National Research University
Higher School of Economics

Presnova Anna Pavlovna

Method of synthesis of suboptimal controls for uncertain nonlinear dynamic systems

PhD Dissertation summary
for the purpose of obtaining academic degree
Doctor of Philosophy in Applied Mathematics

Academic supervisor:
Doctor of Technical Sciences
professor
Afanas’ev Valery Nikolaevich

Moscow - 2020

Statement of the problem

Synthesis of control actions for dynamic objects of different physical nature is a common problem of control theory. Dynamic control objects considered in this dissertation are described by nonlinear differential equations and belong to the class of nonlinear undefined objects. The uncertainty of describing the dynamics of such objects is associated with incomplete information about the state, parameters, and interaction with the environment. Well-known analytical methods for the synthesis of optimal linear object controls for such systems are not applicable. Thus, the design of control systems becomes much more complicated as we move from analysis and control of linear systems to nonlinear systems, since there are still requirements for the stability and optimality of the constructed system. Currently, there is no single method for constructing optimal control of a nonlinear system in the control theory. For the analysis of nonlinear systems, their linearization is often carried out, since for the resulting linearized model, it becomes possible to apply the methods used in the theory of control of linear systems. However, if the linearization is an approximation (for example, the Taylor series linearization when nonlinear terms are discarded), it will usually not be able to provide a synthesis of the global control law for the original non-linearized system. Therefore, it is necessary to use other methods of linearization of systems.

In this paper, the "extended linearization" method will be used to solve this problem. When using the "extended linearization" method, the original system of nonlinear differential equations can be represented in a pseudo-linear form. In this case, the system of equations has a linear structure, but the parameters of the resulting system depend on the state vector. In the literature, this representation of systems is called SDC (State Dependent Coefficient).

Since the main problem in the construction of optimal control in the case of nonlinear systems is the solution of the Hamilton-Jacobi-Bellman partial differential equation, the main search is aimed at constructing its approximate solution. The use of the quadratic quality criterion in problems with unlimited transition time, and the object's model obtained using the "extended linearization" method, allow the control synthesis to move from the need to find a solution to the scalar Hamilton-Jacobi-Bellman partial differential equation to a Riccati-type equation with state-dependent parameters. This method of synthesis of suboptimal control actions for nonlinear control systems is called the SDRE method in the literature (State Dependent Riccati Equation, SDRE [Cimen, 2008]). This method allows you to create effective control structures with nonlinear feedback, preserving the nonlinearities in the system states. However, solving the resulting Riccati equation with parameters that depend on the state of the object is a separate complex task. The computational cost required to accurately implement the SDRE method is its main drawback.

To solve the resulting Riccati equation with state-dependent parameters and search for control actions, a new method for forming optimization algorithms for nonlinear control systems is proposed in this paper.

Relevance

To obtain the most realistic description of the behavior of the control object (aerospace, technological, biomedical), synthesized mathematical models of dynamic processes are systems of nonlinear differential equations. Often such systems do not have complete a priori information about the object parameters and its state at the time of the start of management. Therefore, the problem of control of nonlinear uncertain dynamical systems is relevant for today and has no general solution.

Degree of development of the problem

Currently available methods for analytical design of optimal control systems are of great theoretical importance and are widely used. When using these methods of control synthesis, the assignment of parameters to create an optimal system that minimizes the quality functionality occurs at the design stage. Thus, for analytical methods of designing optimal controls, all information about the object, about external influences on the system, and about the dynamics of internal processes in the system is necessary. In other words, these methods are used in conditions of complete information about the system. An important feature of analytical methods is that when solving a specific problem, a General solution is obtained for an entire class of problems. In addition, these methods are often used for linearized models of initial nonlinear objects.

Linearization of the system can be performed using various methods. However, in order to preserve the properties of nonlinear systems, in recent years, linearization methods have been developed, in which an equivalent representation of the original systems occurs. Such methods include the feedback linearization method [Isidori, 1995; Alvarez-Ramirez et al., 2000; Chetverikov, 2013; Mehra, 2014], as well as the method of "extended linearization" [Pearson, 1962; Cloutier, Cockburn, 2001; Cimen, 2011; Afanasiev, 2015].

