The Concorcet Jury Theorem and Preference Heterogeneity

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Abstract. We consider a committee, board or jury that faces a binary collective decision under uncertainty. Each member holds some relevant private information, and all members agree about what decision should be taken in each state of nature, had this been known. However, the state is unknown and members may attach different values to the two types of mistake that may occur. It is well-known that standard voting rules have a plethora of uninformative equilibria, and that informative voting may even be incompatible with equilibrium. We generalize existing results with respect to preference heterogenity and analyze a randomized majority rule that has a unique equilibrium. We show that this equilibrium is strict, all votes are informative, and that the equilibrium implements the collectively optimal decision with a probability that approaches 1 as the committee size tends to infinity.

Keywords: Voting, Condorcet, committee, heterogeneity, judgement aggregation.

JEL-codes: D71, D72.

1. Introduction

Many important decisions are not taken by individuals but by groups, committees, boards or electorates. We here analyze a class of such situations. The decision is binary and there are only two states of nature. All group members agree which decision is optimal in each state. However, the true state of nature is unknown. Group members have a common prior probability over these states, a prior that may

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be based on *ex ante* public information, such as evidence or expert reports presented to the whole group, committee, board or electorate. In addition, each committee member also has some private information, a private "signal" about the true state of nature. Group members may also differ in their valuations of the costs associates with the two types of mistake that may occur.[1] What decision rule should the group use in order to aggregate their private information and valuations? How should each member act under such a group decision rule? These are the questions that we here address, within a simple and abstract game-theoretic framework.

The topic is not new. Condorcet's (1785) so-called jury theorem essentially establishes that if (a) each member's information is positively correlated with the true state of nature, (b) distinct members' information is conditionally independent (given the state of nature), and (c) all jury members base their votes on their own private information and the public information, with no regard to other jury members' potential information and votes, then aggregation by way of the majority rule is asymptotically efficient in the sense that the probability for a mistaken collective decision tends to zero as the number of voters tends to infinity. However, as is by now well-known, Austen-Smith and Banks (1996) pointed out a weakness of this classical result. While Condorcet's hypothesis (c) may seem innocuous, a careful game-theoretic analysis shows that such voting behavior may not be consistent with Nash equilibrium play, even when all members have (ex ante) identical preferences (valuations of the two types of mistake). More exactly, if the number of voters is large, informative voting is generically not a Nash equilibrium of a Bayesian game that formally represents Condorcet's setting. The game-theoretic reasoning runs as follows: an individual vote affects the collective decision only if the vote is pivotal.[2] Hence, a rational voter under majority rule should reason *as if* the other votes were in a tie. But if all the others vote informatively, the fact that they are tied is very informative, perhaps "drenching" the individual voter's own private information. Hence, it may be rational not to vote according to one's own information. If the number of voters is large enough, this argument against informative voting becomes overwhelming. Consequently, informative voting is then not a Nash equilibrium and Condorcet's judgement-aggregation argument fails. However, the conclusion of the Condorcet jury theorem, that the out-

[1] They may actually also differ in their prior beliefs, see Remark 1 in Section 2.

[2] The notion of a pivotal event for a player is not restricted to voting games; Al-Najjar and Smorodinsky (2000) defined, in a general setting, the *influence* of a player in a mechanism as the maximum difference this player's action can make to the expected value of a collective result. They show that, in a precise sense, the mechanisms that maximise the number of influential players are closely related to majority rule.

come is asymptotically efficient, has been shown to be compatible with equilibrium by McLennan (1998) and Wit (1998), who provide conditions under which there exists a sequence of mixed-strategy Nash equilibria (hence, containing insincere voting) with this efficiency property for electorates with (ex ante) identical preferences.

Another seminal paper on incentives for informative voting is Feddersen and Pesendorfer (1996), in which the so-called swing voter's curse is analyzed. It refers to the following phenomenon. If voters, among whom there are partisans for each alternative as well as non-partisans, are allowed to abstain from voting, then poorly informed non-partisans may use the following mixed strategy. They probabilistically balance their votes in such a way that they collectively compensate for the presence of partisan voters (who support a given candidate in any case) and leave room for the better informed non-partisan voters. This mixed strategy of poorly informed non-partisan voters involves abstention with positive probability. Subsequent theoretical research on committee behavior has mainly concern the relative merits of different voting rules, see Feddersen and Pesendorfer (1998), and the role of straw-votes or debates before voting, see Coughlan (2000) and Austen-Smith and Feddersen (2005). When voters are identical, the picture is very different with and without debate or straw vote. If voters with identical preferences share their private information in the debate or straw vote, which they are in certain equilibria, then votes are unanimous in the decisive vote, and all majoritarian voting rules (including unanimity) are equivalent, see Gerardi and Yariv (2007). However, in general there is a plethora of other, uninformative equilibria. Moreover, truthful reporting in the straw vote is incompatible with equilibrium if committee members differ sufficiently in terms of preferences (their valuations of the costs associates with the two types of mistake).

We here generalize Austen-Smith's and Bank's (1996) model to allow for preference heterogeneity within the group or committee in question. More exactly, we consider a group, committee or board consisting of n members who have to take a binary collective decision: either $x = 0$ or $x = 1$. There are two possible states of nature, $\omega = 0$ and $\omega = 1$, and all committee members agree that decision $x = \omega$ is the right one. However, the state of nature is unknown when the decision is to be made. All committee members have some relevant private information, that takes the form of conditionally independent binary signals s_i, where q_ω is the conditional probability that a signal is correct ($s_i = \omega$), for $\omega = 0, 1$ and all members i. The signal precision is thus the same for all committee members. However, they may disagree about the cost or disutility associated with the two types of mistake that may be made. We assume that the signal is sufficiently informative for each member to optimally follow his or her private signal if making the decision single-handedly (the signal-informativeness condition).

In this setting, we characterize (utilitarian) optimality of deterministic collective decision rules and show that there, for any number n of committee members, exits some k-majority rule that is optimal, where a k-majority rule is one that takes the collective decision $x = 1$ if and only if at least k of the private signals favor this alternative. We also characterize those k-majority rules under which sincere voting is a Nash equilibrium, and we show that sincere voting is a Nash equilibrium under a k-majority rule if and only if the rule is optimal and the committee members have sufficiently similar (but not necessarily identical) preferences. The similarity condition is identical to that in Coughlan (2000), and generically differs from the signal-informativeness condition. This result generalizes Theorem 1 in Austen-Smith and Banks (1996) from the case when both signals are equally precise and all voters have identical and symmetric preferences. For committees that violate the similarity conditions — to be called heterogeneous committees — sincere voting is not a Nash equilibrium under any k-majority rule. Moreover, in general there is also a plethora of non-informative equilibria even when sincere voting is a Nash equilibrium. We here define a class of randomized majority rules under which sincere voting is the unique Nash equilibrium, irrespective of whether the committee is heterogeneous or not, and we show that, for suitably chosen sequences of such rules, the asymptotic efficiency claim in Condorcet's theorem holds for the associated sequence of equilibria. The unique equilibrium is strict, and hence meets all refinements. Our efficiency result complements McLennan's (1998) result that, for committees with ex ante identical members, there exists a sequence of strategically stable sets of equilibria that is asymptotically stable.

The rest of the paper is organized as follows. In section 2 we set up the model. Section 3 analyzes optimality of deterministic collective decision rules, and Section 4 is analyzes equilibrium voting under majoritarian voting rules. Section 5 makes a comparison between the signal-informativeness and preference similarity assumptions. Section 6 develops the mentioned randomized majority rule. In section 7 we analyze the robustness of the above results with respect to the rationality of the committee members. Section 8 concludes. Mathematical proofs are provided in an appendix at the end of the paper.

2. The model

2.1. Notation and basic setup. There are n committee members, where n is a positive integer, $n \in \mathbb{N}$. The committee has to make a binary decision, $x \in \{0,1\} = X$. All committee members agree what is the right decision in each state of nature. However, they do not know the state of nature $\omega \in \{0,1\} = \Omega$. Each committee member i receives a private "signal" $s_i \in \{0,1\}$, a random variable that is positively correlated with the true state of nature ω:

$$\begin{cases} \Pr[s_i = 0 \mid \omega = 0] = q_0 \\ \Pr[s_i = 1 \mid \omega = 1] = q_1 \end{cases}$$

for $q_0, q_1 > 1/2$. Hence, all committee members are "equally competent" in the sense of having the same conditional probability of receiving the "correct" signal. Signals received by different committee members are, however, conditionally independent, given the state of nature. The committee members share a prior belief about the actual state of nature (prior to the receipt of their private signals). This common prior may be their shared posterior after they have received a common signal (and held a common prior before that). Such a common signal may in practice take the form of document shared, hearings, a public debate etc. Let $\mu = \Pr[\omega = 1]$ be the common prior, and assume that $0 < \mu < 1$.

All committee members agree that the right decision in state ω is $x = \omega$. However, they may differ in the von Neumann-Morgenstern utilities that they assign to the four possible decision-state pairs. For each committee member i, these utilities are given by the following table:

	$\omega = 0$	$\omega = 1$
$x = 0$	u_{00}^i	u_{01}^i
$x = 1$	u_{10}^i	u_{11}^i

(1)

where $u_{01}^i = u_{11}^i - \alpha_i$ and $u_{10}^i = u_{00}^i - \beta_i$ for $\alpha_i, \beta_i > 0$. For each committee member i, these two parameters are the disutilities or "costs" that the committee member attaches to the two types of mistake, namely, of taking the wrong decision in each of the two states. A committee member's von Neumann-Morgenstern utilities may represent his or her personal values or those of some constituency that the member represents. We will sometimes refer to the first mistake (decision $x = 0$ in state $\omega = 1$) as a mistake of type I (accepting the false hypothesis that the state is 0) and the second mistake (decision $x = 1$ in state $\omega = 0$) as a mistake of type II (rejecting the true hypothesis that the state is 0). For many purposes, the relevant data about each committee member's values, as given in (1), can be summarized in a single number, namely

$$\gamma_i = \frac{\mu\alpha_i}{(1-\mu)\beta_i} \tag{2}$$

where $\gamma_i > 0$ follows from our assumptions. Note that $\gamma_i = 1$ if and only if committee member i attaches the same *ex ante* expected "cost" to both types of mistake. Before receiving his or her signal, the probability that a committee member attaches to state 1 is μ and the "cost" of a mistake in that state (a mistake of type I) is α_i. Hence, the *ex-ante* expected cost of a mistake of type I, according to committee member i's values, is $\mu\alpha_i$. Likewise, the probability attached to state 0 is $1-\mu$ and the "cost" of a mistake then, that is, of type II, is, in i's view, β_i. Hence, the *ex-ante* expected cost of a mistake of type II, according to committee member i, is $(1-\mu)\beta_i$. The summary parameter γ_i is the ratio between these two *ex-ante* expected costs, as evaluated by committee member i.

In the base-line setting, each committee member i casts a vote $v_i \in \{0,1\}$, a vote which may, but need not, be guided by i's private signal, and the collective decision x is determined by way of some pre-specified rule f that maps each *vote profile* $v = (v_1, ..., v_n)$ to a probability $f(v) \in [0,1]$ that the decision will be $x = 1$. The probability for decision $x = 0$ is $1 - f(v)$.

Formally, a *voting rule,* for a committee of arbitrary given size $n \in \mathbb{N}$, is a function $f : \{0,1\}^n \to [0,1]$. In particular, *majority rule* is defined by $f(v) = 1$ if $\sum_{i=1}^n v_i > n/2$, $f(v) = 0$ if $\sum_{i=1}^n v_i < n/2$ and $f(v) = 1/2$ otherwise. For any $k \in N \cap [0, n+1]$, let $f^k : \{0,1\}^n \to \{0,1\}$ be the *k-majority rule,* defined by $f^k(v) = 1$ if and only if $\sum_{i=1}^n v_i \geq k$. In particular, f^0 is the rule to take decision 1 irrespective of the votes, f^1 is the unanimity rule to take decision 0 only if all votes are for that alternative, f^n is the opposite unanimity rule, to take decision 1 only if all votes are for that alternative, and f^{n+1}, finally, is the rule to take decision 0 irrespective of the votes.

A *voting strategy* for committee member i in the base-line setting is a function $\sigma_i : \{0,1\} \to [0,1]$ that maps i's signal s_i to a probability $\sigma_i(s_i)$ for a vote v_i on alternative 1: $\Pr[v_i = 1 \mid s_i] = \sigma_i(s_i)$.[3] In others words, a voting strategy prescribes with what probability the committee member will vote for decision alternative 1. We assume that abstention is not an alternative, so the probability that i will vote on alternative 0 is $1 - \sigma_i(s_i)$.[4] By a *pure* voting strategy we mean a strategy σ_i such that $\sigma_i(s_i) \in \{0,1\}$ for both signals s_i. In this case, $v_i = \sigma_i(s_i)$. In the voting literature, the pure strategy to always vote according to one's signal, $\sigma_i(s_i) \equiv s_i$, is

[3] We will later analyze behavioral voting strategies under two-stage voting rules.

