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Polina V. Pogorelova

MODELING CRYPTOCURRENCY VOLATILITY USING FINANCIAL MARKET VOLATILITY

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Anatoly A. Peresetsky

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Motivation

In the last fifteen years, cryptocurrencies have emerged and developed rapidly, the most famous of them, Bitcoin, which appeared in 2008, has a capitalization approximately equal to the capitalization of Apple. In addition to Bitcoin, dozens of other cryptocurrencies have emerged, with slightly different properties. Currently, Ethereum is the second-largest cryptocurrency by market capitalization. Cryptocurrencies are decentralized digital currencies that are recorded in accounts called crypto wallets. They are used for conducting certain transactions and operations. The creation and circulation of these currencies are based on mathematical algorithms and asymmetric public key cryptography. This ensures the security and privacy of users (Andryushin, 2020). The main advantages of cryptocurrencies include the anonymity of their owners, the speed of transactions, and decentralization. These features make them resistant to interference from third parties and ensure the integrity of the currency system.

However, anonymity and decentralization also have disadvantages when it comes to cryptocurrency, especially if we look at this issue from the perspective of the state and its legislative framework. This makes many investors question the reliability of cryptocurrency as a means of payment. Despite the fact that the cryptocurrency market has become a full participant in the global financial system for several years, studying its relationship to the behavior of the financial market remains an urgent task for both public authorities and investors in order to better understand and diversify risks associated with this new form of payment.

The increasing importance of cryptocurrencies in the financial sector has also led to an increase in the number of works devoted to the analysis of the dynamics of cryptocurrency exchange rates and volatility and their modeling. Due to the fact that volatility is an unobservable quantity, there is a need to find some approximation for this measure of dispersion of asset returns.

A nonparametric estimate called realized volatility is often used as a measure of the volatility of financial assets for which high-frequency intraday data is available. There is a family of models called HAR-RV (The Heterogeneous Autoregressive model of the Realized Volatility), in which a daily nonparametric estimate of volatility is modeled as dependent on its average values over certain time periods (usually weekly and monthly).

Another popular family of models used to model the volatility of both traditional financial market instruments and digital currencies is the GARCH (Generalized AutoRegressive Conditional Heteroskedasticity). Models from the HAR and GARCH families are actively used to forecast volatility, however, as noted in studies in this area (Aganin et al., 2023; Bergsli et al., 2022; Caporale, Zekokh, 2019), classical econometric models have low predictive ability for cryptocurrencies. These results lead to the question: what metrics can be used to model cryptocurrency volatility and what could potentially improve the quality of its forecast? Probably, the growth of capitalization and the increasing importance of digital currencies indicate the possibility of using indicators from the financial market as predictors of Bitcoin volatility.

In recent years, researchers have increasingly investigated the using of factors that reflect economic, market, and geopolitical uncertainty in modeling and forecasting financial asset returns and volatility. These factors include well-known indicators such as the VIX, as well as indices based on social media data and news headlines.

Another group of factors that can be used in financial modeling and forecasting tasks includes indicators that reflect the degree to which investors are attracted to a financial asset. For example, services for searching information about user requests on the Internet, such as Google Trends and Yandex Wordstat provide quantitative information about keyword searches that can be used to model the characteristics of financial instruments. The use of machine learning methods, including large language models, is actively gaining popularity (Large Language Models, LLM), allowing for analysis of investor sentiment for the purpose of its further use in predicting the return or volatility of a financial asset. The authors of the work (Lopez-Lira., Tang., 2023) note the high potential of hybrid models that combine methods of econometric modeling and machine learning in the tasks of forecasting various financial indicators.

Degree of Problem Elaboration

Since 2017, when the value of the Bitcoin cryptocurrency began to grow rapidly, a large number of papers have appeared in the scientific literature devoted to the analysis of processes associated with digital currencies.

Despite the growing capitalization of the cryptocurrency market and the increasing number of transactions with digital currencies in times of global uncertainty, experts have different opinions about the seriousness of cryptocurrencies, particularly Bitcoin, as participants in the financial system. In general, studies on cryptocurrencies can be grouped into two categories. The first category includes studies that aim to understand the significance of the cryptocurrency market within the financial system and its relationship with the traditional financial market. This information is of interest to governments (including for regulatory purposes) and investors, who may use it to diversify their portfolios and reduce risk.