Various names are used for the "extended linearization" method in publications related to this topic, for example, "apparent linearization" [Mracek, Cloutier, 1996], "extended linearization" [Cimen, 2008], "representation as a system with state-dependent parameters, SDC" (State-Dependent-Coefficient factorization [Erdem, Alleyn, 2004; Cloutier, Cockburn, 2001]). An important advantage of this method is that all the nonlinearity of the system is preserved. However, this transformation is not unambiguous. To date, the problem of choosing the optimal SDC-representation for the original nonlinear system has not been solved, and there is no general algorithm for investigating the obtained linearized systems for stability.

Modern formulations of optimal control began with problems considered by the American mathematician Richard Bellman and the Russian mathematician Lev Pontryagin in the XX century (M. Atans, M. Falb, R. Kalman, N. N. Moiseev, A. M. Letov, N. N. Krasovsky, A. A. Krasovsky, A. A. Feldbaum, Ya. Z. Tsypkin, F. L. Chernousko, S. V. Emelyanov, and others).

The central concept of Bellman's dynamic programming is the optimality principle. The goal of dynamic programming is to use the optimality principle to represent the dynamic optimization problem as a recurrent relation. In this method, solving the nonlinear optimal control problem entails solving the Hamilton-Jacobi

Bellman partial differential equation, which is difficult to solve analytically. There are several areas of research aimed at approximating the solution of this equation.

When the system dynamics is linear and the penalty functional is quadratic, the Hamilton-Jacobi-Bellman equation is reduced to the Riccati differential equation. In addition, if the control problem in question is an optimal control problem over an infinite time interval, this means that the Riccati differential equation is further reduced to a stationary or algebraic Riccati equation.

In the case of a nonlinear system, it is possible to obtain realizable solutions using algorithmic procedures [Afanasiev, 2008]. "Algorithmic design "[Petrov, Krutko, 1972; Dorf, Bishop, 2004] allows optimizing the system's operation based on input data and external behavior by creating additional circuits.

Currently, there are a large number of methods that use additional chains to compensate for parametric uncertainty. In most cases, the methods used to build these systems do not explicitly depend on the functional quality of the system. These methods are presented, for example, in the works [Mishkin, Brown, 1963; Petrov et al., 1980], as well as many other methods used for the analysis of the system and the construction of additional chains of regularities inherent in this system.

In contrast to algorithms used in various tasks, the proposed method for forming optimization algorithms for undefined nonlinear control systems is based on the use of functions of acceptable values of control actions, Hamiltonians.

The goals and objectives of the research

The purpose of this dissertation is to synthesize optimal and suboptimal control actions in the problem of controlling a nonlinear undefined dynamic object, represented as an SDC mathematical model, for a fixed and unlimited transition time.

To achieve these goals, the following **research objectives** are formulated:

a) ***for control problems of nonlinear objects with a quadratic quality functional and a fixed time interval of the transition process:***

- solve the problem of conditional optimization for an object with restrictions on control actions;

- obtain the condition of uniform asymptotic stability of a system with synthesized control;
- obtain a condition describing the behavior of the Hamiltonian on an optimal trajectory;
- justify the use of the "extended linearization" method to obtain the SDC-model;
- solve the problem of building control using the SDC-model;

b) ***for problems of constructing control of nonlinear objects with a quadratic quality functional for an unlimited transition time:***

- synthesize a regulator with state-dependent parameters that provides uniform asymptotic stability to the control system;
- establish the value of the Hamiltonian achievable with suboptimal control synthesized using the SDC-model;
- develop a new parametric control algorithm that uses the behavior of the Hamiltonian for the appropriate control and state of the system;
- analyze stability of a nonlinear control system when applying the parametric optimization algorithm;

c) ***checking the effectiveness of the received solutions***

- perform synthesis using the obtained theoretical results of control actions for the model of the human immune system in the presence of the HIV virus in the body;
- check the effectiveness of the proposed control with parametric optimization of the controller using mathematical modeling methods.

Personal contribution of the author to the development of the problem

The dissertation presents the results of research performed by the author himself and with his participation. The author's personal contribution is to develop a method for algorithmic construction of control actions that occur at two different speeds depending on the value of the Hamiltonian. The author compared suboptimal control and control with parametric optimization using a model example. The author also personally investigated and solved the problem of synthesis of suboptimal control that controls the supply of drugs in the model of the human immune system in the presence of HIV.

