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**INNOVATION, FIRM DYNAMICS,  
AND INTERNATIONAL TRADE**

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We present a general equilibrium model of the decisions of firms to innovate and to engage in international trade. We use the model to analyze the impact of a reduction in international trade costs on firms' process and product innovative activity. We first show analytically that if all firms export with equal intensity, then a reduction in international trade costs has no impact at all on firms' investments in process innovation. We then show that if only a subset of firms exports, a decline in marginal trade costs raises process innovation in exporting firms relative to that of non-exporting firms. This reallocation of process innovation reinforces existing patterns of comparative advantage, and leads to an amplified response of trade volumes and output over time. In a quantitative version of the model, we show that the increase in process innovation is largely offset by a decline in product innovation. We find that, even if process innovation is very elastic and leads to a large dynamic response of trade, output, consumption, and the firm size distribution, the dynamic welfare gains are very similar to those in a model with inelastic process innovation.

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Мы представляем модель общего равновесия, в которой фирмы принимают решение о целесообразности инноваций и вовлечения в международную торговлю. Мы используем модель для анализа последствий снижения издержек международной торговли с точки зрения инновационного развития фирмы и ее товарной инновации. Сначала мы показываем аналитически, что если все фирмы производят продукцию на экспорт с одинаковой интенсивностью, то снижение издержек международной торговли не оказывает никакого воздействия на инвестиции фирм в инновационное развитие. Затем мы демонстрируем, что если только часть фирм производит продукцию на экспорт, то сокращение предельных издержек торговли приводит к ускорению инновационного развития фирм, производящих продукцию на экспорт, по сравнению с фирмами, которые не экспортируют свою продукцию. Данное перераспределение инновационной активности приводит к укреплению существующей структуры сравнительных преимуществ и к более сильной реакции в объемах торговли и выпуска во времени. Используя расчетную версию модели, мы показываем, что более бурное инновационное развитие в значительной степени компенсируется сокращением товарных инноваций. Даже если инновационное развитие является очень эластичным и ведет к сильной динамической реакции объема торговли, выпуска, потребления и распределения размера фирмы, то динамические выигрыши в благосостоянии сопоставимы с результатами модели с неэластичным инновационным развитием.

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# 1 Introduction

Over the past several decades, there has been a striking growth in the share of international trade in output, both for the US economy and for the world as a whole. How has this expansion of opportunities for international trade changed firms' incentives to engage in innovative activities?

In this paper, we present a simple general equilibrium model of the decisions of firms to innovate and to participate in international trade. Motivated by the observation that there is large heterogeneity in export behavior across firms even within narrowly defined sectors, recent research in international trade has modeled comparative advantage as an attribute of the firm (see, for example, Melitz 2003 and Bernard, Jensen, Eaton, and Kortum 2003). In these models, each firm has a stock of some firm specific factor that determines its current profit opportunities. Examples of this firm specific factor include productivity, managerial skill, product quality, or brand name. Our model includes two forms of innovation: innovation to increase the stock of this firm specific factor in an existing firm — *process innovation*, and innovation to create new firms with a new initial stock of the firm specific factor — *product innovation*. We use the model to study the dynamic gains from trade that arise as process and product innovation respond to a decline in the costs of international trade.

We start our analysis of the impact of a reduction in the costs of international trade on innovation with a stark analytical result. We show in our model that if all firms export (so the fixed costs of international trade are zero) then, in general equilibrium, a decrease in the marginal costs of international trade has no impact at all, in steady-state, on firms' process innovation decisions and hence no impact on aggregate productivity and output over and above the impact of this trade cost reduction on the volume of trade and production in a version of the model that abstracts from process innovation.

The intuition for this result in our model is as follows. If all firms export, a reduction in the marginal costs of international trade changes the profit opportunities of all firms by the same proportion. The returns to process innovation are proportional to firm profits, so a reduction in the marginal costs of international trade also changes the returns to process innovation by the same proportion for all firms. In general equilibrium with free entry, the expected profits of starting a new firm and the cost of innovative inputs must rise by the same amount to ensure that there are zero profits to product innovation. Because the reduction in trade costs affects the opportunities of all firms proportionally, this free entry condition implies that the ratio of the returns to process innovation to the cost of innovative inputs remains unchanged for all firms. Hence, the equilibrium process innovation decisions are left unchanged.