A significant amount of research into the connection between the cryptocurrency market and the traditional market has confirmed a strong two-way statistical connection between Bitcoin (and other lesser-known digital currencies), on the one hand, and financial indicators, particularly the S&P 500, on the other. It is worth noting that the direction and strength of the connection between stock indices and Bitcoin prices vary. This depends on whether the stock indices belong to developed or developing countries (Ahmed, 2021). Additionally, the different nature of this relationship also depends on whether a country is a member of the G7 or E7 group. Authors of the paper (Aydogan et al., 2022) found unidirectional effects of cryptocurrency markets and stock markets for E7, but bidirectional for G7 (including Bitcoin and S&P 500). A similar conclusion is presented in the paper (Ghorbel, Jeribi, 2021). Using the DCC-GARCH model, they noted the significant effect of the launch of Bitcoin futures in December 2017 and observed an increase in the conditional correlation between stock indices and cryptocurrencies starting in 2020.

Another interesting result was obtained in the article (Uzonwanne, 2021), the author of which shows that in the short-term there was a volatility spillover effect from the S&P 500 to Bitcoin, while in the long-term the volatility spillover effect was bidirectional in both markets.

The second group of studies focuses on obtaining increasingly accurate short-term forecasts for cryptocurrency volatility. This includes further assessment of market risk and Value-at-Risk, the calculation of which is an important task for investors.

The papers (Aganin, 2017; Pichl, Kaizoji, 2017; Bergsli et al., 2022) show the advantage of models from the HAR family compared to models from the GARCH family in the task of short-term forecasting of one-day volatility of financial indicators.

The authors (Caporale, Zekokh, 2019) come to the conclusion that standard GARCH models are low in effectiveness for forecasting the volatility of some popular cryptocurrencies. The dynamics of cryptocurrency market volatility are characterized by long memory and regime switching, as evidenced by research results (Kaya Soylu et al., 2020; Chkili, 2021; Segnon and Bekiros, 2020). These papers demonstrate the effectiveness of the FIGARCH model, which explicitly takes into account long memory, over all other specifications from the GARCH family in the task of forecasting Bitcoin volatility.

Due to the insufficiently high level of accuracy of forecasts of volatility of financial instruments, which is provided by parametric models that use only information about the trajectory of the volatility of an asset at previous time periods, there is a need to obtain new factors for modeling and forecasting the volatility or return of assets.

In a number of works (Wang et al., 2019; Jareño et al., 2020; Simran, Sharma, 2023; Noir, Hamida, 2023), the authors study the impact of various uncertainty indices, such as EPU (Economic Policy Uncertainty Index), VIX (Index CBOE volatility index), TMU (Twitter-based Market Uncertainty Index), TEU (Twitter-based Economic Uncertainty Index) on cryptocurrency returns and volatility. The researchers consider different time periods, some of which include the beginning of the Covid-19 pandemic, which in most cases means there are structural breaks in the data. Most of the studies confirm the statistically significant impact of indices on the returns and volatility of cryptocurrencies, and also show that at different intervals this influence can change both its intensity and direction.

Since 2023, research on related topics has increasingly investigated the potential of machine learning methods. For example, in the article (Lopez-Lira., Tang., 2023), the artificial intelligence chatbot ChatGPT is used to assess sentiment based on the analysis of sentiment headlines, which can be used to obtain more accurate forecasts.

Object and Subject of Research

The object of the thesis research is the volatility of Bitcoin.

The subject of the thesis research is the development of methods for modeling and forecasting the volatility of cryptocurrencies using the volatility of financial markets.

Research Goal and Objectives

The purpose of the research is to model Bitcoin volatility, investigate the relationship between Bitcoin volatility and the volatility of financial market indices and uncertainty indicators, as well as forecasting Bitcoin volatility.