Description of the research methodology

Analytical part of the research work

In this paper, nonlinear objects described by a differential equation of the form were studied

$$\frac{d}{dt}x(t) = f(x(t)) + B(x(t))u(t),\ x(t_0) = x_0, \tag{1}$$

where $x(\cdot) = \{x(t) \in R^n, t \in [t_0, t_f]\}$ - system status; $x(\cdot) \in \Omega_x$, $\Omega_x -$ area (open connected set) R^n, containing the beginning; $x_0 \in X_0 \in \Omega_x -$ set of valid values for initial system conditions; $u(\cdot) = \{u(t) \in R^r, t \in [t_0, t_f]\}$ - control; $f(x(t)), B(x(t))$ - real continuous matrix functions. A type restriction is imposed on the control action $u(t)$

$$\int_{t_0}^{t_f} \|u(t)\|_R^2 \, dt \le E_u, \tag{2}$$

where R - positive definite matrix, E_u - positive number. In the future, the restriction (2) will be written as $u(\cdot) \in U$.

a) The problem of synthesis of control of nonlinear objects with a quadratic quality functional and a fixed time interval of the transition process.

The quadratic quality functional in this problem has the form:

$$J(x(\cdot), u(\cdot)) = \frac{1}{2}x^{\mathrm{T}}(t_f)Fx(t_f) + \frac{1}{2}\int_{t_0}^{t_f} \{x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Ru(t)\}dt, \tag{3}$$

here Q and F - are symmetric, positive semidefinite matrices, and the matrix R is symmetric, positive definite.

Lemma 1. If the optimal control for a nonlinear system (1) exists, then it is unique and is determined by the equation

$$u^0(t) = -R^{-1}B^{\mathrm{T}}(x(t))\left\{\frac{\partial V(t,x(t))}{\partial x}\right\}^{\mathrm{T}}, \tag{4}$$

where the vector $\dfrac{\partial V(t,x(t))}{\partial t}$ corresponds to the solution of the Hamilton-Jacobi-Bellman equation

$$\begin{gathered}\frac{\partial V(t,x(t))}{\partial t} + \frac{\partial V(t,x(t))}{\partial x} f(x(t)) - \\ -\frac{1}{2}\frac{\partial V(t,x(t))}{\partial x} B(x(t))R^{-1}B^{\mathrm{T}}(x(t))\left\{\frac{\partial V(t,x(t))}{\partial x}\right\}^{\mathrm{T}} + \frac{1}{2}x^{\mathrm{T}}(t)Qx(t) = 0, \\ V(t_f, x(t_f)) = \frac{1}{2}x^{\mathrm{T}}(t_f)Fx(t_f).\end{gathered} \tag{5}$$

Here $V(t,x(t))$- is a continuous positive definite scalar function, called the Bellman function $V:\Omega_x \to R^+$, defined as

$$V(s,x) \triangleq \inf_{u(\cdot)\in U}\left[\frac{1}{2}\int_s^{t_f}\left\{x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Ru(t)\right\}dt\right]. \tag{6}$$

Theorem 1. A system (1) with optimal control (4) is uniformly asymptotically stable if and only if:

$$\begin{gathered}\frac{\partial V(t,x(t))}{\partial t} + \\ + \frac{\partial V(t,x(t))}{\partial x}\left[f(x(t)) - B(x(t))R^{-1}B^{\mathrm{T}}(x(t))\left\{\frac{\partial V(t,x(t))}{\partial x}\right\}^{\mathrm{T}}\right] \le \\ \le -\frac{1}{2}x^{\mathrm{T}}(t)Qx(t), \quad \forall x \ne 0.\end{gathered} \tag{7}$$

Consequence from the Hamilton-Jacobi-Bellman equation. Value of the Hamiltonian

$$\begin{gathered}H\left(x(t),u(t),\frac{\partial V(t,x(t))}{\partial x}\right) = \\ = \frac{1}{2}\left[x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Ru(t)\right] + \frac{\partial V(t,x(t))}{\partial x}\left[f(x(t)) + B(x(t))u(t)\right],\end{gathered} \tag{8}$$

corresponding to the managed process $\xi = \left(x(t), u^0(t), t \in \left[t_0, t_f\right]\right)$, defined by the expression

$$H\left(x(t),u^0(t),\frac{\partial V(t,x(t))}{\partial x}\right)=-\frac{\partial V(t,x(t))}{\partial t}. \qquad (9)$$

The paper uses the "extended linearization" method [Erdem, Alleyn, 2004; Cloutier, Cockburn, 2001]. To do this, the necessary assumptions are made, the implementation of which makes it possible to transform the original nonlinear system into a system with a linear structure, but with parameters that depend on the state.

