Committee decisions: optimality and equilibrium

Jean-Francois Laslier and Jörgen Weibull
Ecole Polytéchnique and Stockholm School of Economics

June 3, 2009

or…

Is democracy good?

- The French revolutionary Condorcet answered “yes”!
- Here it will be argued: yes, if we do it right!

Many important decisions are not made by individuals

1. Company: investment opportunity
2. Central-bank: monetary policy, the interest rate
3. University: Appoint a new faculty member, pass a Ph D dissertation
4. Court: Jury decision
5. Hospital: Medical treatment of a patient
6. Environmental emergency unit: Intervention decision

Usually:

1. General agreement in the committee/board/group about the goal
2. But not full information about the state of nature
3. Both public and private information

Questions:

Q1: What collective decision rule is optimal?

Q2: Given a collective decision rule, how will individual members act?

Q3: Will judgement aggregation be efficient?

- These are questions of utmost importance

  - mistaken decisions can have devastating consequences

- And yet the practice is frequently ad hoc and procedures ill understood

  - some theory results, but many open questions

  - is current theory empirically valid?

- Need for:

  1. More behavioral game theory models of group decisions

  2. More laboratory experiments and field studies of group decisions

The pioneering result:

The Condorcet Jury Theorem

1. "*Marie Jean Antoine Nicolas de Caritat,* **marquis de Condorcet** *(1743–1794), was a French philosopher, mathematician, and early political scientist... He advocated a liberal economy, free and equal public education, constitutionalism, and equal rights for women and people of all races... He died a mysterious death in prison after a period of being a fugitive from French revolutionary authorities.*" (Wikipedia)

2. "*If voters have some relevant information and vote according to this, then majority rule will lead to higher and higher probability for the right collective decision as the number of voters increases, and this probability will converge to 1.*" (Condorcet, 1785)

Modern game-theoretic analysis of group decisions

[Austen-Smith and Banks (1996) and followers]

- n members of a *committee* (board, jury, group)

- Binary collective decision: $x \in X = \{0, 1\}$

- Two states of nature: $\omega \in \Omega = \{0, 1\}$

- All members agree that decision $x = \omega$ is right

- The state ω is unknown at the time of decision

- Common prior: $\mu = \Pr[\omega = 1]$

  - The result of shared public information

- Each member i receives a private signal $s_i \in \{0, 1\}$, where

$$\Pr[s_i = \omega \mid \omega] = q_\omega > 1/2$$

- Signals are *conditionally independent*, given ω

- Note q_ω is the same for all committee members: they are "equally competent"

Example 0.1 *Consider* $\mu = 0.5$, $q_0 = 0.9$, $q_1 = 0.8$, *and majority rule. Each member wishes to maximize the probability* $\Pr(x = \omega)$ *for a correct collective decision (equal costs for mistakes of types I and II)*

Case I: $n = 1$

How would you vote if your private signal were $s_1 = 0$*?*

Bayes' law:

$$\Pr[\omega = 0 \mid s_i = 0] = \frac{\Pr[s_i = 0 \mid \omega = 0] \cdot \Pr[\omega = 0]}{\Pr[s_i = 0]}$$
$$= \frac{(1-\mu)\, q_0}{(1-\mu)\, q_0 + \mu\,(1 - q_1)} \approx 0.82 > 0.5$$

Case II: $n = \#$ *participants in this room. Majority rule.*

How would you vote if your signal were $s_i = 0$*?*

Suppose *that all others vote according to their private information*

Would you then do likewise?

Bayesian rationality: *Condition not only on your private signal* $s_i = 0$ *but also on the event that your vote is pivotal (a tie among the others):*

$$\Pr[\omega = 0 \mid \mathcal{T} \wedge s_i = 0] = \frac{(1-\mu)\Pr[\mathcal{T} \ \wedge \ s_i = 0 \mid \omega = 0]}{\Pr[\mathcal{T} \ \wedge \ s_i = 0]}$$

$$[for \quad n = 7:] = \frac{q_0^4 (1-q_0)^3}{q_0^3 (1-q_0)^3 + q_1^3 (1-q_1)^4} \approx 0.42 < \frac{1}{2}...$$

Hence, you should vote $v_i = 1$*, against your private information!*

Conclusion: *Informative voting is* **not** *a Nash equilibrium*

- *not even when committee members have* **identical** *preferences*

Example 0.2 *Replace majority rule by the* **unanimity rule***: collective decision* $x = 1$ *iff* $\sum v_i = n$.