[4] In some committees abstention is indeed not permitted while in others it is. We exclude the latter case for analysitical convenience and brevity.

usually called *informative* voting, while voting for the alternative that maximizes the voter's expected utility, conditional on his or her own signal, and only on that piece of information, is called *sincere* voting.

Remark 1. *The subsequent analysis is also valid for the following more general setting (see Dixit and Weibull (2007)). Let each committee member i initially hold some prior $\upsilon_i = \Pr(\omega = 1)$ concerning the state of nature, and let all committee members receive a common signal s_0 (say, a public hearing or shared documentation). For each committee member, let μ_i be the postierior obtained by Bayes' law from the prior υ_i:*

$$\mu_i = \Pr(\omega = 1 \mid s_0) = \frac{\upsilon_i \Pr(s_0 \mid \omega = 1)}{\upsilon_i \Pr(s_0 \mid \omega = 1) + (1 - \upsilon_i) \Pr(s_0 \mid \omega = 1)}$$

and assume that $\mu_i \in (0, 1)$. For each committee member i, define γ_i as in (2), with μ replaced by μ_i. Let the private signals s_i be defined as in the main text. The results in this paper apply if one sets $\mu = 1/2$ and everywhere replaces α_i by $\mu_i \alpha_i$ and β_i by $(1 - \mu_i) \beta_i$.

2.2. Condorcet's jury theorem. Condorcet's Jury Theorem asserts that if all committee members vote informatively, then the probability of a mistaken collective decision under majority rule tends to zero as the committee size tends to infinity. The result hinges on the assumption that the signals are positively correlated with the true state and that they are conditionally independent. The result does not explicitly depend on committee members' values, since their voting behavior is assumed:

Theorem 1 [Condorcet]**.** *Suppose that all committee members vote informatively. Let $X_n(\omega) \in \{0, 1\}$ be the collective decision under majority rule when there are n committee members and the true state is ω. Then*

$$\lim_{n \to \infty} \Pr[X_n(\omega) \neq \omega] = 0$$

2.3. Signal informativeness. A hypothesis in Condorcet's theorem is thus that all committee members vote informatively. Clearly, this is not always a reasonable assumption, not even for $n = 1$, the case of a single decision-maker. To clarify this aspect, suppose, that one committee member has been selected to make the decision single-handedly, based only on his or her private signal. If the signal is noisy and her prior and valuation of mistake costs favor one alternative over the other, the right decision may well be to disregard the signal. An application of Bayes' rule gives the following posterior probability for state 0 after signal 0 has been received:

$$\Pr[\omega = 0 \mid s_i = 0] = \frac{(1-\mu)\Pr[s_i = 0 \mid \omega = 0]}{\Pr[s_i = 0]} = \frac{(1-\mu)\,q_0}{(1-\mu)\,q_0 + \mu\,(1-q_1)}$$

and likewise for the signal $s_i = 1$. Consequently, the strategy to vote informatively — when the decision is in i's hands — is optimal if and only if $(1-\mu)\,q_0\beta_i \geq \mu\,(1-q_1)\,\alpha_i$ and $\mu q_1 \alpha_i \geq (1-\mu)(1-q_0)\,\beta_i$, or, equivalently, if and only if $(1-q_0)/q_1 \leq \gamma_i \leq q_0/(1-q_1)$. We assume henceforth that both inequalities hold strictly for all committee members:

$$\frac{1-q_0}{q_1} < \gamma_i < \frac{q_0}{1-q_1} \qquad \forall i, \tag{3}$$

a condition we will refer to as *the signal-informativeness condition.* It follows from our assumption $q_0, q_1 > 1/2$ that the lower (upper) bound in (3) is below (above) unity. Moreover, these bounds tend to zero and plus infinity as q_0 and q_1, respectively, tend to one. The signal-informativeness condition thus holds when some or all three parameters γ_i, q_0 and q_1 are close to unity. Figure 1 below shows those combinations of signal precisions q_0 and q_1 that satisfy condition (3). If the minimal and maximal γ_i-values in the committee are 1/2 and 2, respectively, then this is the area north-east of the two solid straight lines. The dashed straight lines are drawn for the case when the minimal γ_i-value in the committee is 1/4 and the maximal value is 4.

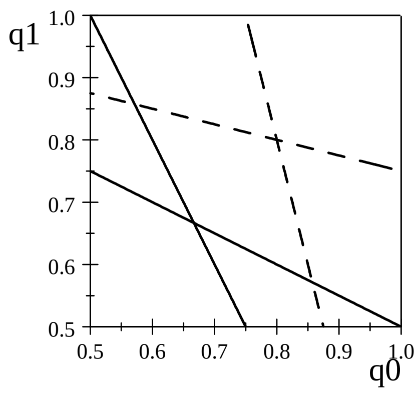

Figure 1: The signal-informativeness condition for $\gamma_i \in (1/2, 1)$.

3. OPTIMAL COLLECTIVE DECISION RULES

What decision rules would be optimal for a committee if all members' private information could be used? This defines an upper bound on what can be achieved by way of voting rules as defined above. We here analyze this question in terms of *deterministic collective decision rules*, defined as functions $d : \{0,1\}^n \rightarrow X$ that map signal profiles $s = (s_1, ..., s_n)$ to collective decisions $x = d(s) \in X = \{0,1\}$. For this consideration to be meaningful, we need to specify a normative criterion by which to rank collective decisions. Two candidates criteria seem relevant: (a) the probability that the decision will be correct ($x = \omega$), and (b) the sum of the committee members' expected utilities from the decision. While the first criterion is independent of committee members' preferences and hence does not discriminate between mistakes of type I and II, the second depends, in a well-defined way, on committee members' preferences.[5]

We here focus on this latter, utilitarian criterion. We thus call a deterministic collective decision rule d optimal if there exists no other such rule that yields higher expected welfare. Formally, let D be the set of deterministic collective decision rules and define $W : D \rightarrow R$ by

$$W(d) = \sum_{x,\omega,i} \Pr[(x,\omega) \mid x = d(s)] \cdot u^i_{x\omega}$$

Let D^* be the set of *optimal deterministic collective decision rules* for a given committee of size n:[6]

$$D^* = \{d^* \in D : W(d^*) \geq W(d) \quad \forall d \in D\}.$$

We note that all k-majority rules are deterministic collective decision rules when applied directly to the vector of private signals: $f^k \in D$, for $k = 0, 1, ..., n, n+1$. The following result follows more or less immediately from our assumptions:

Lemma 1. *Suppose that condition (3) is met. Then $D^* \neq \varnothing$, and there exists a $k \in \{1, ..., n\}$ such that $f^k \in D^*$.*

[5] In the more general case of distinct priors, this welfare criterion is subjectivistic-utilitarian in that it evaluates each voter's expected welfare according to that voter's probabilistic beliefs.

[6] Using another definition of collective welfare, Chwe (2007) analyzes which deterministic *voting rule* maximizes welfare under the constraint that voters should have no incentive to vote insincerely. The optimal voting rule is then non-monotonic (a large majority in favor of one alternative leads to the adoption of the opposite decision) and under this rule all voters are indifferent between sincere and insincere voting.

Hence, without loss of generality we may restrict the quest for optimal decision rules to k-majority rules, for $1 \leq k \leq n$. Let

$$\bar{\alpha}_n = \frac{1}{n}\sum_{i=1}^{n}\alpha_i, \quad \bar{\beta}_n = \frac{1}{n}\sum_{i=1}^{n}\beta_i \quad \text{and} \quad \bar{\gamma}_n = \frac{\mu\bar{\alpha}_n}{(1-\mu)\,\bar{\beta}_n}.$$

The parameter pair $\left(\bar{\alpha}_n, \bar{\beta}_n\right)$ can be though of as the values of a *(synthetic) representative voter*. For arbitrary positive integers n and k, let

$$g\,(k,n) = \left[\frac{(1-q_0)\,(1-q_1)}{q_0 q_1}\right]^k \left(\frac{q_0}{1-q_1}\right)^n$$

and note that the factor in square brackets is less than 1 while the factor in round brackets exceeds 1. Hence, $g\,(k,n)$ is decreasing in k and increasing in n. The following result provides a characterization of the optimal k-majority rule for any committee meeting the signal-informativeness condition:[7]

Theorem 2. *Suppose that condition (3) is met. For any positive integers n and $k \in [1,n]$, $f^k \in D^*$ if and only if*

$$g\,(k,n) \leq \bar{\gamma}_n \leq g\,(k-1,n)\,. \tag{4}$$

It is easily verified that since all γ_i meet condition (3), so does $\bar{\gamma}_n$.[8] Moreover, since $g\,(n,n) \leq (1-q_0)\,/q_1$ and $g\,(0,n) \geq q_0/\,(1-q_1)$, condition (4) holds for at least one $k \in \{1,...,n\}$. Generically, this k-value is unique.

First, consider the case when n is odd, and let m denote "majority:"

$$m = (n+1)\,/2.$$

Then $d = f^m$ is majority rule. By Theorem 2, $f^m \in D^*$ if and only if

$$\frac{1-q_0}{q_1}\left[\frac{q_0\,(1-q_0)}{(1-q_1)\,q_1}\right]^{m-1} \leq \bar{\gamma}_n \leq \frac{q_0}{1-q_1}\left[\frac{q_0\,(1-q_0)}{(1-q_1)\,q_1}\right]^{m-1}. \tag{5}$$

In the special case of equally precise signals, $q_0 = q_1$, the factor in square brackets is unity, and then (5) follows immediately from the signal informativeness condition (3). Formally:[9]

[7] Condition (4) below generalizes inequality (7) in Austen-Smith and Banks (1996) from the case of symmetric identical preferences, $\alpha_i = \beta_i\ \forall i$, to all preferences that meet the signal-informativeness condition (3). Our state 0 corresponds to their state A, our π to their $1-\pi$ and our k to their k^*+1.

[8] By (3): $(1-\mu)\,\beta_i\,(1-q_0)\,/q_1 < \mu\alpha_i < (1-\mu)\,\beta_i q_0/\,(1-q_1)$ for all i. Addition of the n inequalities yields the claimed inequality.

[9] Essentially the same result was obtained in a different model in Sah and Stiglitz (1988).

Corollary 1. *Suppose that condition (3) is met. Majority rule is an optimal deterministic collective decision rule if n is odd and $q_0 = q_1$.*

We note that in that case the optimality of majority rule is independent of committee members' preferences, as long as these do not violate the signal-informativeness condition. The reason is that, with equally precise signals in the two states of nature, committee members have sufficiently similar preferences to agree that one should always take the decision with the largest number of signals, if these were known.

Secondly, consider a sequence of committees of ever larger size $n = 1, 2...$, all with the same signal precisions q_0 and q_1, and assume that the following *uniform preference-boundedness condition* holds: there exists a compact set Θ in the interior of the positive orthant of $\mathbb{R}^2$, such that

$$(\alpha_i, \beta_i) \in \Theta \qquad \forall i \tag{6}$$

This condition is trivially met if all committee members in ever larger committees are identical, and it is also met under replication of a given finite preference profile, and under independent sampling from a fixed probability distribution with support in Θ.[10]

For each positive integer n, let k_n be such that k_n-majority rule is an optimal collective decision rule, and write κ_n for k_n/n. In other words, for each committee size n, $\kappa_n \in [0, 1]$ is an optimal cut-off, in terms of the share of 1's among all n private signals, for decision $x = 1$. It is not difficult to verify that such a sequence $\langle \kappa_n \rangle_{n \in \mathbb{N}}$ is convergent. Perhaps surprisingly, the limit, as $n \to \infty$, is independent of preferences (as long as these meet the signal-informativeness and value-boundedness conditions); it only depends on the precision of the two signals. In particular, the limit value is exactly 1/2 if the two signals are equally precise. Let

$$\kappa^* = \frac{\ln\left(\frac{q_0}{1-q_1}\right)}{\ln\left(\frac{q_0}{1-q_1}\right) + \ln\left(\frac{q_1}{1-q_0}\right)}. \tag{7}$$

Corollary 2. *If conditions (3) and (6) are met, then $\lim_{n\to\infty} \kappa_n = \kappa^*$ and $\kappa^* \in [1 - q_0, q_1]$.*

The reason why the asymptotically optimal collective decision rule is independent of preferences is, roughly, that for large committees the probability of a mistake

[10] In the case of distinct priors, the uniform boundedness condition is $(\mu_i \alpha_i, (1 - \mu_i) \beta_i) \in \Theta$.

is vanishingly small, while the "relative cost" ratio $\bar{\gamma}_n = \bar{\alpha}_n/\bar{\beta}_n$, between the two types of mistake, by hypothesis is bounded away from both zero and plus infinity (uniformly in n). Hence, asymptotically, it does not matter exactly what values the parameters $\bar{\gamma}_n$ take, as long as they all stay above some positive number and below another positive number; only the asymptotic mistake probability ratio matters (see appendix for a formal proof). In the knife-edge case when $q_0 = q_1$, this ratio is exactly one and $\kappa_n \to \kappa^* = 1/2$.