Assumption 1. Suppose that the vector function $f(x(t))$ - is continuously differentiable by $x \in \Omega_x$, that is $f(\cdot) \in C^1(\Omega_x)$, and $B(\cdot) \in C^0(\Omega_x)$.

Assumption 2. Assume that system (1) is managed for all $x(t) \in \Omega_x, t \in R^+$.

Assumption 3. Suppose that $x = 0$ is an equilibrium point of the system at $u = 0$ such that $f(0) = 0$, in addition $B(x(t)) \neq 0,\ \forall x(t) \in \Omega_x$.

Taking into account the assumptions made, the equation of the object model (1) takes the form

$$\frac{d}{dt}x(t) = A(x(t))x(t) + B(x(t))u(t),\ x(t_0) = x_0. \qquad (10)$$

Assumption 4. We assume that the representation of the original nonlinear system as a system with a linear structure, but with parameters that depend on the state (10), is manageable in the range of acceptable values $\left[t_0,t_f\right]\times\Omega_x$, that is, the pair $\left\langle A(x(t)),B(x(t))\right\rangle$ is manageable for all $(t,x)\in\left[t_0,t_f\right]\times\Omega_x$.

Vector $\left[\partial V(t,x(t))/\partial x\right]^{\mathrm{T}}$, required for control synthesis (4), is searched for in the form

$$\left\{\frac{\partial V(t,x(t))}{\partial x}\right\}^{\mathrm{T}} = S(x(t))x(t), \qquad (11)$$

where $S(x(t))$ - positive definite symmetric matrix. Then the optimal control that delivers the minimum to the functional is determined by the relation

$$u^0(t) = -R^{-1}B^{\mathrm{T}}(x(t))S(x(t))x(t), \qquad (12)$$

where the positive definite matrix $S(x(t))$ is the solution of the Riccati type equation

$$\left[\begin{aligned}&\frac{d}{dt}S(x(t))+A^{\mathrm{T}}(x(t))S(x(t))+S(x(t))A(x(t))-\\&-S(x(t))B(x(t))R^{-1}B^{\mathrm{T}}(x(t))S(x(t))+Q+\\&+\left\{\left[\frac{\partial A(x(t))}{\partial x}\right]^{\mathrm{T}}\otimes x(t)+\left[\frac{\partial B(x(t))}{\partial x}\right]^{\mathrm{T}}\otimes u(t)\right\}S(x(t))\end{aligned}\right]x(t)=0, \qquad (13)$$

$$S(x(t_f))=F,$$

here $\otimes$ - is the Kronecker multiplication symbol.

The solution of equation (13) is no less difficult than the solution of the Hamilton-Jacobi-Bellman equation (5), therefore, the construction of an optimal control is an almost impossible task. In further work, a search is made for suboptimal control actions.

b) The problem of synthesis of control of nonlinear objects with a quadratic quality functional for an unlimited time of the transition process.

The quality functionality will look like this

$$\hat{J}(x(.),u(.))=\lim_{t_f\to\infty}\frac{1}{2}\int_{t_0}^{t_f}\left\{x^{\mathrm{T}}(t)Qx(t)+u^{\mathrm{T}}(t)Ru(t)\right\}dt. \qquad (14)$$

Suboptimal management for a system (10) with a quality functional (14) has the form

$$\hat{u}(t)=-R^{-1}B^{\mathrm{T}}(x(t))\hat{S}(x(t))x(t), \qquad (15)$$

here $\hat{S}(x(t))$ - a positively defined symmetric matrix that is a solution to an algebraic Riccati equation whose parameters depend on the state

$$\begin{aligned}A^{\mathrm{T}}(x(t))\hat{S}(x(t))+\hat{S}(x(t))A(x(t))-\hat{S}(x(t))B(x(t))R^{-1}B^{\mathrm{T}}(x(t))\hat{S}(x(t))+\\+Q=0.\end{aligned} \qquad (16)$$