Suppose that all others vote informatively. How would you now vote?

Bayesian rationality*: If you received* $s_i = 0$*, then you should condition on the event of* 1 *signal 0 and* $n - 1$ *signals 1* $\Rightarrow$ *you should vote* $v_i = 1$*, again against your private information!*

Further difficulties:

1. Multiple Nash equilibria

2. Heterogeneity in valuation of errors of type I and II ("hawks" and "doves")

3. Complex motivations (ethical and/or social preferences)

4. Bounded rationality (mistakes possible)

This discussion [based on Laslier and Weibull, 2009]:

1. Allow for preference heterogeneity and distinct individual priors

2. Characterize optimal collective decision rules

3. Characterize Nash equilibria under any given voting rule

4. Propose a new voting rule that

   (i) has a unique and strict equilibrium and

   (ii) which gives asymptotically efficient judgment aggregation

5. Committee members with bounded rationality

6. Committee members with complex motivations

7. Conclusion

Literature

1. Austen-Smith and (1996): “Information aggregation, rationality, and the Condorcet Jury Theorem”, *American Political Science Review*.

2. Glazer Jacob and Ariel Rubinstein (1998): “Motives and implementation: on the design of mechanisms to elicit opinions”, *Journal of Economic Theory*.

3. Myerson, Roger (1998): “Extended Poisson games and the Condorcet jury theorem”, *Games and Economic Behavior*.

4. Coughlan (2000): “In defense of unanimous jury verdicts: mistrials, communication, and strategic voting” *American Political Science Review*.

1 The model

- n committee members facing a binary collective decision

- Two states of nature: $\omega \in \Omega = \{0, 1\}$

- Two decision alternatives $x \in \{0, 1\}$

- Common prior: $\mu = \Pr[\omega = 1] \in (0, 1)$

  - the analysis permits distinct individual priors μ_i [cf. Dixit and Weibull (2007)]

- Each voter i receives a private signal $s_i \in \{0, 1\}$, where

$$\begin{aligned} \Pr[s_i = 0 \mid \omega = 0] &= q_0 > 1/2 \\ \Pr[s_i = 1 \mid \omega = 1] &= q_1 > 1/2 \end{aligned}$$

- Signals are *conditionally independent*, given ω

- i's *von Neumann - Morgenstern utilities*:

	$\omega = 0$	$\omega = 1$
$x = 0$	u_0^i	$u_1^i - \alpha_i$
$x = 1$	$u_0^i - \beta_i$	u_1^i

  $\alpha_i = i$'s valuation of *error of type I*

  $\beta_i = i$'s valuation of *error of type II*

- Only α_i and β_i will matter. *Summary parameter* for each member i:

$$\gamma_i = \frac{\mu \alpha_i}{(1 - \mu) \beta_i} > 0$$

- A *voting rule:* a function $f : \{0, 1\}^n \to [0, 1]$ that maps each *vote profile* $v = (v_1, ..., v_n)$ to a probability $f(v)$ for decision $x = 1$

- A *voting strategy* for committee member i: a function $\sigma_i : \{0,1\} \rightarrow [0,1]$ that maps i's signal s_i to a probability $\sigma_i(s_i)$ for a vote on alternative 1

- *Informative* voting: to always vote according to one's signal, $\sigma_i(s_i) \equiv s_i$

- *Sincere* voting: to always vote for the alternative that maximizes one's expected utility, conditional upon one's signal

1.1 Signal informativeness

- Suppose that committee member i were to make the decision single-handedly

- If the signal is noisy and the prior and mistake costs favor one alternative over the other, the right decision may well be *not* to follow one's signal

- By Bayes' law: optimal to *vote informatively* if and only if

$$\frac{1-q_0}{q_1} \leq \gamma_i \leq \frac{q_0}{1-q_1} \tag{1}$$

- This *signal-informativeness* condition will henceforth be assumed to hold strictly $\forall i$

- Under signal-informativeness:

$$\text{sincere voting} = \text{informative voting}$$

1.2 Condorcet's theorem

- An odd number of voters n

- Majority rule:

$$f(v) = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} v_i > n/2 \\ 0 & \text{if } \sum_{i=1}^{n} v_i < n/2 \end{cases}$$

Theorem 1.1 (Condorcet) *Suppose that all vote informatively. Let* $X_n(\omega) \in \{0,1\}$ *be the collective decision under majority rule in state* ω*. Then*

$$\lim_{n \to \infty} \Pr[X_n(\omega) = \omega] = 1 \quad \forall \omega \in \{0,1\}$$

- *Asymptotically efficient judgement aggregation*

- **But** is informative voting compatible with equilibrium?