We note that if one of the two signals is more precise than the other, then the asymptotically optimal decision rule requires more of that signal: $q_1 > q_0 \Rightarrow \kappa^* > 1/2$. Mathematically, this follows from the observation that κ^* is decreasing in q_0 and increasing in q_1. The intuition is that if the signal is more precise in state 1 than in state 0, then observation of signal 1 is more likely to be erroneous than observation of signal 0. Figure 2 below shows the dependence of κ^* on the signal precisions q_0 and q_1. The area north-east of both straight lines are the signal-precision pairs that satisfy the signal-informativeness condition (3) for all γ_i-values between 1/2 and 2, just as in Figure 1. The two curves are isoquants for κ^*. The upper curve is where $\kappa^* = 2/3$, the lower curve where $\kappa^* = 1/3$, and the diagonal is where $\kappa^* = 1/2$.

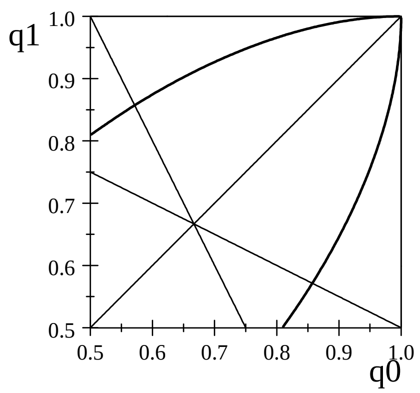

Figure 2: Asymptotically optimal collective decision rules.

4. Equilibrium voting

Suppose that the collective decision is to be taken according to some k-majority voting rule applied to a committee consisting of n members, where $1 \leq k \leq n$. Is sincere voting then a Nash equilibrium? In force of the signal informativeness condition (3), sincere voting is identical with informative voting, and we will use these two attributes interchangeably. In this voting game, each voter i first observes her private signal and then casts her vote $v_i \in \{0, 1\}$, simultaneously with all other voters. The collective decision $x = 1$ results if at least k voters cast the vote 1, while the collective decision $x = 0$ results in the opposite case.

In Nash equilibrium, each voter maximizes his or her expected utility, given his or her private signal, and given all other voters' strategies. Clearly, there is a plethora of (pure and mixed) uninformative Nash equilibria whenever $n \geq 3$. For example, to always vote 0 (or 1), independently of one's private signal, constitutes a Nash equilibrium. For if others vote according to such a strategy, then my vote will never be pivotal and hence I can just as well use the same uninformative voting strategy as the others. Under what conditions, if any, will sincere voting constitute an equilibrium?

Theorem 3. *Suppose that condition (3) is met. For any positive integers n and $k \leq n$, sincere voting under k-majority rule constitutes a Nash equilibrium if and only if*

$$g(k, n) \leq \gamma_i \leq g(k-1, n) \qquad \forall i \tag{8}$$

Two remarks are in place. First, if committee members have identical preferences, then conditions (8) and (4) are identical. Hence, in this special case, a k-majority rule is optimal if and only if sincere voting under this rule is a Nash equilibrium. This was first proved for the case of symmetric preferences ($\alpha_i = \beta_i \; \forall i$) by Austen-Smith and Banks (1996, Lemma 2). See Costinot and Kartik (2006) for more findings under the hypothesis of *ex ante* identical committee members. Secondly, for n odd and equally informative signals, the theorem implies that sincere voting is a Nash equilibrium under majority rule, irrespective of individual valuations γ_i, as long as these meet the signal-informativeness condition:

Corollary 3. *Suppose that condition (3) is met. Sincere voting is a Nash equilibrium under majority rule if n is odd and $q_0 = q_1$.*

This is not surprising, however, since with equally precise signals, a tie among an even number of other votes does not affect the odds for one state over the other.[11] In general, the probability for a mistake of type I may well differ from the probability of a mistake of type II. Suppose, thus, that $q_0 \neq q_1$ and consider majority rule in

[11] Then $\Pr[\omega = 0 \mid \mathcal{T} \ \wedge \ s_i = 0] = \Pr[\omega = 0 \mid s_i = 0]$ and likewise for $\omega = s_i = 1$.

a committee with an odd number n of members. Condition (8) then becomes, with $m = (n+1)/2$:

$$\frac{1-q_0}{q_1}\left[\frac{q_0(1-q_0)}{(1-q_1)q_1}\right]^{m-1} \leq \gamma_i \leq \frac{q_0}{1-q_1}\left[\frac{q_0(1-q_0)}{(1-q_1)q_1}\right]^{m-1} \quad \forall i \tag{9}$$

If q_0 and q_1 differ even the slightest, the factor in square brackets is distinct from unity. Hence, as n tends to infinity, this factor either converges to zero (if $q_0 > q_1$) or to plus infinity (if $q_0 < q_1$). Inevitably, for any given positive γ_i-value, one of the two inequalities in (8) is thus violated for all n sufficiently large. We have obtained the following generalization of Theorem 1 in Austen-Smith and Banks (1996):

Corollary 4. *Suppose that $q_0 \neq q_1$. For any positive sequence $(\gamma_i)_{i\in\mathbb{N}}$ there exists an $n_0 \in \mathbb{N}$ such that sincere voting is a Nash equilibrium under majority rule for no $n \geq n_0$.*

This result is intuitively plausible. For suppose that state 0 is more likely to give rise to signal value 0 than state 1 is likely to give rise to signal value 1, that is, $q_0 > q_1$. In such a case, signal 0 is *less* informative than signal 1 in the sense that signal 0 is more likely in state 1 than signal 1 is in state 0. If n is large, a tie among the others is then quite a strong indication of state 1, even if a voter's own signal is 0, since in total there are just about as many signals 0 as signals 1, quite an unlikely event in state 0. Hence, even if I, as a voter, believed that the others vote sincerely, I should nevertheless vote on alternative 1, irrespective of my own signal.

Remark 2. *The corollary can also be explained in terms of Corollary 2: If $q_0 \neq q_1$, then majority rule is not an asymptotically optimal collective decision rule. Hence, by Theorem 3 there exists a committee size n_0 beyond which sincere voting is not a Nash equilibrium under majority rule.*

We finally explore the relations between (i) optimality of a k-majority rule, f^k, as a collective decision rule, and (ii) sincere voting being a Nash equilibrium under a k-majority rule. It follows immediately from Theorems 2 and 3 that if sincere voting is a Nash equilibrium under some k-majority rule, then this rule is an optimal collective decision rule. What about the converse? We already noted that the converse holds if committee members have identical values/preferences. What if they do not? To this end, it is useful to first ask which collective decision $x \in \{0,1\}$ individual i would like to see taken, if i had known the total (random) number, N_1, of signals 1 received among all committee members. As shown in the appendix, voter i will deem decision $x = 1$ to be better than decision $x = 0$ if and only if :

$$\Pr[N_1 \mid \omega = 0] \leq \gamma_i \cdot \Pr[N_1 \mid \omega = 1] \tag{10}$$

After some algebraic manipulation, this condition can be re-written as $N_1 \geq \tau(\gamma_i)$, where, for any $z > 0$,

$$\tau(z) = n \cdot \kappa^* - \frac{\ln z}{\ln[q_0 q_1] - \ln[(1 - q_0)(1 - q_1)]} \tag{11}$$

We will call $\tau(\gamma_i) \in \mathbb{R}$ the *threshold* for i; voter i needs at least $\tau(\gamma_i)$ signals 1 in order to prefer decision 1. We note that both terms in the denominator are positive and that $\tau(z)$ decreases continuously with z. Without loss of generality, assume that $\gamma_1 \leq \gamma_2 \leq ... \leq \gamma_n$ and hence $\tau(\gamma_1) \geq \tau(\gamma_2) \geq .. . \geq \tau(\gamma_n)$. For generic parameter values q_0, q_1 and γ_i, the real numbers $\tau(\gamma_i)$ are not integers. We here focus on this generic case. Write

$$T(z) = \lfloor \tau(z) \rfloor + 1$$

for the smallest integer $\geq \tau(z)$. Committee member i thus prefers decision 1 (over decision 0) if and only if the number of signals 1 is at least $T(\gamma_i)$. Two committee members i and j will be said to have *similar* preferences if $T(\gamma_i) = T(\gamma_j)$. A committee where all members have similar preferences, that is, where

$$T(\gamma_i) = T(\gamma_j) \qquad \text{for all } i, j \in \{1, 2, .., n\} \tag{12}$$

will be called *homogeneous*, otherwise *heterogeneous*. For generic parameter values we have:

Proposition 1. *Suppose that condition (3) is met. For any positive integer $k \leq n$, sincere voting is a Nash equilibrium under f^k if and only if $f^k \in D^*$ and the committee is homogeneous.*

This result implies that, under the signal-informativeness condition (3), sincere voting is incompatible with Nash equilibrium under all k-majority rules in all heterogeneous committees. By contrast, in every homogeneous committee, sincere voting is compatible with Nash equilibrium under some k-majority rule (by Lemma 1).[12]

[12] For electorates with ex ante identical members, McLennan (1998) and Wit (1998) establish results that connect optimality with equilibrium for mixed-strategy profiles in which all or some voters vote insincerely.

We also note that Proposition 1 implies that for committees that are homogeneous, optimality is invariant under positive affine transformations of individual members' von Neumann-Morgenstern utilities. That optimality of a collective decision rule is invariant under addition of any scalar to all of an individual's von Neumann-Morgenstern utilities is evident. That optimality also is invariant under multiplication of an individual's von Neumann-Morgenstern utilities is perhaps less evident, since this would amount to giving more weight to that individual's utility in the welfare function. To see that optimality is nevertheless invariant, suppose that f^k is optimal for some committee, and, for one of its members, i, multiply α_i and β_i with the factor $c+1$ for some $c>0$. Then $\bar{\alpha}_n$ will be replaced by $\bar{\alpha}_n + c\alpha_i/n$ and $\bar{\beta}_n$ by $\bar{\beta}_n + c\beta_i/n$. By Theorem 2, $\bar{\beta}_n g(k,n) \le \bar{\alpha}_n \le \bar{\beta}_n g(k-1,n)$ and, since the committee is homogeneous, $\beta_i g(k,n) \le \alpha_i \le \beta_i g(k-1,n)$ for all members i, and hence also

$$\left[\bar{\beta}_n + c\beta_i/n\right] g(k,n) \le \left[\bar{\alpha}_n + c\alpha_i/n\right] \le \left[\bar{\beta}_n + c\beta_i/n\right] g(k-1,n)$$

By Theorem 2, f^k is still optimal. By contrast, for heterogeneous committees, optimality is not in general invariant under positive multiplication of individuals' von Neumann-Morgenstern utilities. Then individuals differ sufficiently in preferences for the weights placed upon them in the welfare function to matter.

5. HETEROGENEITY AND SIGNAL INFORMATIVENESS

The homogeneity condition (12) and the signal-informativeness condition (3) both impose bounds on how much committee members can differ from each other in terms of preferences. Here we compare these two bounds. First, let $q_0 = q_1 = q$. Then

$$\tau(z) \equiv \frac{1}{2} \cdot \left[n - \frac{\ln z}{\ln q - \ln(1-q)}\right]$$

If n is odd, then it is not difficult to verify that the signal-informativeness condition implies preference homogeneity:[13]

Proposition 2. *If n is odd and $q_0 = q_1$, then (3) holds if and only if $T(\gamma_i) = (n+1)/2$ for all i.*

[13] To see this, note that $T(\gamma_i) = (n+1)/2$ if and only if

$$-1 \le \frac{\ln \gamma_i}{\ln q - \ln(1-q)} < 1.$$

We also note that they all agree that half of the signals are needed for either decision. By contrast, if n is even, then $T(\gamma_i) \geq n/2+1$ for all $\gamma_i < 1$ and $T(\gamma_i) \leq n/2$ for all $\gamma_i \geq 1$, a discontinuity at $\gamma_i = 1$. See Figure 3 below, showing the graphs of τ (dashed) and T (solid), with z on the horizontal, for $n = 6$ and $q = 0.8$. In this numerical example, the signal-informativeness conditions requires that $0.25 < \gamma_i < 4$. Hence, such a committee, with γ_i-values scattered around 1 in this interval, is heterogenous although the signal-informativeness condition is met.