Lemma 3. If the matrix $\hat{S}(x(t))$ - is a solution of the Riccati equation with parameters that depend on the state (16), then the value of the Hamiltonian

$$\begin{aligned}&H\left(x(t),\hat{u}(t),\frac{\partial V(x(t))}{\partial x}\right)=\\&=\frac{1}{2}\left[x^{\mathrm{T}}(t)Qx(t)+\hat{u}^{\mathrm{T}}(t)R\hat{u}(t)\right]+\frac{\partial V(x(t))}{\partial x}\left[f(x(t))+B(x(t))\hat{u}(t)\right],\end{aligned} \qquad (17)$$

will be zero, i.e. $H = 0$, which corresponds to at least the local minimum of the quality functional (14).

Theorem 2. A nonlinear system described by equation (10) with control (15), in which the matrix $\hat{S}(x(t))$ is symmetric positive definite and is a solution of a Riccati-type equation with parameters depending on the state (16), is locally asymptotically stable.

Algorithmic part of the research work

For the synthesis of suboptimal controls of a nonlinear object, a new method for forming optimization algorithms based on the use of functions of acceptable values of control actions is proposed.

The general design of the optimization algorithm is obtained for a nonlinear controlled object described by a differential equation of the form

$$\begin{aligned} &\frac{d}{dt}x(t) = \widetilde{f}(x(t),u(t),\eta(t),a(t)), \\ &x(t_0) = x_0, \end{aligned} \tag{18}$$

here $x(t) \in R^n$ - object state; $u(t) \in R^r$ - control action; $\eta(t) \in \Delta \subset R^k$ - vector of parameters of an object exposed to uncontrolled disturbances, and it is known that $\left|d\eta(t)/dt\right| \le \max\limits_{\eta \in \Delta}\left|d\eta(t)/dt\right| = \sigma > 0$; $a(t) \in \mathrm{A} \subset R^k$ - vector of object parameters that optimize the system operation.

We introduce a scalar function $\Re$ such that

$$\begin{aligned} \Re(t,x(t),u(t),\lambda(t),\eta(t),a(t)) = H(t,x(t),u(t),\lambda(t),\eta(t),a(t)) - \\ - H(t,x^0(t),u^0(t),\lambda^0(t)), \end{aligned} \tag{19}$$

here $H(t,x^0(t),u^0(t),\lambda^0(t))$ - the Hamiltonian for optimal control and the corresponding trajectory of the system.

Thus, the condition

$$\Re(t,x(t),u(t),\lambda(t),\eta(t),a(t)) = 0 \tag{20}$$

there is a necessary condition for the optimal control system.

In our problem, we assume that the necessary optimality conditions (20) are also sufficient optimality conditions. This is done when:

a) the problem has only one minimum and there are no other points of stationarity of the Hamiltonian (for example, a linear object and a quadratic quality criterion);

b) the researcher has information about the acceptable range of changes in control actions, controls from which limit the range of changes in the values of the quality functional so that it is possible to consider the problem with a single minimum.

Theorem 3. Let there be an undefined nonlinear dynamic system (18). It is assumed that in the inner region of the set of parameter values A, if appropriate $u^0(t) \in U$, there are values $a^0(t) \in \mathrm{A}$ at which the specified goal of parametric control is achieved, which corresponds to the fulfillment of the condition

$$\Re(t, x^0(t), u^0(t), \lambda^0(t), \eta(t), a^0(t)) = 0. \tag{21}$$

Then the algorithm

$$\begin{aligned} &\frac{d}{dt}a(t) = -\left\{\frac{\partial H(t, x(t), u(t), \lambda(t), \eta(t), a(t))}{\partial a}\right\}^{\mathrm{T}} \Re(t, x(t), u(t), \lambda(t), \eta(t), a(t)), \\ &a(t_0) = a_0 \end{aligned} \tag{22}$$

provides the initial system with asymptotic stability under parametric optimization, if the condition

$$\begin{aligned} &\left|\Re(t, x(t), u(t), \lambda(t), \eta(t), a(t)) \left\| \frac{\partial H(t, x(t), u(t), \lambda(t), \eta(t), a(t))}{\partial \eta} \right\| \sigma \right| - \\ &-\Re^2(t, x(t), u(t), \lambda(t), \eta(t), a(t)) \left\| \frac{\partial H(t, x(t), u(t), \lambda(t), \eta(t), a(t))}{\partial a} \right\|^2 < 0. \end{aligned} \tag{23}$$