2 Optimality

- Let D be the set of *deterministic collective decision rules*, functions d that map signal vectors, $s = (s_1, ..., s_n)$, to collective decisions.

- Example: for each $k = 1, ..., n$, let

$$f^k(s) = \begin{cases} 1 & \text{if } \sum_{i=1}^n s_i \geq k \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.1 *A rule $d \in D$ is* **optimal** *if it maximizes expected committee welfare,*

$$W(d) = \sum_{x,\omega,i} \Pr\left[(x,\omega) \mid x = d(s)\right] \cdot u^i_{x\omega}$$

Lemma 2.1 $\exists k$ *such that f^k is optimal.*

- Let $\bar{\alpha}_n = \frac{1}{n}\sum_{i=1}^{n}\alpha_i$, $\bar{\beta}_n = \frac{1}{n}\sum_{i=1}^{n}\beta_i$ and $\bar{\gamma}_n = \mu\bar{\alpha}_n / \left(1-\mu\right)\bar{\beta}_n$

  "a *fictitious representative member*"

Theorem 2.2 *f^k is optimal if and only if*

$$g\left(k,n\right) \leq \bar{\gamma}_n \leq g\left(k-1,n\right) \quad (2)$$

where

$$g\left(k,n\right) = \left[\frac{\left(1-q_0\right)\left(1-q_1\right)}{q_0 q_1}\right]^k \left(\frac{q_0}{1-q_1}\right)^n$$

- Note: The inequality is met by at least one k, and generically by only one k

Corollary 2.3 *If n is odd and $q_0 = q_1$, then majority rule is optimal.*

3 Equilibrium

- Consider simultaneous voting by all n members of the committee

- The rule f^k applied to vectors $v = (v_1, ..., v_n)$ of votes is called *k-majority rule*

Theorem 3.1 *Sincere voting under k-majority rule is a Nash equilibrium if and only if*

$$g(k, n) \leq \gamma_i \leq g(k-1, n) \qquad \forall i \tag{3}$$

- Note similarity with optimality condition!

- For n odd and majority rule: the condition becomes

$$\frac{1-q_0}{q_1}\left[\frac{q_0\,(1-q_0)}{(1-q_1)\,q_1}\right]^{\frac{n-1}{2}} \leq \gamma_i \leq \frac{q_0}{1-q_1}\left[\frac{q_0\,(1-q_0)}{(1-q_1)\,q_1}\right]^{\frac{n-1}{2}} \quad \forall i$$

- Hence, sincere voting is a NE if $q_0 = q_1$ (by signal-informativeness). Moreover:

Corollary 3.2 *Suppose $q_0 \neq q_1$ and consider any sequence of ever increasing committees. Then $\exists\ n_0 \in \mathbb{N}$ such that sincere voting is a NE under majority rule for* **no** *$n \geq n_0$.*

- Austen-Smith and Banks (1996) showed this for the special case $\alpha_i = \beta_i = 1\ \forall i$.

- For a general committee, let m_i be the # signals 1 that member i needs to be convinced of $x = 1$.

Definition 3.1 *A committee is* **homogeneous** *if* $m_i = m_j \ \forall i, j$. *Otherwise, it is called* **heterogeneous**.

For generic parameter values:

Theorem 3.3 *For any integer* $k \in [1, n]$, *sincere voting is a Nash equilibrium under* f^k *if and only if the committee is homogeneous and* f^k *is optimal.*

4 A new voting rule

- For any $\varepsilon \in [0, 1]$, let the voting rule f_ε be defined by:

  step 1: all committee members vote simultaneously

  step 2:

  With probability $1 - \varepsilon$: majority rule is applied to all n votes

  With probability ε: let a randomly sampled individual vote decide

- "*Ex post delegation to dictator*"

- Note: If $\varepsilon = 1$, then Bayesian rationality $\Rightarrow$ vote sincerely!