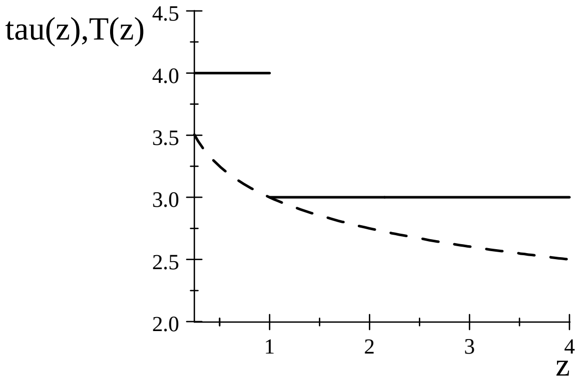

Figure 3: Heterogeneity when n is even and the two signals are of identical precision.

Second, consider the case $q_0 \neq q_1$, with n even or odd. As z increases from the lower to the upper bound in the signal informativeness condition (3), $\tau(z)$ decreases continuously from $\tau^{\max} = (n-1)\kappa^* + 1$ to $\tau^{\min} = (n-1)\kappa^*$. We note that $\tau^{\max} - \tau^{\min} \equiv 1$, and that, for generic signal precisions q_0 and q_1, $\tau^{\max}$ and $\tau^{\min}$ are not integers. Hence, generically, the interval of γ_i-values that meet the signal-informativeness condition is partitioned into two subintervals with distinct $T(\gamma_i)$-values. This is illustrated in the diagram below, drawn for $n = 5$, $q_0 = 0.7$ and $q_1 = 0.8$. In this case, the signal-informativeness condition is met for all γ_i in the open interval $(0.375, 3.5)$. See Figure 4 below, showing the graphs of τ (dashed) and T (solid), with z on the horizontal.

Suppose that the five committee members are independently drawn from an infinite population in which the parameter values γ_i are distributed according to a cumulative distribution function Φ. Then the probability that the signal-informativeness condition will be met in this numerical example is $\Phi(3.5) - \Phi(0.375)$ and the probability that the committee will meet the signal-informativeness condition and also be homogenous is

$$[\Phi(0.646) - \Phi(0.375)]^5 + [\Phi(3.5) - \Phi(0.646)]^5 .$$

Hence, while the first probability is independent of the committee size, the second is decreasing quite fast in the committee size. For instance, if Φ would be uniform on the interval from 0 to 5, then the first probability would be 62.5% while the second would be only 6%.

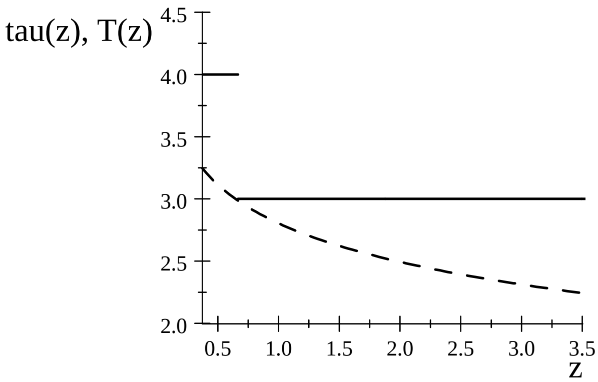

Figure 4: Heterogeneity when the signal precisions differ.

To deal with committees of arbitrary size, it is useful to make the hypothesis that there exists some $\eta < 1$ such that

$$\frac{1-q_0}{\eta q_1} < \gamma_i < \frac{\eta q_0}{1-q_1} \qquad \forall i \tag{13}$$

The following remark shows how weak this *uniform signal-informativeness condition* in comparison with the homogeneity condition.

Remark 3. *Standard techniques for constructing committees of ever larger size produce committees that are heterogeneous. Suppose that, starting from a given committee of size $n_o > 1$, one replicates it ad infinitum to obtain committees of sizes $2n_o, 3n_o, ...$ Suppose that the initial committee satisfies the uniform signal informativeness condition (13) for some $\eta < 1$. Then also all replica meet this condition. However, for generic initial committee, and even if this is homogeneous, the sequence will contain infinitely many heterogeneous committees. To see this, note that equation (11) defines τ as an affine function of the committee size n, with a positive* *slope less than one. Moreover, for each n, its range (as γ_i varies from the lower to the upper bound in (13)) is an interval of length close to* 1 *when $\eta < 1$ is close to* 1*. Hence, for many multiples of n_o, this range will contain an integer and thus the associated committee will be heterogeneous (have some members' $\tau(\gamma_i)$-values below that integer and others above it). The same reasoning is valid if the sequence is not obtained by replication of an initial committee, but by independently drawing new members according to a fixed probability distribution.*

6. Randomized majority rule

We have seen that, for homogeneous committees, sincere voting is an equilibrium under a k-majority rule if and only if it is optimal, while for heterogeneous committees, sincere voting is not an equilibrium under any k-majority rule, not even under an optimal such rule. Moreover, for all committees, homogeneous and heterogeneous alike, there is a plethora of non-informative equilibria. For example, under majority rule in any committee of size $n \geq 3$, it is an equilibrium to always vote 0 (another equilibrium is to always vote 1, etc.). We here define a class of randomized majority rules under which sincere voting is the unique Nash equilibrium, irrespective of whether the committee is homogeneous or heterogeneous, and we show that, for suitably chosen sequences of such rules, the asymptotic efficiency claim in Condorcet's theorem holds for the associated sequence of equilibria.

Consider first the simple randomized voting rule according to which all n members of the committee simultaneously cast their votes, whereafter a random sample of $n^* \leq n$ of these votes is drawn and the collective decision is made by way of some k-majority rule applied to this random sample of size n^*, for $k \leq n^*$. If each vote has a fixed positive probability of being sampled (say $1/n$) and the sample size n^* is small enough (say $n^* = 1$), then sincere voting will be a Nash equilibrium. More precisely, consider a committee of n members for which the signal informativeness condition (3) holds. Let $n^* \leq n$ be such that condition (8) holds for all subsets of the committee of size n^* (that is, with n^* in the place of n in (8)). From the signal-informativeness condition it follows that such an integer $n^* \geq 1$ exists.[14] Let f^* be the voting rule according to which all n committee members vote simultaneously and the collective decision x is determined by majority rule applied to a random sample of size n^* of these votes, the sample being drawn with equal probability for each subset of size n^*, and this draw being statistically independent of the state of nature and of all private signals and votes. From Theorem 3 we immediately obtain:

[14] For $n^* = 1$, (8) follows directly from (3).

Corollary 5. *Sincere voting is a Nash equilibrium under voting rule f^*.*

The voting rule is clearly anonymous in the sense that it treats all n votes equally, without respect to the identities of the voters. However, an evident drawback of this voting rule is that it does not aggregate the private information in an efficient way when n is large, since the sample size n^* generically needs to remain bounded as n increases, in order to keep up the incentive for sincere voting. Hence, the collective decision under f^*, applied to ever larger committees, may remain bounded away from full informational efficiency even in the limit as $n \to \infty$.

However, there is a straight-forward remedy: combine this randomized majority rule with the usual majority rule. Instead of always letting a randomly selected subset of votes determine the collective decision, use a two-stage randomization device to (a) determine whether the collective decision x be determined by the majority of the random sample of size n^* or by the majority of all n votes, and (b) also to select the sample when this is called for. For the sake of clarity and definiteness, we henceforth focus on the special case of majority rule, n odd and $n^* = 1$. For any $\varepsilon \in [0,1]$, let f_ε be the voting rule according to which all votes are cast simultaneously, and then, with probability $1-\varepsilon$, the collective decision is taken according to majority rule applied to all n votes, while with the residual probability, ε, the collective decision is taken according to a randomly sampled single vote, with equal probability, $1/n$, for each vote to be so drawn. Hence, for an individual voter, the probability is ε/n that his or her vote will be selected to single-handedly determine the collective decision. Similar randomized mechanisms have been used in the literature on virtual implementation, see Abreu and Matsushima (1992) and Glazer and Rubinstein (1998).[15]

The question that we will now attack is whether ε can be reduced towards zero as the committee size n grows towards infinity, in such a way that the incentive remains for sincere voting. At first sight, this may appear impossible, since it is known from probability theory that the probability for equally many "heads" as "tails" among n statistically independent throws of a fair coin approaches zero only at the rate proportional to $1/\sqrt{n}$, a slower rate than $1/n$, the probability weight placed on "my" vote in a random draw of the votes. It thus seems that the incentive effect from the randomization might be too weak to dominate the disincentive effect against sincere voting under majority rule.[16] However, we will proceed to show that this is not the case in the present setting.

[15] However, we have (so far) not seen any analysis of the randomized voting rule proposed here.

[16] We are grateful to Sergiu Hart for raising this point.

Clearly sincere voting is a strict Nash equilibrium under f_1, since then i's vote either determines the outcome, $x = v_i$, and this happens with positive probability $1/n$, or else i's vote does not affect the outcome at all (and this happens with probability $1 - 1/n$). By continuity, sincere voting is still a strict Nash equilibrium under f_ε for all $\varepsilon \leq 1$ sufficiently close to 1. However, as the following example shows, even if the probability ε for random delegation to a single vote is large enough to render sincere voting a strict equilibrium, there may still exist other, less informative, equilibria. This constitutes an additional difficulty to overcome.[17]

Example 1. *Consider a committee with three members, with a uniform prior, $\mu = 1/2$, equally precise signals, $q_0 = q_1 = q$, distinct values, $\gamma_1 = 1/c$, $\gamma_2 = 1$ and $\gamma_3 = c$ for some $c > 1$, such that the signal-informativeness condition (3) is met for all committee members. The number n being odd and the signals being equally precise, majority rule is optimal, sincere voting under majority rule is a Nash equilibrium, and the committee is homogeneous (Proposition 2). Let $\varepsilon \in [0, 1]$ and consider the associated randomized majority rule f_ε. Since sincere voting is an equilibrium for $\varepsilon = 0$ and an increase in ε enhances all voters' incentive for sincere voting, sincere voting is an equilibrium for all $\varepsilon \in [0, 1]$. However, for small ε there also exists a mixed equilibrium in which (a) voter 1 votes sincerely for sure when receiving signal 0, but only with probability $x \in (0, 1)$ when receiving signal 1, (b) voter 2 always votes sincerely, and (c) voter 3 votes sincerely for sure when receiving signal 1, but only with probability $y \in (0, 1)$ when receiving signal 0. In other words, the two "extreme" voters randomize when they obtain an "unfavorable" signal. The probability x makes voter 3 indifferent when receiving signal 0, and the probability y makes voter 1 indifferent when receiving signal 1. It is not difficult to verify that these indifference conditions amount to the following requirements:*

$$x = \frac{3\,(1-\varepsilon)\,(c-1)\,q\,(1-q) - \varepsilon\,[(c+1)\,q - c]}{3\,(1-\varepsilon)\,(c+1)\,(2q-1)\,(1-q)\,q}$$

$$y = \frac{3\,(1-\varepsilon)\,(c-1)\,q\,(1-q) - \varepsilon\,[(c+1)\,q - 1]}{3\,(1-\varepsilon)\,(c+1)\,(2q-1)\,(1-q)\,q}$$

For instance, for $c = 2$, $q = 0.7$ and $\varepsilon = 0.2$, $x \approx 0.80$ and $y \approx 0.47$. We note that $x, y \in (0, 1)$ if and only if

$$\varepsilon < \frac{3\,(c-1)\,q\,(1-q)}{3\,(c-1)\,q\,(1-q) + (c+1)\,q - 1},$$

[17] See also Wit (1998), who establish the existence of mixed Nash equilibria of a similar kind but under the standard majority rule as applied to voters with identical and symmetric preferences ($\alpha_i = \beta_i = 1$ for all i).

where the quantity on the right-hand side is positive but less than 1.

