of two alternatives, the probability of selecting received by (4) will equal the probability received by (3).

Example

Let us consider a set of three alternative prospects a = ($50, 0.5; $10, 0.5), b = ($40, 0.7; $0, 0.3), c = ($20, 1.0), and d= ($10, 0.5; $0, 0.5). The graphs of the prospects are shown in Fig. 5a.



a) b)

Fig. 5

For simplicity, assume that $W(p) = p$ and $V(x) = x$. Comparative utility are equal to:

$\psi(a,b)=8,\quad \psi(b,a)=6,\quad \psi(a,c)=15,\quad \psi(c,a)=5,\quad \psi(b,c)=14,\quad \psi(c,b)=6,$
$\psi(a,d)=25,\quad \psi(d,a)=0,\quad \psi(b,d)=19,\quad \psi(d,b)=0,\quad \psi(c,d)=15,\quad \psi(d,c)=0.$

The graph of a Markov random walk is shown in Fig.5b.[5] The defining system of equations for the stationary distribution for (4) is:

$$\begin{cases} (6+5)p_a = 15p_c + 8p_b + 25p_d \\ (8+6)p_b = 6p_a + 14p_c + 19p_d \\ (15+14)p_c = 5p_a + 6p_b + 15p_d \\ (25+19+15)p_d = 0 \\ (\textit{one of the equations can be omitted}) \\ p_a + p_b + p_c + p_d = 1 \end{cases}$$

and has a solution of (0.48, 0.36, 0.16, 0).

NOTE In this example, the order of probabilities of choice alternative coincide with the order of cumulative utilities for alternatives. But this is not necessarily the case in all examples.

In the CMCC model, most probability of choice can be possessed by an alternative of the non-maximal utility. This could occur in a situation where some of the best alternatives are very similar and some are fundamentally different. For example, among the nominees for the film of the year award, there may be two thrillers, a romance movie, and a comedy. And in the struggle for the jury's attention, the similar alternatives (thrillers) may lose to the less useful (interesting), but very different alternative (romance movie).

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[5] In choice theory, it is acceptable to draw the arrow from the edges of a better alternative to the worst, and this tradition is respected. The individual in the process of a random walk goes from the worst alternative to the best alternative. In other words, it moves against the direction of the arrows.

Conclusion

This paper proposed:

• A probabilistic extension of the cumulative prospect theory with its graphic form.

We present a simple visual graphic model of lotteries. In this graphic model it is clear that the concept of stochasticity dominates one lottery over another. If an individual chooses one alternative over two lotteries and no one particular lottery stochastically dominates the other, we can state probabilistic preferences. In the framework of CPT, we calculate comparative utilities; the advantage of the first lottery over another and advantage of the second lottery over the first. These values determine the probabilities of choice for each lottery.

• A model of choice using a continuous Markov random walk for a probabilistic extension of CPT.

Here we consider the task an individual undertakes when selecting one alternative from a set of alternatives between which he has binary probabilistic preferences. In case of a probabilistic extension of CPT, it is proposed to use a model of continuous Markov random walk. The transition probabilities were set equal to comparative utilities, which were already received.

The proposed methods are easy to understand and intuitively clear. Although the models are based on rather crude assumptions and exaggerate the actual process of choosing the individual, they do not require a large amount of initial data for researchers. The obtained results may be quite acceptable for practical use, for example in financial analysis or marketing.

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