This result implies that when all firms export, our model has steady-state implications for the impact of changes in marginal trade costs on *product* innovation closely related to several existing results in the literature building on the work of Krugman (1980) and Grossman and Helpman (1991). The models in this literature typically abstract from process innovation within ongoing firms and focus instead only on product innovation measured in terms of the introduction of new goods. Our result also echoes the findings of Eaton and Kortum (2001). They find the same result that changes in the costs of international trade have no impact on innovative effort in a model of quality ladders embedded in a multi-country Ricardian model of international trade, with a research sector that produces new ideas randomly across goods. Our result differs from theirs in that in our model, process innovation is directed toward reducing the firm's marginal cost of producing a specific product and firms do not compete over technological leadership in producing any particular product.

Our analytical result that changes in the costs of trade have no impact on firm level process innovation holds only if all firms have equal exposure to opportunities for international trade. In the data,

however, firms vary greatly in their participation in international trade — exports are highly concentrated among large firms. Here we build on the insight of Melitz (2003) and add to our model a fixed cost of trade at the firm level so that only the more productive (and hence larger) firms choose to export. In this case, a reduction in the marginal costs of trade leads in general equilibrium to an increase in profits of exporting firms relative to the profits of firms that do not export. We show analytically in our model that a reduction in marginal trade costs leads in equilibrium to an increase in process innovation in exporting firms relative to that in firms that do not export. Hence, over time, this reduction in trade costs leads to an amplification of the initial reallocation effects of a decrease in trade costs — exporting firms grow over time and increase their exports while firms that do not export shrink. In other words, this reallocation of process innovation amplifies existing patterns of comparative advantage and amplifies the response of trade and output over time.

We then develop a quantitative version of our model and show that it reproduces many salient features of the U.S. data on firm dynamics and export behavior. We use this model to assess the impact of a reduction in the costs of international trade on firms' innovative activity and the associated dynamic gains from trade. One challenge we face in making our model quantitative is to infer how elastic process innovation effort is to changes in the incentives to innovate. If investments in process innovation are highly inelastic, then the dynamic responses of exports, output, consumption and the firm size distribution do not vary substantially from those of a model that abstracts from endogenous process innovation. In contrast, if process innovation is highly elastic, then these dynamic responses can be quite large. We examine our model's implications for a wide range of this process innovation elasticity parameter.

We find two main results. Our first result is that in response to a decline in marginal trade costs, the increase in process innovation is largely offset by a decline in product innovation, and these changes

in innovation are larger the more elastic is process innovation.<sup>1</sup> This result follows from the free entry condition governing the creation of new firms. In equilibrium, prices and entry have to adjust following a decline in international trade costs so as to leave the value of starting a new firm equal to the entry cost. To the extent that a decline in international trade cost increases process innovation and hence productivity in large exporting firms, it drives up wages and drives down the value of entrants, which tend to be small non-exporting firms. Hence, entry falls to restore the free entry condition.

Our second result is that consideration of elastic process innovation does not substantially alter the dynamic welfare implications of a reduction in international trade costs. We find that, even when elastic process innovation leads to very large steady state changes in export volumes, output, consumption, and substantial changes in the firm size distribution, the dynamic welfare gains from trade are only slightly higher than the gains achieved with inelastic process innovation. This finding follows because process innovation is an investment — the long-run productivity gains that result from increased innovation require an investment of current resources — and, in our model, the output gains from this investment come only slowly. We show in particular that the transition in our economy from one steady-state to another takes a lot of time.

Our model is closely related to several papers in the literature. If we assume that firms' process innovation choices are inelastic, then our model is a variant of Hopenhayn's (1992) model in which firms' experience exogenous random shocks to their productivity. In this case, our model is specifically an open economy version of the model of firm dynamics in Luttmer (2006) and hence is quite similar to Irarrazabal and Opromolla (2006). Our inclusion of fixed and marginal costs of exporting also follows Melitz (2003) and Bernard, Eaton,

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<sup>1</sup>Baldwin and Robert Nicoud (2007) present a related result, but our finding differs from theirs in that we find that changes in process and product innovation largely offset: the decline in product innovation is larger the more elastic is the response of process innovation.