Objectives include:

1. to analyze the degree of knowledge of the problem;
2. to analyze the dynamics of the relationship between realized volatilities of Bitcoin and E-mini S&P 500 futures;
3. to formulate and estimate the specifications of one-dimensional models from the GARCH and HAR families for forecasting the volatility of Bitcoin and E-mini S&P 500 futures;
4. to determine for each asset the best set of model specifications (in terms of the minimum loss function) for forecasting one-step (day) ahead using the MCS test;
5. to estimate and compare the effects of short- and long-term market and economic uncertainty, as measured by indices calculated from data from the social media platform X, on Bitcoin's realized volatility in the pre- and post-COVID periods.

Research Methods and Data

The study uses econometric and time series analysis methods. Models are estimated using the programming language R and the statistical software package EViews. Data processing is implemented using the programming language Python.

The first chapter of the research examines the relationship between the traditional financial market and the cryptocurrency market. The main results of Chapter 1 were published in the article (Manevich, Peresetsky, Pogorelova, 2022).

The E-mini futures and Bitcoin have been chosen as representatives of the traditional financial market and the cryptocurrency market, respectively. The choice of Bitcoin is due to its status as the first and most widely used cryptocurrency. As of 2024, it accounts for more than 40% of the total market capitalization of all cryptocurrencies. The E-mini futures contract for the S&P 500 index is a financial derivative traded on the Chicago Mercantile Exchange (CME) that represents one-fifth of the value of a standard S&P futures contract. This contract is useful for research as it is linked to one of the most well-known and widely followed indices and is actively traded throughout the trading day, providing a large number of data points for studying the relationship between Bitcoin and the E-mini S&P 500. The "state–space" model is used to analyze the global stochastic trend in the dynamics of Bitcoin and E-mini futures S&P 500 in order to compare the behavior of the cryptocurrency and financial markets. This model was also used in (Peresetsky, Pogorelova, 2020) to analyze the dynamics of the volatilities of various financial indices, taking into account their asynchrony due to the different time zones of the financial markets.

According to the state–space model, the logarithmic realized volatility can be represented as the sum of the global and local components:

$$\ln RV_t = \ln RV_{G,t} + \ln RV_{L,t},$$

where $\ln RV_t$ — logarithm of the daily realized volatility per day t; $\ln RV_{G,t}$ — global volatility occurs under the influence of news and events that simultaneously affect both the financial market and the cryptocurrency market; $\ln RV_{L,t}$ — local volatility, which occurs under the influence of news and events that affect only the financial market or the cryptocurrency market.

To estimate the unknown parameters of the model, the Kalman filter method is used. The general formulation of this method in matrix form is:

$$y_t = \alpha + Hs_t + \varepsilon_t, \text{ (observation equation)}$$

$$s_t = Fs_{t-1} + u_t, \text{ (equation of states)}$$

where y_t — $(n \times 1)$ vector of observations; s_t — $(m \times 1)$ vector of states; H, F — matrices of dimensions $(n \times m)$ and $(m \times m)$, respectively; $u_t \sim N(0, \Sigma_u)$ — $(m \times 1)$ random vector; $\varepsilon_t \sim N(0, \Sigma_\varepsilon)$— $(n \times 1)$ random vector.

The linear Kalman filter method assumes that the variable y_t in the observation equation represents the observable component and the variable s_t in the state equation represents the unobservable component.

The logarithmic realized volatility of Bitcoin and E-mini S&P 500 futures are used as an observable value. The logarithms of the realized volatilities of assets are decomposed into a global component, which is a linear function of the unobserved stochastic trend x_t, and local (residuals) components for the volatility of Bitcoin (ε_{1t}) and E-mini S&P 500 (ε_{2t}):

$$\begin{bmatrix} \ln RV_{BTC,t} \\ \ln RV_{S\&P,t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} x_t + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \quad (1)$$

$$x_t = x_{t-1} + u_t . \quad (2)$$

where x_t is the global stochastic trend; random variables $\varepsilon_{1t} \sim N\left(0,\sigma_1^2\right)$, $\varepsilon_{2t} \sim N\left(0,\sigma_2^2\right)$ and $u_t \sim N\left(0,\sigma_u^2\right)$ are normally distributed and independent for all t, and for all t,s it is assumed that $E\left(\varepsilon_{1t}\varepsilon_{2s}\right) = E\left(u_t\varepsilon_{1s}\right) = E\left(u_t\varepsilon_{2s}\right) = 0$. To make the model identifiable, the variance of the random variable u_t is set to 1, i.e. $Var\left(u_t\right) = 1$.