We first investigate when sincere voting is a Nash equilibrium under such a voting rule f_ε, for a given ε and committee size n. Suppose that committee member i has received the signal $s_i = 0$. Denote by $\Delta u_i^0(\varepsilon)$ the difference in expected utility, for that member, when casting the sincere vote $v_i = 0$ rather than the insincere vote $v_i = 1$ (under the hypothesis that all others vote sincerely). Committee member i will become the "ex-post dictator" with probability ε/n. If instead another committee member's vote is sampled, then i's vote does not matter. It follows from the proof of Theorem 3 in the appendix that, for n odd and with $n = 2t + 1$, we have

$$\Delta u_i^0(\varepsilon) = A_i^0(t) \cdot \varepsilon + B_i^0(t) \cdot (1 - \varepsilon) \tag{14}$$

where

$$A_i^0(t) = \frac{1}{2t+1} \cdot \frac{(1-\mu)\beta_i q_0 - \mu\alpha_i(1-q_1)}{(1-\mu)q_0 + \mu(1-q_1)}$$

and

$$B_i^0(t) = \binom{2t}{t} \cdot \frac{(1-\mu)\beta_i q_0^{t+1}(1-q_0)^t - \mu\alpha_i q_1^t(1-q_1)^{t+1}}{(1-\mu)q_0 + \mu(1-q_1)}$$

Here $A_i^0(t)$ is the probability that i's vote will be randomly sampled, multiplied with the conditionally expected utility difference (from sincere voting compared with insincere voting) when this happens. This difference is positive if and only if $\gamma_i < q_0/(1-q_1)$. The corresponding inequality for signal 1, $A_i^1(t) > 0$, is met if and only if $\gamma_i > (1-q_0)/q_1$. Hence, under the signal-informativeness condition (3), $A_i^0(t), A_i^1(t) > 0$ for all committee members i and all committee sizes $n = 2t+1$. Moreover, both $A_i^0(t)$ and $A_i^1(t)$ decrease with committee size n at the rate $1/n$. The second factor, $B_i^0(t)$, is the probability that majority rule will be applied to all n votes, multiplied with the conditional utility difference (again between sincere and insincere voting) when this happens. Unlike $A_i^0(t)$, $B_i^0(t)$ may be negative, zero or positive, depending on parameter values. However, it is not difficult to show that both $B_i^0(t)$ and $B_i^1(t)$ tend to zero as t tends to infinity. This is not surprising, since the probability for a tie among $2t$ voters converges to zero, and the expected utility difference is bounded. It remains to show that this convergence is faster that $1/n$, the rate at which the first term in (14) diminishes. It follows from the observations made above, concerning coin tossing, that if both signals would be uninformative ($q_0 = q_1 = 1/2$), then $B_i^0(t)$ would tend to zero slower than $A_i^0(t)$ and the randomization would not help to give incentives for informative voting. However, for the case of informative signals ($q_0, q_1 > 1/2$), as we here assume, one can show that $B_i^0(t)$ and $B_i^1(t)$ in fact tend to zero at an exponential rate. This implies that $\varepsilon > 0$ can be made arbitrarily small when t is sufficiently large. More precisely, there exists a decreasing sequence $(\varepsilon_t)_{t\in\mathbb{N}}$ such that $\Delta u_i^0(\varepsilon), \Delta u_i^1(\varepsilon) > 0$ for each $t \in N$ and yet $\varepsilon_t \to 0$ as $t \to \infty$. The incentive for voting sincerely is then strict for each committee member, if all others vote sincerely, so sincere voting is a strict equilibrium for each $t \in N$.

The full curve in the diagram below shows how large ε needs to be in order to keep $\Delta u_i^0(\varepsilon)$ positive for different values of t, in the special case when $\mu = 1/2$, $q_0 = 0.8$, $q_1 = 0.7$ and $\alpha_i = \beta_i = 1$ for all i. We see that the maximal ε-value needed is about 0.27 and that this occurs when n is about 17. However, as the committee size grows beyond that size, the required ε-value decreases towards zero. The dashed curve corresponds to the case when $q_1 = 0.75$, ceteris paribus. As expected, since q_0 and q_1 are closer together, less randomization is needed.

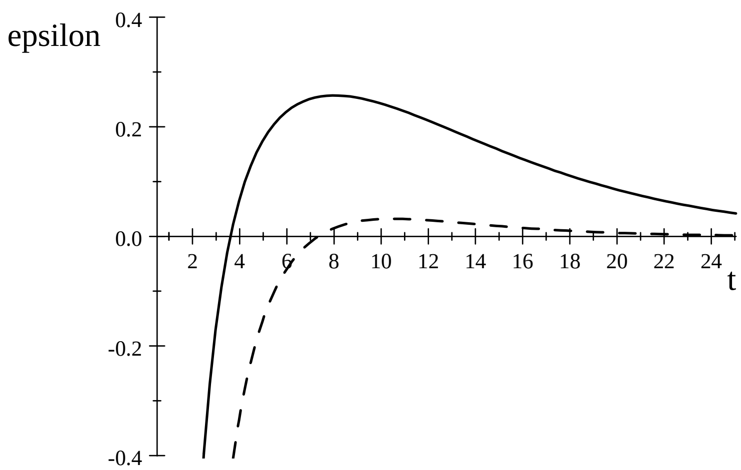

Figure 5: The minimal epsilon for sincere voting to be a Nash equilibrium.

As we saw in the Example 1, there may also exist other Nash equilibria, alongside sincere voting. However, under the uniform signal-informativeness assumption (13) (but without invoking the preference homogeneity assumption), one can show that if ε_t is not reduced too fast as t increases, then the strict and informative equilibrium can in fact be made unique. No other Nash equilibrium — neither pure nor mixed, neither symmetric nor asymmetric — then exists. Formally:

Theorem 4. *Consider a sequence of committees of sizes $n = 2t+1$, for $t = 0, 1, 2, 3....$ Suppose that conditions (6) and (13) are met. There exist a sequence of positive $\bar{\varepsilon}_t \to 0$ such that, for each t and any voting rule f_ε with $\varepsilon \geq \bar{\varepsilon}_t$:*
(i) sincere voting is a strict Nash equilibrium
(ii) there exists no other Nash equilibrium.

It follows from this result that the claim in Condorcet's jury theorem is valid for sequences of randomized majority rules of the type described above. More precisely, let $(\bar{\varepsilon}_t)_{t\in\mathbb{N}}$ be a sequence as specified in Theorem 4. For each $t \in \mathbb{N}$, let $\varepsilon_t \geq \bar{\varepsilon}_t$ and $\varepsilon_t \to 0$. The associated sequence of randomized voting rules, $\langle f_{\varepsilon_t}\rangle_{t\in\mathbb{N}}$, is then asymptotically efficient:

Corollary 6. *Suppose that (6) and (13) hold. Let $X_t \in \{0,1\}$ be the committee decision for a committee of size $2t+1$ under the above described voting rule f_{ε_t}. Then*

$$\lim_{t\to\infty} \Pr[X_t \neq \omega] = 0$$

Although sincere voting is a strict and unique Nash equilibrium for all t under the hypotheses of the theorem, sincere voting is *not* a dominant strategy for all t and $\varepsilon \geq \bar{\varepsilon}_t$. We show this in five steps. First, for any voter type $(\alpha, \beta) \in \Theta$, let $k^*(\alpha, \beta)$ be the minimal $k \in \mathbb{N}$ such that the conditional expected utility to a committee member of type (α, β) from decision $x = 1$ is higher than from decision $x = 0$, conditional upon k signals 1 and 1 signal 0. Let $k^* = \max_{(\alpha,\beta)\in\Theta} k^*(\alpha, \beta)$. Since Θ is compact with positive lower bounds, $k^* \in \mathbb{N}$. Second, fix $k \geq k^*$ and, consider committees of odd sizes $n = 2t + 1$, such that $t > k$. For each n, consider any committee member i and let $\tilde{\sigma}_{-i}^{n,k}$ be the following strategy combination for the others: t of them always vote 0, $t-k$ always vote 1, and the remaining k voters vote sincerely. Third, according to the voting rule f_ε, a committee member's vote is randomly selected to be decisive with probability ε/n and it is pivotal with probability $(1-\varepsilon)\cdot p_i$, where p_i is the probability for a tie among the other $2t$ votes. Under $\tilde{\sigma}_{-i}^{n,k}$, p_i is the probability that the k sincere voters all receive signal 1, a probability that depends on k but not on n (as long as $t > k$). Under $\tilde{\sigma}_{-i}^{n,k}$, i being pivotal is thus a strong indication that the state of nature is $\omega = 1$, so strong that the conditionally expected utility, given that i is pivotal, is maximized when i votes 1 irrespective of his or her own signal. Fourth, by continuity there exists an $\tilde{\varepsilon}_t > 0$ such that, against $\tilde{\sigma}_{-i}^{n,k}$, always voting 1 is a better reply for i than voting sincerely, for all $\varepsilon \in (0, \tilde{\varepsilon}_t)$. Moreover, $\tilde{\varepsilon}_t$ is increasing in t. Fifth, and finally: there exists a $t^* \in \mathbb{N}$ such that $\bar{\varepsilon}_t < \tilde{\varepsilon}_t$ for all $t \geq t^*$. For each $\varepsilon \in (\bar{\varepsilon}_t, \tilde{\varepsilon}_t)$, voting on alternative 1 irrespective of i's signal is a better reply for i, against $\tilde{\sigma}_{-i}^{n,k}$, than sincere voting. Hence, for such n and ε, sincere voting is not a dominant strategy or voter i under f_ε.

We conclude by noting that claim (i) in Theorem 4 holds, *mutatis mutandis*, also under incomplete information. For each committee size n (odd), let "nature" first draw the preference vector $\theta = \langle (\alpha_1, \beta_1), ..., (\alpha_n, \beta_n) \rangle \in \Theta^n$ according to some probability measure ν_n on Θ^n, such that, for some $\eta < 1$, inequality (13) holds with probability one. After this, each committee member i gets to know his or her own "type" (α_i, β_i) (only), receives his or her private signal $s_i \in \{0, 1\}$ and has to give his or her vote $v_i \in \{0, 1\}$. Other committee members' values are irrelevant for a committee member's voting decision when all others vote informatively. In order to know the expected utility associated with each of i's four pure local strategies, given her value pair, i only needs to know everybody's signal precisions (which are taken to be commonly known and the same for all committee members).

7. Robustness against irrationality

Experimental evidence from laboratory studies suggests that human subjects in committee decision problems of the kind analyzed here sometimes make mistakes, see Guarnaschelli *et al.* (2000). Awareness of a positive error rate in others' voting clearly influences the voting incentives of a rational committee member. We here extend our model to allow for this possibility.[18] Consider a committee consisting of n members. Assume that with probability $\lambda \in [0, 1]$ exactly one of these members suddenly becomes a *noise voter*, defined as a voter who votes randomly according to an exogenous probability distribution, irrespective of his or her private signal. Assume, moreover that such a noise voter's vote is statistically independent of the state of nature and all private signals. A committee member who is not a noise voter is called a rational voter.

We analyze a special case of this set-up. Let the number n of committee members be odd, assume that the two signals are equally precise, and assume that the voting rule is majority rule (the optimal rule in the unperturbed model, see Corollary 1). Assume also that the prior is uniform and that a noise voter randomizes uniformly. It is not difficult to then show that the potential presence of a noise voter in the committee increases each rational committee member's incentive to vote sincerely, if this voter expect all other rational committee members vote sincerely. The reason for

[18] Eliaz (2002) analyses the mechanism-design implementation problem with k faulty players among n players. Blais et al. (2008) use the 1 faulty player model to analyse experimental data on voting.

this is two-fold: a noise voter among the others increases the probability for a given rational voter of becoming pivotal, and it also increases the conditionally expected net utility gain from sincere voting (over insincere voting) when the informed voter is pivotal. Formally:

Proposition 3. *Consider majority rule in a committee with n odd, $\mu = 1/2$, $1/2 < q_0 = q_1 < 1$, and with a probability $\lambda \in [0, 1]$ for the presence of a noise voter who randomizes uniformly. The probability that a given committee member's vote will be pivotal under sincere voting is increasing in λ. Moreover, conditional upon being pivotal, the expected-utility difference between sincere and insincere voting is increasing in λ.*

8. Conclusion

The above analysis is restricted to a committee of equally "competent" members who receive private information of exogenously fixed precision and face a binary collective decision problem with no possibility of abstention. Despite these heroic simplifications, we believe that the qualitative conclusions hold more generally. First, suppose that the committee members are unequally "competent" in the sense that some members receive more precise signals than others. If the competence differences are known by all members, then weighted majoritarian rules, whereby more competent voters are given higher weights than less competent ones, may be superior the k-majority rules studied here. For a survey of results of this sort, see Grofman, Owen and Feld (1983), Owen, Grofman and Feld (1989) and Ben-Yashar and Milchtaich (2007). Under the usual majority rule, but with differing competence among the committee members, what can be said about equilibrium voting? Two main cases appear relevant for such a consideration. In the first case, each member i has precision parameters $q_0^i, q_1^i > 1/2$ and these are known by all committee members. In the second case, each member i has precision parameters $q_0^i, q_1^i > 1/2$ but these are know only by member i himself.[19] Let us briefly re-consider the statement and proof of Theorem 3. The quantities $g(k, n)$ have to be re-defined and will, in general, also depend on i. More precisely, each such quantity $g_i(k, n)$ will no longer be a simple product of two factors raised to powers k and n, but will be a complex multinomial sum; instead of just counting the number of signals of each type, one has to keep track of which signal was received by which member, and consider all permutations. With so defined quantities $g_i(k, n)$ in condition (3), the claim of Theorem 3 would remain true, and the

[19] Visser and Swank (2007) assume that committee members do not even know their own competence.

quantities $g_i(k,n)$ would be continuous in the parameter vector $\langle (q_0^1, q_1^1), ..., (q_0^n, q_1^n) \rangle$. Hence, Theorem 3 would be approximately correct for approximately equally competent committee members. Similar considerations apply to other equilibrium results. The second case, that of incomplete information concerning competence, appears to be particularly interesting for analyses of the incentive effects of transparency — that is, ex post revelation of individual votes. For studies of such settings, see Visser and Swank (2007), Gersbach and Hahn (2008), Swank, Visser and Swank (2008) and Hahn (2008).