Jensen, and Kortum (2003), among many others. We study such a version of our model as a benchmark to show that it can reproduce many of the same features of the US data on firm dynamics, the firm size distribution, and export decisions studied in these papers.<sup>2</sup> One of the most important of these features is Gibrat's Law — the observation that, at least for large firms, firm growth is independent of firm size.

Our model of firms' process innovation follows Griliches' (1979) knowledge capital model of firm productivity, and the work of Ericson and Pakes (1995). The assumption that the fruits of innovative activity are stochastic in our model means it can account for simultaneous growth and decline, and entry and exit of firms in steady-state.<sup>3</sup> Our model is also related to Bustos (2005), who provides empirical support (using firm level data from Argentina) for a model of trade and heterogeneous firms in which exporters choose to pay a one-time fixed cost to upgrade their technology. It is also related to the work of Costantini and Melitz (2007), that studies how the dynamics of trade liberalizations shapes the pace of one-time technology upgrading and productivity improvements across firms.

Our paper complements the work of Klette and Kortum (2004) and Lentz and Mortensen (2006) in constructing a model of innovation that abstracts from strategic interactions across firms and is also consistent with data on firm dynamics. While their framework is a quality ladders model a la Grossman and Helpman (1991) where firms engage in undirected innovation and their dynamics are governed by creative destruction, ours is a model in which monopolistically competitive firms engage in product innovation and process innovation to shape the stochastic process of their production cost.

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<sup>2</sup>In related work, Alessandria and Choi (2007) study the welfare gains of trade in a dynamic version of Melitz' model that abstracts from process innovation.

<sup>3</sup>Doraszelski and Jaumandreu (2006) estimate a Griliches' knowledge capital model in which innovative investments within the firm also lead to stochastic productivity improvements.

The paper is organized as follows. Section 2 presents our model and Section 3 characterizes the equilibrium. Section 4 describes the analytic result that if all firms export then a change in trade costs has no effect on the firms' process innovation decisions and characterizes the impact of a change in trade costs on product innovation. Section 5 examines analytically the reallocation of process innovation that occurs in response to a change in trade costs when not all firms export. Section 6 discusses the implications for firm growth. Section 7 presents the quantitative model and describes how we calibrate the model to match salient features of the US data on firm dynamics and export behavior. Section 8 presents the model's quantitative implications for the response of innovation, output, consumption, and welfare to a decline in trade costs, both across steady states and along the transition. Section 9 concludes. An Appendix includes the characterization of equilibrium, proofs, and solution methods.

## 2 The Model

Time is discrete and labelled  $t = 0, 1, 2, \dots$ . There are two countries: home and foreign. Variables pertaining to the foreign country are denoted with a star. Households in each country are endowed with  $L$  units of time. Production in each country is structured as follows. There is a single final nontraded good that can be consumed or used in innovative activities, a continuum of differentiated intermediate goods that are produced and can be internationally traded subject to a fixed and a variable trade cost, and a nontraded intermediate good that we call the *research good*. This research good is produced using a combination of final output and labor, and is used to pay the costs associated with both process and product innovation, as well as the fixed costs of exporting. The productivities of the firms producing the differentiated intermediate goods are determined endogenously through equilibrium process innovation, and the measure



of differentiated intermediate goods produced in each country is determined endogenously through product innovation. To simplify the presentation of the analytical results in the paper, the benchmark model abstracts from fixed operating costs (that give rise to endogenous exit), and time varying fixed export costs (to account for the extent of firm switching between exporting and non-exporting status). These considerations are introduced in the quantitative model in Section 7.

Households in the home country have preferences of the form  $\sum_{t=0}^{\infty} \beta^t \log(C_t)$ , where  $C_t$  is the consumption of the home final good at date  $t$ . Households in the foreign country have preferences of the same form over consumption of the foreign final good  $C_t^*$ . Each household in the home country faces an intertemporal budget constraint of the form

$$\sum_{t=0}^{\infty} Q_t (P_t C_t - W_t L) \leq \bar{W} , \quad (1)$$

where  $Q_t$  are intertemporal prices,  $W_t$  is the wage in the home country,  $P_t$  is the price of the home final good,  $C_t$  is the household's consumption, and  $\bar{W}$  is the initial stock of assets held by the household. Households in the foreign country face similar budget constraints with the same intertemporal prices  $Q_t$  and wages, prices, and assets all labelled with stars.