The estimate of the unobserved global component $\hat{x}_t$ is calculated using the Kalman filter as an estimate of the conditional expectation of x_t with all the information available at the time t. Model parameters (1)–(2) are estimated using the maximum likelihood method.

The model is estimated using five-minute data on the Bitcoin exchange rate (GDAX exchange) and the E-mini S&P 500 futures (Chicago Exchange). The data source is the financial portal *finam.ru.*[1] The data includes the period from 01/01/2018 (00:00) to 12/29/2021 (24:00) (UTC, Coordinated Universal Time).

Initially, model parameters (1)–(2) are estimated over the entire time interval under consideration. Next, in a rolling window with a width of 120 observations with a step of 7 observations, the share of the global component in the variance of the logarithm of the realized volatility of each asset is calculated. To take into account the possible heterogeneity of the period under consideration, the model is also estimated in a rolling window with a width of 120 observations and the share of the variance of the common global component in the variance of the logarithm of the realized volatility of each asset is calculated.

Analysis of the dynamics of the global stochastic component and its contribution to the volatility of E-mini S&P 500 and Bitcoin futures has led us to propose a hypothesis regarding the existence of volatility connections between the cryptocurrency market and the traditional stock market.

In the second chapter of the thesis, parametric models from two families, GARCH and HAR, are used to get a one-step ahead forecast of the realized volatility of Bitcoin

[1]https://www.finam.ru/

and E-mini S&P 500. The main findings of Chapter 2 are presented in the article (Aganin, Manevich, Peresetsky, Pogorelova, 2023).

In all specifications of GARCH models, the return of a financial instrument is described by an AR(1) process without a constant:

$$r_t = \phi r_{t-1} + \varepsilon_t, \tag{3}$$

where r_t is the return at the moment t; ϕ is a parameter.

The standard GARCH (p,q) model (Bollerslev, 1986) is specified by

$$\varepsilon_t = \sigma_t z_t, \tag{4}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{5}$$

where σ_t^2 is the conditional variance at the moment t; r_t is the return at the moment t; $\phi, \theta, \alpha, \beta$ are parameters estimated by the maximum likelihood method; z_t is random noise (in the classical model, the standard normal distribution is used for z_t).

In addition to the standard model, 9 more GARCH specifications are considered in this work. For each of the specifications, the model is estimated with each of nine conditional normalized error distributions $z_t = \varepsilon_t / \sigma_t$. Thus, a total of 810 models from the GARCH family are estimated.

HAR models use daily, weekly, and monthly volatility components to account for the long-term volatility memory and market heterogeneity.

The standard HAR (*w*,*m*) model (Corsi, 2009) defined by

$$RV_{t+1}^d = \beta_0 + \beta_1 RV_t^d + \beta_2 RV_t^w + \beta_3 RV_t^m + \varepsilon_{t+1}, \tag{6}$$

where RV_t^d is the realized volatility per day t;

$RV_t^w = \frac{1}{w} \sum_{j=0}^{w-1} RV_{t-j}^d$ — weekly component of realized volatility per day t, calculated as the average for the current and previous $w-1$ days;

$RV_t^m = \frac{1}{m} \sum_{j=0}^{m-1} RV_{t-j}^d$ — monthly component of realized volatility per day t, calculated as the average for the current and previous $m-1$ days;

ε_{t+1} — random error at the moment $t+1$.

In model (6) for the stock index, the parameters are used $w = 5, m = 21$. Since cryptocurrency trading differs from regular exchange trading, this work considers all pairs (m, w), where $4 \leq w \leq 7$ and $21 \leq m \leq 27$.

In addition to the standard HAR model, the work also considers its various specifications, which allow taking into account the features of the series. Moreover, in HAR models, each specification is considered for realized volatility, its logarithm value and the square root of realized volatility. In total, 138,936 models from the HAR family were evaluated in the study. The GARCH (1,1) model with normally distributed errors and HAR (5;21) were chosen as a benchmarks.