A specific type of algorithm for the synthesis of suboptimal controls of nonlinear systems linearized using the “extended linearization” method can be obtained. To do this, we define the matrix $\hat{S}(x(t))$ in the form

$$\hat{S}(x(t)) = S_0 + s(t), \tag{24}$$

here the matrix S_0 is from the solution of the Riccati equation with constant parameters (at $x(t_0) = x_0$), the matrix $s(t)$ is the matrix of parametric optimization,

$$S(x_0)A(x_0) + A^{\mathrm{T}}(x_0)S(x_0) - S(x_0)B(x_0)R^{-1}B^{\mathrm{T}}(x_0)S(x_0) + Q = 0. \tag{25}$$

The parametric optimization algorithm is described by the following equation:

$$\begin{aligned}
&\frac{d}{dt}s_1(t) = -\left\{\frac{\partial H\left\{x_1(t), u_1(t), \left[S_0 + s_1(t)\right]x_1(t)\right\}}{\partial s_1}\right\}^{\mathrm{T}} \times \\
&\times\left(H\left\{x_1(t), u_1(t), \left[S_0 + s_1(t)\right]x_1(t)\right\} - H\left\{x^0, u^0, S(x^0)x^0\right\}\right), \\
&s_1(t_0) = 0,
\end{aligned} \tag{26}$$

where the Hamiltonian is defined by the expression

$$\begin{aligned}
&H\left\{x_1(t), u_1(t), \left[S_0 + s_1(t)\right]x_1(t)\right\} = \\
&= \frac{1}{2}x_1^{\mathrm{T}}(t)Qx_1(t) + \frac{1}{2}u_1^{\mathrm{T}}(t)Ru_1(t) + x_1^{\mathrm{T}}(t)\left[S_0 + s_1(t)\right]^{\mathrm{T}}\frac{d}{dt}x_1(t).
\end{aligned} \tag{27}$$

Control with parametric optimization has the form

$$u_1(t) = -R^{-1}B^{\mathrm{T}}(x(t))\left[S_0 + s(t)\right]x(t). \tag{28}$$

Study of the effectiveness of the proposed solutions

Verification of the effectiveness of the theoretical results was conducted using mathematical modeling methods. For a model of the human immune system in the presence of the HIV virus in the body [Zurakowski, Teel, 2003, 2006]:

$$\begin{aligned}
&\frac{d}{dt}i(t) = \lambda - di(t) - \beta(1-\eta u)i(t)y(t), \\
&\frac{d}{dt}y(t) = \beta(1-\eta u)i(t)y(t) - ay(t) - p_1z_1(t)y(t) - p_2z_2(t)y(t), \\
&\frac{d}{dt}z_1(t) = c_1z_1(t)y(t) - b_1z_1(t), \\
&\frac{d}{dt}w(t) = c_2i(t)y(t)w(t) - c_2qy(t)w(t) - b_2w(t), \\
&\frac{d}{dt}z_2(t) = c_2qy(t)w(t) - hz_2(t),
\end{aligned} \tag{29}$$

where i - uninfected T-cells of the immune system, T-helpers; y - infected T-helpers (viruses); z_1 - T-killers; w - B-lymphocytes; z_2 - immunoglobulins, memory cells; u - the dose of the injected drug, that is, our effect. The SDC representation was obtained and the control effect was synthesized with parametric optimization. The computer simulation was performed using the MATLAB Simulink package.

The simulation was performed for a given initial state in two modes: in the treatment synthesized by an algorithmic method, and in the absence of any treatment, that is, when $u = 0$. Figure 1 shows the dynamics of healthy T-helper cells (i), cells of the immune system. For the case when the initial state of the patient is critical, the synthesized controls cope with the task and bring the concentration of healthy cells of the human immune system to an acceptable level. In the absence of treatment, the number of healthy cells in a weakened body is reduced to a low level. Figure 2 shows the dynamics of infected immune system cells (y). The constructed control over a short period of time can reduce the concentration of infected cells to 0. Figure 3 shows the behavior of the Hamiltonian when applying a control synthesized using a new algorithm. Transients in the system end quickly enough, and the value of the Hamiltonian is set to 0, which corresponds to Lemma 3.