Intermediate goods are differentiated products each produced by heterogeneous firms indexed by  $z$ , which indicates their productivity. A firm in the home country with productivity index  $z$  has productivity equal to  $\exp(z)^{1/(\rho-1)}$  and produces output  $y_t(z)$  with labor  $l_t(z)$  according to the CRS production technology

$$y_t(z) = \exp(z)^{1/(\rho-1)} l_t(z). \quad (2)$$

We rescale firm productivity using the exponent  $1/(\rho-1)$  so that, as explained below, labor and profits are proportional to  $\exp(z)$ .

The output of this firm can be used in the production of the home final good, with the quantity of this domestic absorption denoted

$a_t(z)$ . Alternatively, some of this output can be exported to the foreign country to be used in the production of the foreign final good. International trade is subject to both fixed and iceberg type costs of exporting. The fixed, per-period cost of exporting in terms of the research good is denoted by  $n_x$ . The iceberg type marginal cost of exporting is denominated in terms of the intermediate good being exported. The firm must export  $Da_t^*(z)$  units of output, with  $D \geq 1$ , to have  $a_t^*(z)$  units of output arrive in the foreign country for use in the production of the foreign final good.

Let  $x_t(z) \in \{0, 1\}$  be an indicator of the export decision of home firms with productivity index  $z$  (it is 1 if the firm exports and 0 otherwise). Then, feasibility requires that

$$a_t(z) + x_t(z)Da_t^*(z) = y_t(z) , \quad (3)$$

and that  $x_t(z)n_x$  units of the research good be used to pay fixed costs of exporting.

A firm in the foreign country with productivity index  $z$  has the same production technology, with output denoted  $y_t^*(z)$ , labor  $l_t^*(z)$ , and domestic absorption  $b_t^*(z)$ . Exports to the home country,  $b_t(z)$ , are subject to both fixed and marginal costs and hence feasibility requires that  $x_t^*(z)Db_t(z) + b_t^*(z) = y_t^*(z)$ , and that  $x_t^*(z)n_x$  units of the foreign research good be used to pay the fixed costs of exporting.

The home final good is produced from home and foreign intermediate goods with a constant returns production technology of the form

$$Y_t = \left[ \int a_t(z)^{1-1/\rho} dM_t(z) + \int x_t^*(z)b_t(z)^{1-1/\rho} dM_t^*(z) \right]^{\rho/(\rho-1)} \quad (4)$$

where  $M_t(z)$  is the measure of operating firms in the home country with productivity less than or equal to  $z$ , and  $M_t^*(z)$  the corresponding measure in the foreign country. Production of the final good in

the foreign country is defined analogously. It will be useful below to distinguish between the total measure of operating firms given by  $N_t = M_t(\infty)$ , and the distribution of  $z$  across operating firms which has a cumulative distribution function given by  $M_t(z)/N_t$ .

The final good in the home country is produced by competitive firms that choose output  $Y_t$  and inputs  $a_t(z)$  and  $b_t(z)$  subject to (4) to maximize profits taking prices  $P_t$ ,  $p_{at}(z)$ ,  $p_{bt}(z)$ , export decisions  $x_t(z)$ ,  $x_t^*(z)$ , and measures of operating intermediate goods firms  $M_t$  and  $M_t^*$  as given. Standard arguments give that equilibrium prices must satisfy

$$P_t = \left[ \int p_{at}(z)^{1-\rho} dM_t(z) + \int x_t^*(z) p_{bt}(z)^{1-\rho} dM_t^*(z) \right]^{1/(1-\rho)} \quad (5)$$

and quantities

$$\frac{a_t(z)}{Y_t} = \left( \frac{p_{at}(z)}{P_t} \right)^{-\rho} \quad \text{and} \quad \frac{b_t(z)}{Y_t} = \left( \frac{p_{bt}(z)}{P_t} \right)^{-\rho} \quad (6)$$

Analogous equations hold for prices and quantities in the foreign country.