Models are estimated during the period from January 1, 2018 to December 29, 2021 (inclusive), all data with a frequency of 5 minutes are taken from the website *finam.ru*. To select the optimal (in terms of the loss function) model, MCS is used (Model Confidence Set) test. The results obtained in the second chapter allowed us to identify a class of models that can provide the most accurate forecasts both for a representative of the financial market and for Bitcoin. Optimal parameters w and m were found for models from the HAR family for the cryptocurrency market as well.

The third chapter of the study focuses on the exogenous factors that may affect the volatility of cryptocurrencies. The issue of the difference in the relationship between the realized volatility of Bitcoin and the factors under consideration in the periods before and after the beginning of the Covid-19 pandemic is also being investigated. The main results of Chapter 3 are presented in the article (Pogorelova, 2024). Given the increased interest in recent years in various uncertainty indices and their use in forecasting the characteristics of financial assets, their impact on the Bitcoin's volatility is investigated using the ARDL model.

As an estimate of volatility, as in previous parts of the study, realized volatility is used, calculated from five-minute data on digital currency prices. The paper considers three uncertainty indices: VIX, TEU_ENG and TMU_ENG. The CBOE Volatility Index (VIX) is an indicator of market conditions and reflects investor sentiment. The other two indices, TEU_ENG and TMU_ENG, measure the level of economic and market uncertainty, respectively, using information from the social network X (formerly Twitter, blocked in Russia).

The logarithmic return of the E-mini S&P 500 futures $\left(SPMINI\_RET_t\right)$, the price of WTI crude oil $\left(WTI_t\right)$, and the EUR/USD exchange rate $\left(EUR\_USD_t\right)$ are considered as control (exogenous) variables. These variables were chosen based on the results of the research (Nouir, Hamida, 2023).

ARDL model ($p, q_1, q_2, \ldots, q_6$) is specified by

$$BTC\_RV_t = \nu + \sum_{i=1}^{p}\delta_i BTC\_RV_{t-1} + \sum_{i=1}^{q_1}\beta_j^1 SPMINI\_RET_{t-j} + \sum_{i=1}^{q_2}\beta_i^2 WTI_{t-i} +$$

$$+\sum_{i=1}^{q_3}\beta_i^3 VIX_{t-i} + \sum_{i=1}^{q_4}\beta_i^4 TEU\_ENG_{t-i} + \sum_{i=1}^{q_5}\beta_i^5 TMU\_ENG_{t-i} +$$

$$+\sum_{i=1}^{q_6}\beta_i^6 EUR\_USD_{t-i} + \varepsilon_t, \tag{7}$$

where ε_t is a random error at time t. The parameters $p, q_1, q_2, \ldots, q_6$ are selected based on the Akaike information criterion.

Model (7) can be represented as ECM (Error Correction Model) as follows:

$$\Delta BTC\_RV_t = \mu + \alpha_1 \Delta SPMINI\_RET_t + \alpha_2 \Delta WTI_t + \alpha_3 \Delta VIX_t + \alpha_4 \Delta TEU\_ENG_t +$$

$$+\alpha_5 \Delta TMU\_ENG_t + \alpha_6 \Delta EUR\_USD_t + \sum_{i=1}^{p-1}\delta_i \Delta BTC\_RV_{t-i} + \sum_{i=1}^{q_1-1}\beta_i^1 \Delta SPMINI\_RET_{t-i} +$$

$$+\sum_{i=1}^{q_2-1}\beta_i^2 \Delta WTI_{t-i} + \sum_{i=1}^{q_3-1}\beta_i^3 \Delta VIX_{t-i} + \sum_{i=1}^{q_4-1}\beta_i^4 \Delta TEU\_ENG_{t-i} + \sum_{i=1}^{q_5-1}\beta_i^5 \Delta TMU\_ENG_{t-i} +$$

$$+\sum_{i=1}^{q_6-1}\beta_i^6 \Delta EUR\_USD_{t-i} + \gamma_0 BTC\_RV_{t-1} + \gamma_1 SPMINI\_RET_{t-1} + \gamma_2 WTI_{t-1} + \gamma_3 VIX_{t-1} +$$

$$+\gamma_4 TEU\_ENG_{t-1} + \gamma_5 TMU\_ENG_{t-1} + \gamma_6 EUR\_USD_{t-1} + \varepsilon_t. \tag{8}$$

The coefficients γ_j are the long-term effects; short-term effects correspond to coefficients α_j.