A second direction for generalization, which would be valuable and challenging to explore, concerns the binary nature of both signals and choices. What can be said if the choice is binary but there are more than two signal values, perhaps just three, or a whole continuum? What if there are more than two choice alternatives? New results have recently been obtained for more general collective decision problems of this sort, see McLennan (2007).

A third direction would be to analyze equilibrium outcomes if abstention is an option and/or the number of voters is unknown by the voters. Such aspects may be less relevant for some committees but may play a major role in other committees and certainly in general elections. Krishna and Morgan (2007) undertake an investigation of precisely these two aspects, in a setting where the number of voters is a Poisson distributed random variable and each voter draws a random cost for casting a vote. Each voter only observes his or her own signal and voting cost. Krishna and Morgan assume that the voters are *ex-ante* identical, that the two states of nature are equally likely and that the two signals are equally precise. They show that sincere voting then is the unique Nash equilibrium under super-majority rules when the expected number of voters is large. Moreover, equilibrium participation rates are such that the outcome is asymptotically efficient. While their model thus is cast more in the mold of general elections, it would be interesting to explore whether (strategic) abstention, allowed for in their framework, can be introduced in our framework for a committee of fixed and know size, and whether the kind of preference heterogeneity that we here permit can be introduced into their framework. Here, we only note that sincere voting under our randomized majority rule will remain a strict Nash equilibrium also when abstention is allowed. However, our uniqueness claim may then fail.

A fourth avenue for future work would be to endogenize voters' signal precision. Before a committee meets, individual members usually make (typically unobserved) efforts to study the question at hand, so that they will be well informed at the meeting. However, as is well-known both by practitioners and theorists, this gives rise to a free-rider problem, whereby committee members tend to under-invest and arrive at the meeting less informed than what would be collectively desirable. For recent analyses of this moral hazard phenomenon in various models, see Mukhopadhaya (2003), Persico (2004), Gerardi and Yariv (2008) and Koriyama and Szentes (2007).

Finally, it would be useful to test the robustness of the conclusions to perturbations of committee members' objectives in empirically plausible directions, such as a (slight) preference for voting according to one's conviction (that is, voting sincerely). Of interest is also to analyze situations in which committee members to some extent care about others' esteem of their "competence" (signal precision), partially revealed by their voting, in case individual votes and the true state becomes publicly known ex post.

9. Appendix

We here provide mathematical proofs of claims not proved in the main text.

9.1. Theorem 1. Suppose that $\omega = 0$ and consider any positive integer n. The probability that voter i votes $v_i = s_i = 1$, when voting informatively, is $1 - q_0$. Under majority rule, the probability of a wrong decision in this state is thus

$$\Pr\left[X_n = 1 \mid \omega = 0\right] \leq \Pr\left[\frac{1}{n}\sum_{i=1}^{n} s_i \geq \frac{1}{2} \mid \omega = 0\right]$$

Conditional upon $\omega = 0$, the random variables $\{s_i\}_{i=1}^n$ are independent, with the same Bernoulli distribution. Hence, according to the Central Limit Theorem (see, for example, Theorem 27.1 in Billingsley, 1995), their average, $\frac{1}{n}\sum_{i=1}^{n} s_i$ (given $\omega = 0$), converges in distribution towards the normal distribution with mean $1 - q_0$ and variance $q_0(1 - q_0)/n$. Since $1 - q_0 < \frac{1}{2}$:

$$\lim_{n\to\infty} \Pr\left[\frac{1}{n}\sum_{i=1}^{n} s_i \geq \frac{1}{2} \mid \omega = 0\right] = 0$$

The same argument applies to the state $\omega = 1$, and the result follows from the identity

$$\Pr\left[X_n(\omega) \neq \omega\right] = (1 - \mu)\Pr\left[X_n = 1 \mid \omega = 0\right] + \mu \Pr\left[X_n = 0 \mid \omega = 1\right]$$

9.2. Lemma 1. To see this, first note that existence follows from the finiteness of the set of alternatives, D. Secondly, suppose that $d^* \in D^*$. Since all signals have the same precision, there exists some symmetric function $d \in D$ such that $W(d) = W(d^*)$. Then $d(s_1, ..., s_n)$ is a function $h : \{0, 1, ..., n\} \to \{0, 1\}$ of the signal sum: $d(s) \equiv h(\sum s_i)$. Clearly $d \in D^*$, which implies that h is increasing. For if $h(k) = 1$ then also $h(k+1) = 1$ since by assumption $q_0, q_1 > 1/2$. Since h is increasing, d is a k-majority rule for some $k \in \{0, 1, ..., n, n+1\}$. If the signal informativeness condition (3) holds strictly, and $\alpha_i, \beta_i > 0$, it is never optimal to disregard all signals, so then $k \in \{1, ..., n\}$.

9.3. Theorem 2. Write $W(f^k)$ in the following way, where the random variable N_1 is the number of signals 1 received, $U_0 = (1-\mu)\sum_{i=1}^n u_{00}^i$ and $U_1 = (1-\mu)\sum_{i=1}^n u_{11}^i$, two real numbers:

$$\begin{aligned} W(f^k) &= -(1-\mu)\bar{\beta}_n \Pr[x=1 \mid \omega=0] - \mu\bar{\alpha}_n \Pr[x=0 \mid \omega=1] + U_0 + U_1 \\ &= -(1-\mu)\bar{\beta}_n \Pr[N_1 \geq k \mid \omega=0] - \mu\bar{\alpha}_n \Pr[N_1 < k \mid \omega=1] + U_0 + U_1 \end{aligned}$$

Hence,

$$W(f^{k+1}) - W(f^k) = (1-\mu)\bar{\beta}_n \Pr[N_1 = k \mid \omega=0] - \bar{\alpha}_n \Pr[N_1 = k \mid \omega=1]$$

and thus

$$\begin{aligned} W(f^{k+1}) \leq W(f^k) &\iff \bar{\gamma} \geq \frac{\Pr[N_1 = k \mid \omega=0]}{\Pr[N_1 = k \mid \omega=1]} \\ &\iff \bar{\gamma} \geq \frac{(1-q_0)^k q_0^{n-k}}{q_1^k (1-q_1)^{n-k}} = g(k,n) \end{aligned}$$

Likewise:

$$\begin{aligned} W(f^{k-1}) \leq W(f^k) &\iff \bar{\gamma} \leq \frac{\Pr[N_1 = k-1 \mid \omega=0]}{\Pr[N_1 = k-1 \mid \omega=1]} \\ &\iff \bar{\gamma} \leq \frac{(1-q_0)^{k-1} q_0^{n-k+1}}{q_1^{k-1} (1-q_1)^{n-k+1}} = g(k-1,n) \end{aligned}$$

Since $g(k,n)$ is decreasing in k, $W(f^k) \geq W(f^h)$ for all $h = k+1, k+2, ..., n$ if and only if $\bar{\gamma} \geq g(k,n)$. Likewise, $W(f^k) \geq W(f^h)$ for all $h = k-1, k-2, ..., 1$ if and only if $\bar{\gamma} \leq g(k-1,n)$. Hence, as k increases from 1 to n, $W(f^k)$ reaches its maximum value either at a unique k or (non-generically) at two adjacent values, $k-1$ and k. As noted in footnote 6, $k=0$ and $k=n+1$ are never optimal.

9.4. Corollary 1. Condition (4) is equivalent with

$$\left[\frac{(1-q_0)(1-q_1)}{q_0 q_1}\right]^k \left(\frac{q_0}{1-q_1}\right)^n \leq \bar{\gamma}_n \leq \left[\frac{(1-q_0)(1-q_1)}{q_0 q_1}\right]^k \left(\frac{q_1}{1-q_0}\right)\left(\frac{q_0}{1-q_1}\right)^{n+1}$$

or

$$\left(\frac{q_0}{1-q_1}\right)^n \leq \bar{\gamma}_n \cdot \left[\frac{q_0 q_1}{(1-q_0)(1-q_1)}\right]^k \leq \left(\frac{q_1}{1-q_0}\right)\left(\frac{q_0}{1-q_1}\right)^{n+1}$$

Taking logarithms and dividing through with n, we obtain

$$\begin{aligned} \ln\left(\frac{q_0}{1-q_1}\right) &\leq \frac{1}{n}\ln\bar{\gamma}_n + \frac{k}{n}\ln\left[\frac{q_0 q_1}{(1-q_0)(1-q_1)}\right] \\ &\leq \frac{1}{n}\ln\left(\frac{q_1}{1-q_0}\right) + \left(1+\frac{1}{n}\right)\ln\left(\frac{q_0}{1-q_1}\right) \end{aligned}$$

As $n \to \infty$, the upper bound converges to the lower bound, $\ln\left(\frac{q_0}{1-q_1}\right)$, and $\frac{1}{n}\ln\bar{\gamma}_n$ tends to zero since, in force of the value-boundedness condition (6). This establishes that $\lim_{n\to\infty}\kappa_n = \kappa^*$.

In order to establish the claimed bounds on κ^* (defined in equation (7)), first suppose that $q_1 \leq q_0$. Then $\frac{q_1}{1-q_0} \leq \frac{q_0}{1-q_1}$, from which we deduce that $\kappa^* \geq 1/2$ and hence $1 - q_0 \leq \kappa^*$. This establishes the lower bound. To obtain the upper bound, $\kappa^* \leq q_1$, note that this inequality can be re-written, after some manipulation, as

$$q_1 \ln(1-q_0) + (1-q_1)\ln q_0 \leq q_1 \ln q_1 + (1-q_1)\ln(1-q_1)$$

The right-hand side is independent of q_0, while the left-hand side is decreasing in q_0. Thus, the claimed inequality $\kappa^* \leq q_1$ holds for all $q_0 \in [q_1, 1]$ if and only if it holds for $q_0 = q_1$. Writing the inequality for that special case, one obtains

$$q_1 \ln(1-q_1) + (1-q_1)\ln q_1 \leq q_1 \ln q_1 + (1-q_1)\ln(1-q_1)$$

or, equivalently,

$$(2q_1 - 1)\ln(1-q_1) \leq (2q_1 - 1)\ln q_1$$

an inequality which clearly holds, since $q_1 \geq 1/2$. This establishes the upper bound.

Secondly, suppose that $q_1 \leq q_0$. Then the above reasoning, switching q_0 and q_1, gives us $1 - q_1 \leq 1 - \kappa^* \leq q_0$, which is equivalent with $1 - q_0 \leq \kappa^* \leq q_1$.

9.5. Theorem 3. For any $n, k \in \mathbb{N}$ such that $1 \leq k \leq n$, consider voter i and denote by T the event of a tie among the others, that is, that exactly $k-1$ of the other voters receive the signal 1 and exactly $n-k$ receive the signal 0.