The research good in the home country is produced with a constant returns to scale production technology  $F$  that uses  $X_t$  units of the home final good and  $L_{mt}$  units of labor to produce  $Y_{mt}$  units of the research good according to  $Y_{mt} = F(X_t, L_{mt})$ . As is standard, the equilibrium price of the home research good, denoted by  $W_{mt}$ , is given as a function of the price of the home final good  $P_t$  and the home wage  $W_t$ , by the unit cost function associated with the production function  $F$ . We write this cost function  $W_{mt} = W_m(P_t, W_t)$ . Likewise, in the foreign country we have  $Y_{mt}^* = F(X_t^*, L_{mt}^*)$  and  $W_{mt}^* = W_m(P_t^*, W_t^*)$ .

Intermediate goods firms in each country are monopolistically competitive. A home firm with productivity  $\exp(z)^{1/(\rho-1)}$  faces a

static profit maximization problem of choosing labor input  $l_t(z)$ , prices  $p_{at}(z)$ ,  $p_{at}^*(z)$ , quantities  $a_t(z)$ ,  $a_t^*(z)$ , and whether or not to export  $x_t(z)$ , to maximize current period profits taking as given wages for workers  $W_t$ , the price of the home research good  $W_{mt}$ , and prices and output of the final good in both countries  $P_t, P_t^*, Y_t$ , and  $Y_t^*$ . This problem is written

$$\Pi_t(z) = \max_{y, l, p_a, p_a^*, a, a^*, x \in \{0,1\}} p_a a + x p_a^* a^* - W_t l - W_{mt} x n_x \quad (7)$$

subject to (2), (3), and the demand functions

$$a = \left(\frac{p_a}{P_t}\right)^{-\rho} Y_t \text{ and } a^* = \left(\frac{p_a^*}{P_t^*}\right)^{-\rho} Y_t^*. \quad (8)$$

Productivity at the firm level evolves over time depending both on idiosyncratic productivity shocks hitting the firm and on the level of investment in productivity improvements undertaken within the firm. We model the evolution of firm productivity as follows. At the beginning of each period  $t$ , every existing firm has probability  $\delta$  of exiting exogenously, and corresponding probability  $1 - \delta$  of surviving to produce. A surviving firm with current productivity  $\exp(z)^{1/(\rho-1)}$ , and that invests  $H(z, p) = h \exp(z) c(p)$  units of the research good in improving its productivity in the current period  $t$ , has probability  $p$  of having productivity  $\exp(z + s)^{1/(\rho-1)}$  and probability  $1 - p$  of having productivity  $\exp(z - s)^{1/(\rho-1)}$  in the next period  $t + 1$ . We assume that  $c(p)$  is increasing and convex in  $p$ , and  $h$  is a positive constant. In the next sections, we show that this form of  $H(z, p)$  allows us to obtain analytical results for process innovation.

With this evolution of firm productivity, the expected discounted present value of profits for the firm satisfies the Bellman equation

$$V_t(z) = \max_{p \in [0,1]} \Pi_t(z) - W_{mt} H(z, p) + (1 - \delta) \frac{Q_{t+1}}{Q_t} [p V_{t+1}(z + s) + (1 - p) V_{t+1}(z - s)]. \quad (9)$$

Let  $p_t(z)$  denote the optimal choice of investment in improving productivity in the problem (9). We refer to  $p_t(z)$  as the *process innovation* decision of the firm. Note that if the time period is small, our binomial productivity process approximates a geometric Brownian motion in continuous time (as in Luttmer 2006) in which the firm controls the drift of this process through investments of the research good.

New firms (or new products) are created with investments of the research good. Investment of  $n_e$  units of the research good in period  $t$  yield a new firm in period  $t + 1$  with initial productivity  $\exp(z)^{1/(1-\rho)}$  drawn from a distribution over  $z$  given by  $G$ . New firms are not subject to exogenous exit in their entering period. In any period in which there is entry of new firms, free entry requires that

$$W_{mt}n_e = \frac{Q_{t+1}}{Q_t} \int V_{t+1}(z)dG. \quad (10)$$

Let  $M_{et}$  denote the measure of new firms entering in period  $t$ , that start producing in period  $t + 1$ . The analogous Bellman equation holds for the foreign firms as well. We refer to  $M_{et}$  as the *product innovation* decision as this is the mechanism through which new differentiated products are produced.