This part of the study is based on data from 01/02/2018 to 12/30/2022. The source of five-minute data on Bitcoin prices, daily data on WTI oil prices, E-mini S&P 500 futures prices, as well as VIX index values is website *finam.ru*. Daily data on the values of the uncertainty indices TEU_ENG and TMU_ENG were obtained from *policyuncertainty.com*.

For substantive reasons, the entire period is divided into two parts: before and after the beginning of the Covid-19 pandemic. As an auxiliary tool, the Bai–Perron test for the presence of structural breaks (Bai, Perron, 2003) was conducted, which indicated the date of the structural break — March 1, 2020. Note that the using of this test is not entirely correct, since it assumes the stationarity of time series. However, the test showed a reasonable break date. Considering that the power of tests for dates of structural breaks is not high, a completely satisfactory result has been obtained.

The period under consideration was divided into two parts – the pre-COVID period (from January 2nd, 2018 to February 28th, 2020) and the post-COVID period (March 2nd, 2020 – December 31st, 2022).

Using representation (8), we analyzed the short- and long-term effects of the CBOE volatility index (VIX) and indices of economic and market uncertainty, calculated based on data obtained from the social network X (formerly Twitter, blocked in Russia), on the realized volatility of Bitcoin at two intervals – pre-Covid and post-Covid.

Scientific Novelty

1. Using the “state–space” model, the global stochastic component of the financial market and the cryptocurrency market was simulated and analyzed (using the example of the largest representatives of these markets – the E-mini S&P 500 futures and Bitcoin).
2. Based on the results of the "state–space" model estimation, a hypothesis about volatility spillover effect between the financial and cryptocurrency markets was proposed and confirmed.
3. Using the MCS test, a comparison was made of a large number of model specifications from the GARCH and HAR families to forecast the volatility of Bitcoin and E-mini S&P 500 futures at one step (day ahead). The optimal values of the parameters of the weekly and monthly components in models from the HAR family for the cryptocurrency market were identified.
4. Using the ECM model, the short- and long-term impact of the volatility index (VIX), as well as market and economic uncertainty indices, on the realized volatility of Bitcoin in the pre- and post-Covid periods were analyzed.

Main Findings and Contributions

1. A “state-space” model is proposed to isolate the common part of the realized volatility of two financial assets that are traded 24 hours a day – Bitcoin and the E-mini S&P 500 futures, which in this work represent the cryptocurrency and traditional financial markets, respectively. These assets are traded 24 hours a day. Analysis of the results from the estimation of the model using a rolling window has allowed us to formulate a hypothesis regarding the existence of volatility spillover effect between financial markets and the cryptocurrency market. It has been concluded that during times of sharp increases in the price of Bitcoin, the proportion of the global component in the realized volatility of Bitcoin rises, which suggests that during such times, the cryptocurrency market's behavior and structure resemble that of the traditional stock market. Thus, it can be assumed that as the popularity of Bitcoin and other cryptocurrencies continues to grow, their capitalization will also increase. This will lead to a closer similarity between the relatively new cryptocurrency market and the traditional financial market. This similarity could simplify the analysis of cryptocurrencies and the economic processes they represent, in the event that they become more widespread.

2. In order to predict volatility for Bitcoin and E-mini S&P 500, models from the GARCH and HAR families were considered. Taking into account all specification options, a total of 810 GARCH models and 46,312 HAR models were estimated. Each HAR model specification was estimated for realized volatility, its logarithm value, and the square root of realized volatility, resulting in a total of 138,936 HAR models in the comparison. Using the MCS test, which allows comparing a large number of models in terms of the loss function, the best (in terms of the minimum loss function) models for one-step forecasting for both assets were selected. It is shown that models from the HAR family selected by the MCS test are superior to models from the GARCH family selected by the MCS test for realized volatility. in the accuracy of forecasting realized volatility one-step ahead for both Bitcoin and the E-mini S&P 500 futures. It is worth noting that for both time series under study, the best results were shown by models from the HAR family for logarithmized realized volatility (HAR-ln(RV)), which is consistent with the lognormal nature of realized volatility.