First, suppose that i has received the signal $s_i = 0$. Should i then vote on alternative 0? The probability for the joint event that $s_i = 0$ and that there is a tie among the others, conditional on the state $\omega = 0$, is

$$\Pr\left[\mathcal{T} \ \wedge \ s_i = 0 \mid \omega = 0\right] = \binom{n-1}{k-1} q_0^{n-k}(1-q_0)^{k-1}$$

Likewise, conditional on the state $\omega = 1$, we have

$$\Pr[\mathcal{T} \ \wedge \ s_i = 0 \mid \omega = 1] = \binom{n-1}{k-1} q_1^{k-1} (1-q_1)^{n-k}$$

Therefore, the probability for the joint event that i receives the signal 0 and there is a tie among the others is

$$\Pr[\mathcal{T} \ \wedge \ s_i = 0] = \binom{n-1}{k-1} \left[(1-\mu) q_0^{n-k} (1-q_0)^{k-1} + \mu q_1^{k-1} (1-q_1)^{n-k}\right]$$

Since the probability of receiving the signal 0 is $\Pr[s_i = 0] = (1-\mu) q_0 + \mu (1-q_1)$, committee member i attaches the following conditional probability of a tie among the others:

$$p_0(k) = \Pr[\mathcal{T} \mid s_i = 0] = \binom{n-1}{k-1} \frac{(1-\mu) q_0^{n-k} (1-q_0)^{k-1} + \mu q_1^{k-1} (1-q_1)^{n-k}}{(1-\mu) q_0 + \mu (1-q_1)}$$

We are now in position to compute the difference in expected utility for voter i between casting the sincere vote $v_i = 0$ instead of the insincere vote $v_i = 1$, when $s_i = 0$:

$$\Delta u_i = \mathbb{E}[u_i \mid s_i = v_i = 0] - \mathbb{E}[u_i \mid s_i = 0 \ \wedge \ v_i = 1]$$

Because i's vote affects the collective decision x only in the event $\mathcal{T}$, we have

$$\Delta u_i = p_0(k) \cdot (\mathbb{E}[u_i \mid \mathcal{T} \ \wedge \ s_i = v_i = 0] - \mathbb{E}[u_i \mid \mathcal{T} \ \wedge \ s_i = 0 \ \wedge \ v_i = 1])$$

where

$$\begin{aligned} \mathbb{E}[u_i \mid \mathcal{T} \ \wedge \ s_i = v_i = 0] &= \nu_i - \alpha_i \Pr[\omega = 1 \mid \mathcal{T} \ \wedge \ s_i = 0] \\ \mathbb{E}[u_i \mid \mathcal{T} \ \wedge \ s_i = 0 \ \wedge \ v_i = 1] &= \nu_i - \beta_i \Pr[\omega = 0 \mid \mathcal{T} \ \wedge \ s_i = 0] \end{aligned}$$

and ν_i is the conditionally expected utility of taking the right decision, $x = \omega$, conditional on the event $\mathcal{T} \wedge s_i = 0$.[20] By Bayes' law (factorials cancel):

$$\begin{aligned} \Pr[\omega = 0 \mid \mathcal{T} \ \wedge \ s_i = 0] &= \frac{(1-\mu) \Pr[\mathcal{T} \ \wedge \ s_i = 0 \mid \omega = 0]}{\Pr[\mathcal{T} \ \wedge \ s_i = 0]} \\ &= \frac{(1-\mu) q_0^{n-k+1} (1-q_0)^{k-1}}{(1-\mu) q_0^{n-k} (1-q_0)^{k-1} + \mu q_1^{k-1} (1-q_1)^{n-k+1}} \end{aligned}$$

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$$\kappa_i = u_{00}^i \Pr[\omega = 0 \mid \mathcal{T} \ \wedge \ s_i = 0] + u_{11}^i \Pr[\omega = 1 \mid \mathcal{T} \ \wedge \ s_i = 0]$$

Hence:

$$\Delta u_i = \binom{n-1}{k-1} \frac{(1-\mu)\, q_0^{n-k+1} (1-q_0)^{k-1} \beta_i - \mu q_1^{k-1} (1-q_1)^{n-k+1} \alpha_i}{(1-\mu)\, q_0 + \mu (1-q_1)} \tag{15}$$

The condition for $\Delta u_i \geq 0$, that is, for vote $v_i = 0$ to be optimal for i, given signal $s_i = 0$, is thus

$$(1-\mu)\, q_0^{n-k+1} (1-q_0)^{k-1} \beta_i \geq \mu q_1^{k-1} (1-q_1)^{n-k+1} \alpha_i$$

which can be written as

$$\gamma_i \leq \left(\frac{1-q_0}{q_1}\right)^{k-1} \left(\frac{q_0}{1-q_1}\right)^{n-k+1} = g(k-1, n) \tag{16}$$

Hence, if all others vote sincerely, sincere voting on alternative 0 (that is, to chose $v_i = 0$ when $s_i = 0$) is a best reply for i if and only if the right inequality in (8) is met.

Secondly, suppose that i has received the signal $s_i = 1$. By the same logic, if all others vote sincerely, sincere voting on alternative 1 is a best reply for i if and only if

$$\gamma_i \geq \left(\frac{1-q_0}{q_1}\right)^{k} \left(\frac{q_0}{1-q_1}\right)^{n-k} = g(k, n) \tag{17}$$

9.6. Proposition 1. We prove the proposition by establishing three claims.

Claim I: If sincere voting is an equilibrium under some f^k, then $f^k \in D^*$.

To establish this claim, suppose that sincere voting is an equilibrium under f^k, where $1 \leq k \leq n$. By Theorem 3, $g(k,n) \leq \gamma_i \leq g(k-1,n)$ for all i. Hence

$$(1-\mu)\, \beta_i g(k,n) \leq \mu \alpha_i \leq (1-\mu)\, \beta_i g(k-1,n) \qquad \forall i$$

so, by summation, also $\bar{\beta}_n g(k,n) \leq \bar{\alpha}_n \leq \bar{\beta}_n g(k-1,n)$, or, equivalently, $g(k,n) \leq \bar{\gamma}_n \leq g(k-1,n)$. Thus, by Theorem 2, $f^k \in D^*$.

Claim II: If the committee is homogeneous and $f^k \in D^*$, then sincere voting is an equilibrium under f^k.

To establish this claim, suppose that the committee is homogeneous and that $f^k \in D^*$. Then $\bar{\beta}_n g(k,n) \leq \bar{\alpha}_n \leq \bar{\beta}_n g(k-1,n)$ by Theorem 2. Moreover, by homogeneity, there exists a positive integer t such that $t-1 \leq \tau(\gamma_i) < t$ for all i. Hence, by definition (11),

$$\left[\frac{q_0}{1-q_1}\right]^{n-t} \left[\frac{1-q_0}{q_1}\right]^{t} < \gamma_i \leq \left[\frac{q_0}{1-q_1}\right]^{n-t-1} \left[\frac{1-q_0}{q_1}\right]^{t-1} \qquad \forall i \tag{18}$$

Equivalently: $g(t,n) < \gamma_i \leq g(t-1,n)$ for all i. For generic parameter values, this implies that $t = k$, so sincere voting is an equilibrium under f^k, by Theorem 3.

Claim III: If sincere voting is an equilibrium under some f^k, then the committee is homogeneous.

To establish this claim, suppose again that sincere voting is an equilibrium under f^k, where $1 \leq k \leq n$. By Theorem 3, $g(k,n) \leq \gamma_i \leq g(k-1,n)$ for all i. For generic parameter values, both inequalities are strict for all i, and hence, by the same calculation as above, $k-1 < \tau(\gamma_i) < k$ for all i, implying that $T(\gamma_i) = k$ for all i, and hence the committee is homogeneous.

9.7. Theorem 4.

Claim (i). Write $n = 2t+1$, that is, for any committee member, t is half of the rest of the committee. To see that sincere voting under f^ε is a strict Nash equilibrium, first note that $\Delta u_i(\varepsilon) > 0$ if and only if

$$\frac{\varepsilon}{1-\varepsilon} > \frac{2t+1}{B_i} \cdot \binom{2t}{t} \left[\alpha_i \mu q_1^t (1-q_1)^{t+1} - \beta_i (1-\mu) q_0^{t+1} (1-q_0)^t\right] \tag{19}$$

where the factor $B_i = \beta_i (1-\mu) q_0 - \alpha_i \mu (1-q_1)$ is positive by (3). By Stirling's formula,

$$\binom{2t}{t} = \frac{(2t)!}{(t!)^2} = \frac{4^t}{\sqrt{\pi t}}(1+o(t))$$

so the right-hand side of (19) is approximated by

$$\begin{aligned} & (1+o(t)) \cdot \frac{(2t+1)4^t}{B_i\sqrt{\pi t}} \cdot \left[\alpha_i \mu q_1^t (1-q_1)^{t+1} - \beta_i (1-\mu) q_0^{t+1} (1-q_0)^t\right] \\ \leq \; & (1+o(t)) \cdot \frac{2\alpha_i \mu (1-q_1)}{B_i\sqrt{\pi}} [4q_1(1-q_1)]^t \sqrt{t} \leq (1+o(t)) \cdot \frac{C_i}{B_i} \cdot a^t \sqrt{t} \end{aligned}$$

where $C_i = 2\alpha_i \mu (1-q_1)/\sqrt{\pi}$ and $a = 4q_1(1-q_1) < 1$. Hence, (19) is met if

$$\frac{\varepsilon}{1-\varepsilon} > \frac{C_i}{B_i}(1+o(t))a^t\sqrt{t}$$

A sufficient condition for this to hold is that

$$\varepsilon > \frac{C_i}{B_i}(1+o(t))a^t\sqrt{t} \tag{20}$$

The preference boundedness condition (6) together with condition (13) implies that C_i/B_i is uniformly bounded: there exists a $D \in \mathbb{R}$ such that $C_i/B_i < D$ for all i.[21] Let $\varepsilon = b^t$ with $a < b < 1$. Then $\varepsilon \to 0$ as $t \to +\infty$. Moreover, since

$$\left(\frac{b}{a}\right)^t \frac{1}{\sqrt{t}} \to +\infty \quad \text{as} \quad t \to \infty$$

(20) holds for all t large enough, irrespective of how large D is.

The same reasoning applies to the expected utility upon receiving the signal $s_i = 1$. This proves claim (i) for $\varepsilon = b^t$, for any b such that

$$\max\{4q_0(1-q_0), 4q_1(1-q_1)\} < \quad b \quad < \quad 1$$

where we note that lower bound indeed is less than 1 since $q_0, q_1 > 1/2$.

Claim (ii). We consider voter strategies $\sigma_i : \{0,1\} \to [0,1]$, for $i = 1, ..., n$, that map voter i's signal s_i to a probability $\sigma_i(s_i)$ for i voting on alternative 1 (and voting on alternative 0 with the complementary probability, $1 - \sigma_i(s_i)$). Sincere voting thus is the strategy $\sigma_i(s_i) \equiv s_i$.

Consider first a voter i who has received signal $s_i = 0$. Denote by $\mathcal{T}_i$ the event of a tie among all other votes. Such a tie may arise by chance, even for given signals, if other voters randomize their votes. However, since signals, and hence also votes, are statistically independent, conditionally upon the state ω, we have, under any strategy profile $(\sigma_1, ..., \sigma_n)$:

$$\Pr[\mathcal{T}_i \ \wedge \ s_i = 0 \mid \omega] = \Pr[\mathcal{T}_i \mid \omega] \cdot \Pr[s_i = 0 \mid \omega]$$

for each $\omega \in \{0,1\}$. In the base-line model (simple majority rule), the difference in expected utility for voter i between voting 0 and 1, conditional on having received signal 0, is

$$\begin{aligned} \Delta u_i^0(0) &= \beta_i \Pr[\mathcal{T}_i \ \wedge \ \omega = 0 \mid s_i = 0] - \alpha_i \Pr[\mathcal{T}_i \ \wedge \ \omega = 1 \mid s_i = 0] \\ &= \beta_i \frac{(1-\mu)\, q_0}{\Pr[s_i = 0]} \Pr[\mathcal{T}_i \mid \omega = 0] - \alpha_i \frac{\mu(1-q_1)}{\Pr[s_i = 0]} \Pr[\mathcal{T}_i \mid \omega = 1] \end{aligned}$$

[21] To see this, note that $C_i/B_i \leq D$ iff

$$\frac{1}{\gamma_i} \cdot \frac{q_0}{1-q_1} - 1 \geq \frac{1}{D}$$

and let $\eta = D/(D+1)$.

Thus, sincere voting in this case is optimal if and only if

$$\beta_i (1-\mu) q_0 \Pr[\mathcal{T}_i \mid \omega = 0] - \alpha_i \mu (1-q_1) \Pr[\mathcal{T}_i \mid \omega = 1] \geq 0 \tag{21}$$

and mixing is optimal if and only if this equation is an equality.

Likewise, the difference in expected utility for voter i between voting 1 and 0, conditional on having received signal 1, is

$$\Delta u_i^1(0) = \alpha_i \frac{\mu q_1}{\Pr[s_i = 1]} \Pr[\mathcal{T}_i \mid \omega = 1] - \beta_i \frac{(1-\mu)(1-q_0)}{\Pr[s_i = 1]} \Pr[\mathcal{T}_i \mid \omega = 0]$$

and thus sincere voting in this case is optimal if only if

$$\alpha_i \mu q_1 \Pr[\mathcal{T}_i \mid \omega = 1] - \beta_i (1-\mu)(1-q_0) \Pr[\mathcal{T}_i \mid \omega = 0] \geq 0 \tag{22}$$

and mixing is optimal if and only if this last equation is an equality.

Summing the left hand sides of (21) and (22) yields:

$$\beta_i (1-\mu)(2q_0 - 1) \Pr[\mathcal{T}_i \mid \omega = 0] + \alpha_i \mu (2q_1 - 1) \Pr[\mathcal{T}_i \mid \omega = 1]$$

Because $q_0, q_1 > 1/2$, this is a strictly positive number as soon as at least one of the two conditional probabilities $\Pr[\mathcal{T}_i \mid \omega]$ is positive. Therefore, at least one of the two inequalities (21) and (22) is necessarily strict in such cases, which means that each voter must be voting sincerely on (at least) one signal. In particular no voter can be strictly mixing on both signals. The same argument hold for all $\varepsilon > 0$. Moreover, the probability for a tie is positive in all Nash equilibria when $\varepsilon > 0$, because if the probability for a tie were zero, then a voter i could unilaterally deviate by voting sincerely, and thereby increase his or her conditionally expected utility, since the vote will matter only when vote i is singled out to determine the collective decision, an event with positive probability ε/n.