Feasibility requires that for the final good

$$C_t + X_t = Y_t \quad (11)$$

in the home country and the analogous constraint holds in the foreign country. The feasibility constraint on labor in the home country is given by

$$\int l_t(z)dM_t(z) + L_{mt} = L, \quad (12)$$

and likewise in the foreign country. The feasibility constraint on the research good in the home country is

$$M_{et}n_e + \int [x_t(z)n_x + H(z, p_t(z))] dM_t(z) = F(X_t, L_{mt}), \quad (13)$$

and likewise in the foreign country.

The evolution of  $M_t(z)$  over time is determined by the exogenous probability of exit  $\delta$ , the decisions of operating firms to invest in their productivity  $p_t(z)$ , and the measure of entering firms in period  $t - 1$ ,  $M_{et-1}$ . The measure of operating firms in the home country with productivity less than or equal to  $z'$ , denoted by  $M_{t+1}(z')$ , is equal to the sum of three inflows: new firms founded in period  $t$ , firms continuing from period  $t$  that draw positive productivity shocks (and hence had productivities lower than  $z' - s$  in period  $t$ ), and firms continuing from period  $t$  that draw negative productivity shocks (and hence had productivities below  $z' + s$  in period  $t$ ). We write this as follows:

$$M_{t+1}(z') = G(z')M_{et} + (1 - \delta) \int_{-\infty}^{z'-s} p_t(z) dM_t(z) + (1 - \delta) \int_{-\infty}^{z'+s} (1 - p_t(z)) dM_t(z). \quad (14)$$

The evolution of  $M_t^*(z)$  for foreign firms is defined analogously.

We assume that the households in each country own those firms that initially exist at date 0. Thus we require that the initial assets of the households in both countries adds up to the total value of these firms

$$\bar{W} + \bar{W}^* = \int V_0(z) dM_0(z) + \int V_0^*(z) dM_0^*(z). \quad (15)$$

An equilibrium in this economy is a collection of sequences of prices and wages  $\{Q_t, P_t, P_t^*, W_t, W_t^*, W_{mt}, W_{mt}^*, p_{at}(z), p_{at}^*(z), p_{bt}(z), p_{bt}^*(z)\}$  a collection of sequences of quantities  $\{Y_t, Y_t^*, C_t, C_t^*, X_t, X_t^*, L_{mt}, L_{mt}^*, a_t(z), a_t^*(z), b_t(z), b_t^*(z), l_t(z), l_t^*(z)\}$  initial assets  $\bar{W}, \bar{W}^*$ , and a collection of sequences of firm value functions and profit, export, and investment decisions  $\{V_t(z), V_t^*(z), V_t^o(z), V_t^{o*}(z), \Pi_t(z), \Pi_t^*(z), x_t(z), x_t^*(z), p_t(z), p_t^*(z)\}$  together with measures of operating and entering firms  $\{M_t(z), M_{et}, M_t^*(z), M_{et}^*\}$  such that household in each country are maximizing their utility subject to their budget

constraints, intermediate goods firms in each country are maximizing within period profits, final goods firms in each country are also maximizing profits, all of the feasibility constraints are satisfied, and the measures of operating firms evolve as described above.

A steady-state of our model is an equilibrium in which all of the variables are constant.<sup>4</sup> In what follows, we omit time subscripts when discussing steady-states.

In some sections of the paper we focus our attention on equilibria that are symmetric in the following sense. First, we assume that the distribution of initial assets is such that expenditure is equal across countries at date 0 and hence in every period. Second, we assume that each country starts with the same distribution of operating firms by productivity and hence, because prices and wages are equal across countries, continue to have the same distribution of operating firms by productivity in each subsequent period. In such a symmetric equilibrium, we have  $Y_t = Y_t^*$ ,  $P_t = P_t^*$ , and  $W_t/P_t = W_t^*/P_t^*$ .