- [Cf. virtual implementation in mechanism design, Abreu and Matsushima (1992)]

Theorem 4.1 *Suppose that the signal-informativeness condition is uniformly met and that preferences are uniformly bounded. There exist a sequence of positive $\bar{\varepsilon}_n \downarrow 0$ such that for each n, $\varepsilon \geq \bar{\varepsilon}_n$ and voting rule f^{ε}:*

(i) sincere voting is a strict Nash equilibrium

(ii) there is no other (pure or mixed) Nash equilibrium

- *Uniform signal-informativeness*: $\exists\ \eta \in (0,1)$ such that

$$\frac{1-q_0}{\eta q_1} < \gamma_i < \frac{\eta q_0}{1-q_1} \qquad \forall i$$

- *Uniform preference boundedness*: $\exists$ compact set $\Theta \subset (0,+\infty)^2$ such that

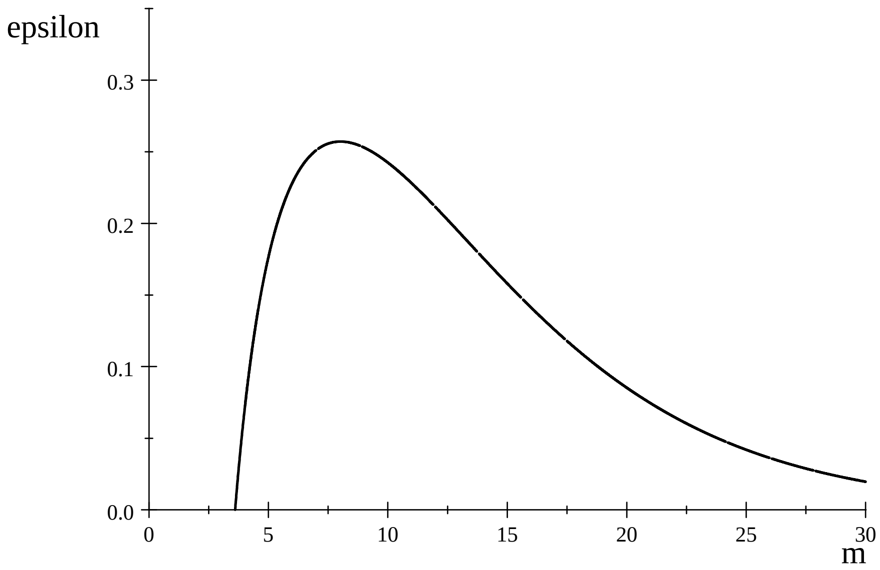
$$(\alpha_i, \beta_i) \in \Theta \qquad \forall i$$

- The claim in Condorcet's theorem can now be restored:

Corollary 4.2 *Under the hypothesis of the theorem, let* $\varepsilon_n \downarrow 0$ *such that* $\varepsilon_n > \bar{\varepsilon}_n \; \forall n$, *let* (f^{ε_n}) *be the associated sequence of randomized majority rules, and let* $X_n \in \{0, 1\}$ *be the equilibrium committee decision under* f^{ε_n}. *Then*

$$\lim_{n \to \infty} \Pr\left[X_n\left(\omega\right) = \omega\right] = 1$$

- How big ε is needed to make sincere voting an equilibrium?



5 Committee members making mistakes

A result on robustness to bounded rationality:

- Suppose that there is a probability $\lambda > 0$ that (exactly) one voter will make a mistake, will become a "noise voter" [c.f. Kfir (2002) in the context of mechanism design]

Proposition 5.1 *Consider majority rule in a committee with n odd, $\mu = 1/2$, $q_0 = q_1$, and with a probability $\lambda \in [0, 1]$ for a single noise voter. The probability that a rational committee member's vote will be pivotal under sincere voting is increasing in λ. Moreover, conditional upon being pivotal, the expected-utility difference between sincere and insincere voting is increasing in λ.*

6 Committee members with complex motivations

6.1 A slight disutility from voting against one's conviction

- Most humans arguably feel some discomfort when acting against their own conviction