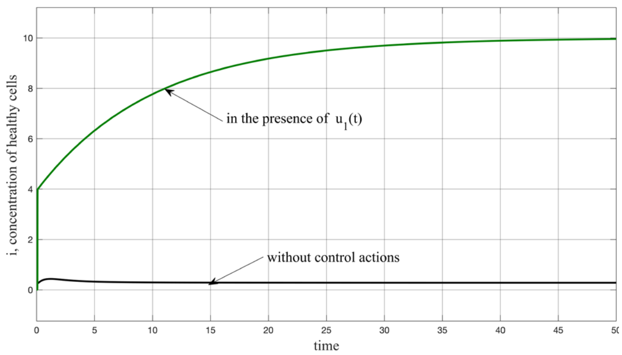

Figure 1. Concentration of healthy cells of the human immune system in the presence of HIV in the body with controls synthesized using a new algorithm, and without control

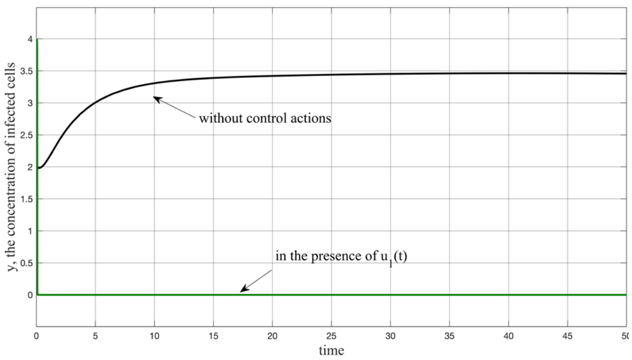

Figure 2. Concentration of HIV-infected cells in the human body under controls synthesized using the new algorithm, and without controls

Figure 4 shows that the matrix $s(t)$, that optimizes the system operation, takes certain values after the end of transients.

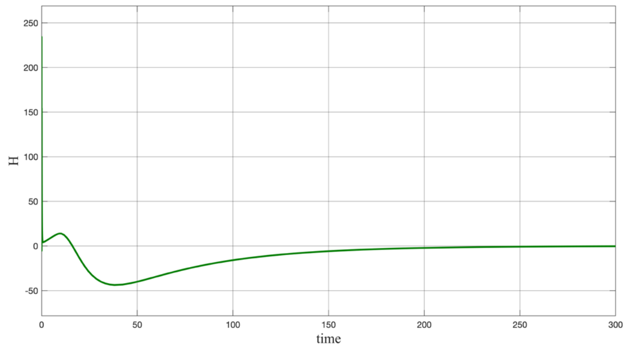

Figure 3. Behavior of the Hamiltonian under control synthesized using the algorithm (26)

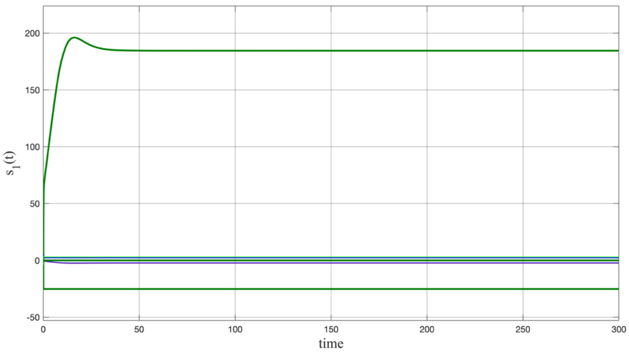

Figure 4. Changing the values of the matrix obtained using the algorithm (26)

The obtained simulation results for the selected mathematical model of the immune system of a person with HIV demonstrate the success of the constructed control using the algorithmic method.

The main results for the thesis defense

A new method is presented for the formation of optimization algorithms for non-linear indefinite control systems based on the property of the Hamilton function to take certain values on an optimal path. A general view of the algorithms is obtained and a condition is obtained under which parametric optimization provides asymptotic stability to the original control system.

For problems of control synthesis by nonlinear dynamic systems represented by mathematical models with a linear structure and state-dependent parameters (SDC-model), with a quadratic quality functional and an undefined control time interval:

1) the value of the Hamiltonian is established, achievable with suboptimal control synthesized using the SDC model;

2) a suboptimal coordinate control algorithm is developed with optimization of the controller parameters, which ensures the stability of a non-linear system with control;

3) the effectiveness of the constructed suboptimal control with parametric optimization of the regulator was verified by mathematical design and subsequent mathematical modeling of the medical treatment system for a person in the presence of HIV virus.