In sum: For each $\varepsilon > 0$, and in each equilibrium under the associated voting rule f_ε, each voter is voting sincerely on at least one signal.

From this it follows that there exists one signal value, say 0, such that at least half of the population vote sincerely when receiving this signal. Without loss of generality, we may take the point of view of individual $i = 2t + 1$ and suppose that individuals $j = 1, ..., t$ vote $v_j = 0$ when receiving signal $s_j = 0$.

Let $\mathcal{N}_0$ denote the random number of votes 0 among voters $1, ..., 2t$, conditional upon $\omega = 0$. Then:

$$\Pr[\mathcal{T}_i \mid \omega = 0] = \Pr[\mathcal{N}_0 = t]$$

One can decompose the random variable $\mathcal{N}_0$ as $\mathcal{N}_0 = \mathcal{X}_0 + \mathcal{Y}_0$, where

$$\mathcal{X}_0 = \sum_{j=1}^{t} \mathbf{1}_{\{s_j=0|\omega=0\}}$$

and

$$\mathcal{Y}_0 = \sum_{j=1}^{t} \mathbf{1}_{\{s_j=1\wedge v_j=0|\omega=0\}} + \sum_{j=t+1}^{2t} \mathbf{1}_{\{v_j=0|\omega=0\}}$$

Notice that

$$\begin{aligned} \Pr[\mathcal{N}_0 = t] &= \sum_{k=0}^{t} \Pr[\mathcal{X}_0 = k] \cdot \Pr[\mathcal{Y}_0 = t-k] \\ &\leq \max_{0\leq k\leq t} \Pr[\mathcal{X}_0 = k] \end{aligned}$$

We do not know the probability distribution of $\mathcal{Y}_0$, because of possible mixing. However, we know that $\mathcal{X}_0$ is binomial with parameters q_0 and t. Therefore,

$$\max_{0\leq k\leq t} \Pr[\mathcal{X}_0 = k] = \Pr[\mathcal{X}_0 = \lfloor q_0 t \rfloor]$$

where $\lfloor q_0 t \rfloor$ denotes the integer part of $q_0 t$. If $q_0 t$ is itself an integer, then

$$\Pr[\mathcal{N}_0 = t] \leq \Pr[\mathcal{X}_0 = q_0 t] = \binom{q_0 t}{t} q_0^{q_0 t}(1-q_0)^{t-q_0 t}$$

and, using Stirling's formula:

$$\Pr[\mathcal{X}_0 = q_0 t] \sim \frac{1}{\sqrt{2\pi t q_0(1-q_0)}}$$

This last property can be shown to actually hold even if $q_0 t$ is not an integer (we leave a verification of this to the interested reader). To obtain a majorization of this probability, we may note, for instance, that it follows that there exists an A (which only depends on q_0) such that for all $t > A$,

$$\Pr\left[\mathcal{T}_{2t+1} \mid \omega = 0\right] < B/\sqrt{t}$$

where $B = 1/\sqrt{q_0(1-q_0)}$. Thus, for $t > A$ and any

$$\varepsilon > \frac{1}{\Pr[s=1]} \beta_i (1-\mu)(1-q_0) \frac{B}{\sqrt{t}}$$

(22) is a strict inequality, which means that sincere voting on signal $s_i = 1$ is strictly optimal for the considered voter $i = 2t + 1$. We also note that the lower bound on ε equals

$$\frac{\beta_i(1-\mu)(1-q_0)}{(1-q_1)\mu+(1-\mu)(1-q_0)}\frac{B}{\sqrt{t}}$$

It follows that there exists a constant B' (which depends on all the parameters) such that for $t > A$ and $\varepsilon > B'/\sqrt{t}$, sincere voting on signal $s = 1$ is strictly optimal for all voters. The parameter values (α_i, β_i) for different voters all belong to a bounded set Θ, so we can take B' to be a constant, independent of t.

If all voters vote sincerely on signal $s = 1$, then the random number $\mathcal{N}_1$ of votes 1 among voters $j = 1, ...2t$, conditionally on $\omega = 1$ can be decomposed **as** $\mathcal{N}_1 = \mathcal{X}_1 + \mathcal{Y}_1$, where

$$\mathcal{X}_1 = \sum_{j=1}^{2t} \mathbf{1}\{s_j = 1 \mid \omega = 1\}$$

and

$$\mathcal{Y}_1 = \sum_{j=1}^{2t} \mathbf{1}\{s_j = 0 \wedge v_j = 1 \mid \omega = 1\}$$

Here $\mathcal{X}_1$ is binomially distributed with parameters $2t$ and q_1. Again we note that

$$\begin{aligned}\Pr[\mathcal{T}_i \mid \omega = 1] &= \Pr[\mathcal{N}_1 = t] = \Pr[\mathcal{X}_1 + \mathcal{Y}_1 = t] \\ &= \sum_{k=0}^{t} \Pr[\mathcal{X}_1 = k] \cdot \Pr[\mathcal{Y}_1 = t-k] \\ &\leq \max_{0 \leq k \leq t} \Pr[\mathcal{X}_1 = k]\end{aligned}$$

The mode of the binomial distribution of $\mathcal{X}_1$ is reached at $\lfloor 2q_1 t \rfloor$, an integer that exceeds t. It follows that

$$\max_{0 \leq k \leq t} \Pr[\mathcal{X}_1 = k] = \Pr[\mathcal{X}_1 = t] = \binom{2t}{t} q_0^t (1-q_0)^t$$

Using Stirling's approximation formula, one again finds that this quantity is decreasing (this time exponentially) with t.

The same reasoning as above can be applied to equation (21): the negative term, $-\alpha_i\mu(1-q_1)\Pr[\mathcal{T}_i \mid \omega = 1]$, is asymptotically close to zero, and we conclude that there exist numbers A' and B'' such that if $t > A'$ and $\varepsilon > B'/\sqrt{t}$, inequalities (21) and (22) are both strict for all i, which means that all voters vote sincerely on both signals. This establishes claim (ii).

9.8. Corollary 6. Suppose first that $\omega = 0$ and consider sincere voting under a randomized voting rule f_{ε_t}, for a committee of fixed size $n = 2t + 1$. The probability that committee member i then votes $s_i = 1$ is, by definition $1 - q_0$. If the collective decision is taken by majority rule applied to all n votes, the probability of a wrong decision, $X_t = 1$, is some number $Q_t \in [0, 1]$. So the probability of a wrong decision, given $\omega = 0$, is

$$\Pr[X_n = 1 \mid \omega = 0] = \varepsilon_t(1 - q_0) + (1 - \varepsilon_t)Q_t$$

Since $\varepsilon_n \to 0$, this probability tends to 0 if $Q_t \to 0$ as $t \to \infty$. It thus remains to prove that $Q_t \to 0$. We proceed just as in the proof of Condorcet's jury theorem. First note that, since $n = 2t + 1$ is odd:

$$Q_t = \Pr\left[\sum_{i=1}^{2t+1} s_i > t \mid \omega = 0\right]$$

Conditional upon $\omega = 0$, the signals s_i are independent, with the same Bernoulli distribution. Hence, according to the Central Limit Theorem, $\frac{1}{n}\sum_{i=1}^{n} s_i$, given $\omega = 0$, converges in distribution to the normal distribution with mean $1 - q_0$ and variance $q_0(1 - q_0)/n$. Since $q_0 > 1/2$:

$$\lim_{t\to\infty} \Pr\left[\frac{1}{n}\sum_{i=1}^{2t+1} s_i > \frac{1}{2} \mid \omega = 0\right] = 0$$

The same argument applies to the case $\omega = 1$.

9.9. Proposition 3. Let $n = 2t + 1$, $\mu = 1/2$ and $q_0 = q_1 = q$. In order to prove the first claim in the proposition, let $p(\lambda)$ be the conditional probability for any given committee member i's vote to be pivotal, conditional upon the event that i is not a noise voter. For $\lambda = 0$ we have, from the calculations in our baseline model,

$$p(0) = \binom{2t}{t} \cdot [q(1-q)]^t$$

For $\lambda > 0$:

$$p(\lambda) = \frac{\lambda}{(1-\lambda)n + \lambda} p(1) + \frac{(1-\lambda)n}{(1-\lambda)n + \lambda} p(0)$$

so it remains to identify $p(1)$. We obtain

$$\begin{aligned} p(1) &= \frac{1}{2}\binom{2t-1}{t}\left[q^t(1-q)^{t-1}+q^{t-1}(1-q)^t\right] \\ &= \binom{2t-1}{t}\cdot\frac{[q(1-q)]^t}{2}\cdot\left(\frac{1}{1-q}+\frac{1}{q}\right)= \\ &= \binom{2t-1}{t}\cdot\frac{[q(1-q)]^{t-1}}{2} \end{aligned}$$

Hence, for any $t \geq 1$:

$$\frac{p(1)}{p(0)}=\frac{1}{4q(1-q)}$$

and thus $p(\lambda)/p(0) \geq 1$ for all $q \in [1/2, 1]$ with strict inequality when $q > 1/2$. This proves the first claim in the proposition.

In order to prove the second claim, suppose that voter i is an informed voter with signal $s_i = 0$. Conditional upon being pivotal under majority rule, what is the conditional probability for each state? Assume first that $\lambda = 0$. We are then back in the standard model and the conditional probability for state $\omega = 0$, conditional upon i's signal $s_i = 0$ and being pivotal under sincere voting, is q (the t other signals 0 cancel the t other signals 1, because $\mu = 1/2$ and $q_0 = q_1$). Secondly, assume that $\lambda = 1$. Being pivotal, i knows that there are either t signals 0 and $t-1$ signals 1, or $t-1$ signals 0 and t signals 1, with equal probability for both events (since the noise voter randomizes uniformly). The conditional probability for state $\omega = 0$, conditional upon i's signal being $s_i = 0$ and upon i's vote being pivotal under sincere voting, is no less than q. To see this, let $\mathcal{T}$ be the event of a tie among the $2t$ other committee members (including the noise voter), let N_0 and N_1 be the (random) numbers of signals 0 and 1 among the other $2t-1$ informed voters:

$$\begin{aligned} \Pr[\omega=0 \mid \mathcal{T} \ \wedge \ s_i=0] &= \frac{1}{2}\Pr[\omega=0 \mid s_i=0 \wedge N_0=t \wedge N_1=t-1]+ \\ &\quad +\frac{1}{2}\Pr[\omega=0 \mid s_i=0 \wedge N_0=t-1 \wedge N_1=t] \\ &= \frac{1}{2}\Pr[\omega=0 \mid (t+1 \text{ signals } 0) \wedge (t-1 \text{ signals } 1)]+ \\ &\quad +\frac{1}{2}\Pr[\omega=0 \mid (t \text{ signals } 0) \wedge (t \text{ signals } 1)] \\ &= \frac{1}{2}\Pr[\omega=0 \mid (t+1 \text{ signals } 0) \wedge (t-1 \text{ signals } 1)]+\frac{1}{2} \end{aligned}$$

Moreover,

$$\Pr\left[\omega = 0 \mid (t+1 \text{ signals } 0) \wedge (t-1 \text{ signals } 1)\right] = \frac{q^{t+1}\left(1-q\right)^{t-1}}{2\Pr\left[(t+1 \text{ signals } 0) \wedge (t-1 \text{ signals } 1)\right]}$$

and

$$\Pr\left[(t+1 \text{ signals } 0) \wedge (t-1 \text{ signals } 1)\right] = \frac{1}{2}\left[q^{t+1}\left(1-q\right)^{t-1} + \left(1-q\right)^{t+1} q^{t-1}\right]$$

Hence,

$$\begin{aligned}\Pr\left[\omega = 0 \mid \mathcal{T} \ \wedge \ s_i = 0\right] &= \frac{1}{2} + \frac{1}{2} \cdot \frac{q^{t+1}\left(1-q\right)^{t-1}}{q^{t+1}\left(1-q\right)^{t-1} + \left(1-q\right)^{t+1} q^{t-1}} \\ &= \frac{1}{2} + \frac{1}{2} \cdot \frac{q^2}{q^2 + \left(1-q\right)^2}\end{aligned}$$

It is not difficult to verify that the quantity on the right-hand side exceeds q when $1/2 < q < 1$ and equals q when $q = 1$.

A similar calculation holds for the case when $s_i = 1$.

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