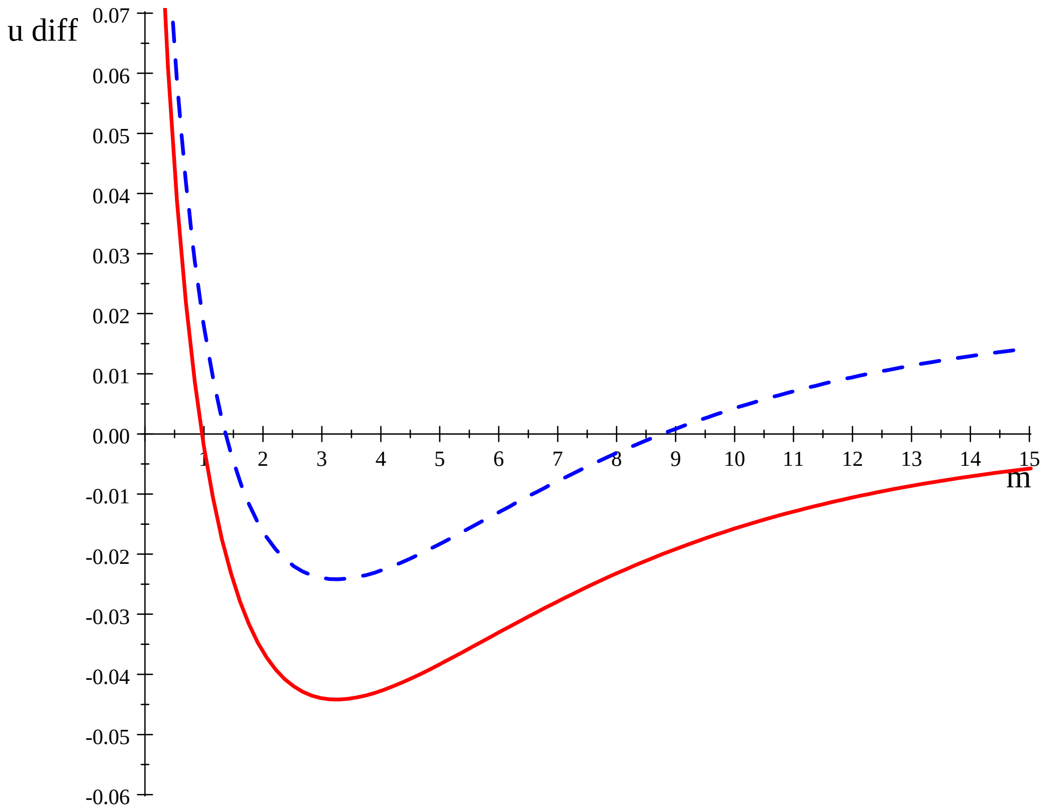
- Assume an arbitrarily small disutility $\delta_i > 0$ from voting against what i believes is the right decision, given one's prior and signal

- Consider simultaneous voting under majority rule

Proposition 6.1 *Suppose that the signal informativeness condition is uniformly met, that preferences are uniformly bounded and that all δ_i are uniformly bounded from below by a positive common bound. There exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$:*

(i) sincere voting is a strict Nash equilibrium

(ii) there is no other (pure or mixed) Nash equilibrium


u diff
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
m

- Again the claim in Condorcet's theorem holds:

Corollary 6.2 *Suppose that the signal informativeness condition is uniformly met and that preference are uniformly bounded and that* $\delta > 0$. *Let* $X_n \in \{0, 1\}$ *be the collective decision in Nash equilibrium under majority rule with* n *voters. Then*

$$\lim_{n \to \infty} \Pr\left[X_n(\omega) = \omega\right] = 1$$

6.2 A slight preference for esteem for competence

- In recent years: increased transparency of central bank committee decisions (Bank of England, the Sveriges Riksbank)

- What is the effect, if any, on voting in these committees?

1. Let there be a positive probability that, *ex post*, the true state of nature and all votes will be revealed to the general public

2. Suppose that with probability $\lambda \in [0, 1]$ exactly one of the committee members is less competent than the others: his or her two signals are less precise (and assume that each member knows his or her competence)

3. Assume that each committee member cares about the public's belief about his or her competence, in case her vote and the true state of nature is revealed in the future.

[...*calculations....calculations*...]

7 Experiments

- Guarnaschelli, Serena, Richard McKelvey and Thomas Palfrey (2000): "An experimental study of jury decision rules" *American Political Science Review* 94: 407-423.

- More data needed!

8 Conclusions

- Results:

  A. Earlier results can be generalized to allow for preference heterogeneity

  B. There is a class of (symmetric) voting mechanisms under which sincere voting is the *unique* equilibrium, a strict equilibrium that is asymptotically efficient

  C. The possibility of irrationality enhances rational voters' incentive for sincere voting

  D. Even a slight preference against voting insincerely has a strong effect on the equilibrium outcome in large committees.

  E. We need more behavioral game theory analysis, controlled laboratory experiments, and field data

- Limitations:

  1. All voters equally competent

  2. Number of voters fixed and known

  3. No abstention

  4. Signal precision exogenous

  5. Binary choice, binary states, binary signals

THE END

Thanks!