3. It is established that due to the heterogeneity of the cryptocurrency market, HAR models provide better relative accuracy in forecasting realized volatility for Bitcoin than for the E-mini S&P 500. The smallest mean absolute percentage error (MAPE) that were achieved in the work when forecasting one-step (day) ahead are 29.51% and 36.12% for Bitcoin and E-mini S&P 500, respectively.

4. Using the ECM model (Error Correction Model), the relationship between Bitcoin's realized volatility and the CBOE volatility index (VIX), as well as economic and market uncertainty indices, was analyzed based on data from the social network X (formerly Twitter, blocked in Russia). The logarithm return of the S&P 500 index, the WTI oil price, and the EUR/USD exchange rate were also included in the model as control variables. The set of control variables was based on published studies on relevant topics. The results of the ECM model estimation differ between the pre-COVID and post-COVID periods, which were identified in the study based on the testing of the hypothesis of a structural change in the data using the Bai–Perron test (Bai, Perron, 2003). In both periods, significant long-term effects of the VIX index, WTI oil price, EUR/USD exchange rate, and cryptocurrency volatility on realized Bitcoin volatility were identified. In the pre-COVID and post-COVID periods, a negative significant effect of the VIX index on Bitcoin volatility was found. Similarly, a negative significant impact of uncertainty on Bitcoin volatility was observed in (Noir, Hamida, 2023). In the short term, during the pre-COVID and post-COVID periods, a significant positive effect has been observed for the market uncertainty indicator (TMU_ENG). Additionally, a significant short-term positive effect of the VIX index has also been found in the pre-COVID period. This is noteworthy, as in the long run, this effect is significantly negative. The results of the study support the existence of a connection between the VIX and TMU_ENG indices and the realized volatility of Bitcoin. The direction of this relationship varies in the short- and long-term, as well as depending on the time period under study. These findings can be applied in models for forecasting the volatility of Bitcoin using additional indicators (uncertainty indices).

Structure of the Thesis

The thesis research consists of an introduction, three chapters, conclusion and bibliography.

The introduction provides an overview of the relevance, object, and subject of the research. It also outlines the purpose and tasks necessary to achieve this goal. The analysis of the development of the stated problem, the methodology used, the scientific novelty, and the main findings are presented.

The first chapter is devoted to the analysis of the connection between the financial and cryptocurrency markets using the example of E-mini S&P 500 and Bitcoin. Chapter 1 reviews the literature in the relevant area and introduces the definition of realized volatility, which is one of the key terms in this work. The chapter contains a detailed description of the state-space model, which allows us to identify the global stochastic trend for Bitcoin and E-mini S&P 500 futures and analysis of the obtained assessment results.

The second chapter of the study is focused on parametric approaches to modeling the volatility of a classical financial instrument and the Bitcoin. The chapter contains a literature review on the using of GARCH and HAR models in forecasting the volatility of various financial instruments. The description of the specifications of the models used is presented in detail, and the conclusions obtained based on the estimation results are contained.

The third chapter of the study focuses on the analysis of the impact of various uncertainty indices, such as economic and market factors, on the volatility of financial assets. It also examines the usefulness of these indices in forecasting the characteristics of financial instruments. Chapter 3 describes the methodology, including the ARDL model applied to Bitcoin's realized volatility. The main findings from the estimation of this model are presented, with a focus on how uncertainty indices affect the volatility of digital currencies in both the pre-COVID and post-COVID periods.

The conclusion contains a discussion of the results obtained during the study.

The text of the work is presented on 90 pages, contains 13 figures and 12 tables. The list of references includes 74 sources.

Approbation of Research Results

The main conclusions of the thesis research are the result of econometric modeling and time series analysis. The instrumental methods used in this study correspond to the academic standards accepted in modern scientific literature.

Conferences

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4. Modern Econometric Tools and Applications – META2023 (Nizhny Novgorod, 2023). Presentation: Investigation of the impact of uncertainty indicators on Bitcoin volatility using ARDL model.

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Publications

1. Peresetsky A. A., Pogorelova P. V. (2020). Extracting the global stochastic trend from non-synchronous data on the volatility of financial indices. *Applied Econometrics*, 57, 53–71. DOI : 10.22394/1993-7601-2020-57-53-71.

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