Validity and reliability of results

The reliability of the theoretical results is ensured by the rigor of the proofs of the corresponding lemmas and theorems, and the efficiency of the algorithm is confirmed by the results of computer modeling.

Scientific novelty

In the dissertation, a number of new results were obtained in the field of synthesis of optimal and suboptimal controls for nonlinear indefinite dynamical systems. The necessary conditions are formulated for the successful use of the “extended linearization” method. The synthesis of the basic structure of an optimal and suboptimal regulator with state-dependent parameters has been carried out.

To search for control actions, a new general suboptimal coordinate control algorithm with optimization of controller parameters has been developed in general form, using information about the behavior of the Hamiltonian on a particular trajectory of the controlled system. The constructed parametric optimization algorithm provides asymptotic stability to the system. A specific type of algorithm is obtained for the problem of synthesizing suboptimal controls by nonlinear systems linearized using the “extended linearization” method. The main scientific results obtained in the work are fixed by the corresponding lemmas and theorems.

List of published articles that reflect the main scientific results of the dissertation

The main results of the dissertation research are presented in the following publications:

Articles published by the author in peer-reviewed scientific journals indexed by the international citation databases Web of Science and Scopus:

1. Afanas’ev, V., Presnova, A. Algorithms for the Parametric Optimization of Nonlinear Systems Based on the Conditions of Optimal System // IFAC-PapersOnLine. – 2018. – Vol. 51. – Is. 32. – Ch. 45. – Pp. 428-433.
2. Presnova, A. Algorithmic method for modeling the optimal treatment of patients with HIV // Journal of Physics: Conference Series. –2019. – No. 1163. – Pp. 1-6.

3. Presnova, A., Afanasiev, V. Suboptimal Control of Nonlinear Dynamic System with Unlimited Transition Process Time // IFAC-PapersOnLine. – 2019. – Vol. 52. – No. 17. – Pp. 42-47.

Article published by the author in a peer-reviewed scientific journal included in the list of recommended journals of THE HSE:

4. Afanas'ev, V.N., Presnova, A.P. Formation of optimization algorithms for non-stationary control systems based on the necessary optimality conditions // Mechatronics. Automation. Management. – 2018. – No. 3. – Pp. 153-159.

Article published by the author in another peer-reviewed publication:

5. Presnova, A.P. Optimal and suboptimal control of a nonlinear object using the extended linearization method / / Automation. Modern technology. – 2018. – No. 12. – Pp. 563-569.

The work was approved at the following international and national conferences:

- 7th IFAC Symposium on Systems Structure and Control (SSSC), «Suboptimal Control of Nonlinear Dynamic System with Unlimited Transition Process Time», Sinaia, Romania, 2019;
- International Conference "Optimal Control and Differential Games" dedicated to the 110th anniversary of L. S. Pontryagin, «A new condition for the existence and uniqueness of minimax solutions in the Hamilton-Jacobi-Bellman-Isaacs problem», Moscow, 2018;
- 17th IFAC Workshop on Control Applications of Optimization, «Algorithms for the Parametric Optimization of Nonlinear Systems Based on the Conditions of Optimal System», Yekaterinburg, 2018;
- III International Conference on Computer Simulation in Physics and beyond, «Algorithmic method for modeling the optimal treatment of patients with HIV», Moscow, 2018;
- Scientific and technical conference of students, postgraduates and young specialists of MIEM HSE E.V.Armyansky, «Construction of guaranteeing controls

using the theory of differential games and the SDRE-method in the task of using antiviral drugs», Moscow, 2015;

Theoretical and practical results were used in research work:

- within the framework of grant No.16-8-00522 of the Federal state budgetary institution "Russian Foundation for Fundamental Research" "Analytical and algorithmic methods for synthesizing regulators for nonlinear undefined objects based on the use of the Bellman-Isaacs equations in differential game problems".
- within the framework of grant No.19-08-00535 of the Federal state budgetary institution "Russian Foundation for Fundamental Research" "Analytical and algorithmic methods of the theory of control of undefined nonlinear objects with coordinate-parametric optimization".