### 3 Characterizing Equilibrium

We start with an analysis of the static profit maximization problem (7) for an operating firm in the home country. All firms choose a constant markup over marginal cost, so equilibrium prices are given by

$$\begin{aligned} p_{at}(z) &= \frac{\rho}{\rho-1} \frac{W_t}{\exp(z)^{1/(\rho-1)}}, \text{ and} \\ p_{at}^*(z) &= \frac{\rho}{\rho-1} \frac{D W_t}{\exp(z)^{1/(\rho-1)}}. \end{aligned} \tag{16}$$

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<sup>4</sup>Since firms in our model grow endogenously through process innovation, there are parameter values for our model in which a steady-state does not exist. Under such parameter values, the equilibrium has no entry and a vanishing set of firms growing endogenously. We focus in the remainder of the paper on cases with positive steady-state entry.

The production employment of home firms is given by

$$l_t(z) = \left( \frac{\rho - 1}{\rho W_t} \right)^\rho [(P_t)^\rho Y_t + x_t(z) (P_t^*)^\rho Y_t^* D^{1-\rho}] \exp(z). \quad (17)$$

Note that (17) implies that there is a simple relationship between the productivity index  $\exp(z)$  and firm size measured as workers employed in production of output for domestic consumption. In contrast, there is no relationship between productivity in the model, measured as  $\exp(z)^{1/(\rho-1)}$  and standard measures of labor productivity. This is because the average productivity of workers in the firm (measured as sales per worker) is given by  $\rho W_t / (\rho - 1)$ , and hence is constant across firms. Differences in productivity across firms in the model, measured by  $\exp(z)^{1/(\rho-1)}$ , are manifest in measures of firm size and not in measures of labor productivity.<sup>5</sup> Hence, firms' innovation decisions,  $p_t(z)$ , together with the stochastic shocks to firm productivity and firms' decisions to export determine the dynamics of firm size and the long run distribution of firms by size in our economy.

Home firms have variable profits  $\Pi_{dt} \exp(z)$  on their home sales, with

$$\Pi_{dt} = \frac{(W_t)^{1-\rho} (P_t)^\rho Y_t}{\rho^\rho (\rho - 1)^{1-\rho}}, \quad (18)$$

and variable profits  $\Pi_{xt} \exp(z)$  on their foreign sales, with

$$\Pi_{xt} = \frac{(W_t)^{1-\rho} (P_t^*)^\rho Y_t^*}{\rho^\rho (\rho - 1)^{1-\rho}} D^{1-\rho}. \quad (19)$$

Therefore, there is a cutoff firm productivity index  $\bar{z}_{xt}$  such that firms with productivity index below  $\bar{z}_{xt}$  do not export and those

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<sup>5</sup>Our model gives this stark result because intermediate goods producing firms choose a constant markup of price over marginal cost. Note that the presence of fixed costs of production, if counted as workers, can only generate differences in value-added per worker for small firms. Bernard, Eaton, Jensen, and Kortum (2003) develop a model in which the markup that firms charge rises with size and hence measured labor productivity rises with firm size.



with productivity index above  $\bar{z}_{xt}$  do export. Hence, we have that firms' static profits are given by

$$\Pi_t(z) = \Pi_{dt} \exp(z) + \max(\Pi_{xt} \exp(z) - W_{mt} n_x, 0). \quad (20)$$

The decision of a firm to invest research goods in improving productivity, if interior, must satisfy the first order condition

$$W_{mt} \frac{\partial}{\partial p} H(z, p) = (1 - \delta) \frac{Q_{t+1}}{Q_t} [V_{t+1}(z + s) - V_{t+1}(z - s)]. \quad (21)$$

This first order condition must satisfy the obvious inequality if the optimal choice of  $p_t(z)$  is equal to either 0 or 1. We discuss the implications of this first order condition (21) for the impact of changes in the costs of trade on process innovation in the next several sections.

In Appendix 1, we describe the aggregate equilibrium conditions of the model.

## 4 Trade and innovation when all firms export

In this section we show that in an economy with no fixed costs of international trade, once and for all changes in the marginal costs of trade have no impact at all on the incentives of firms in steady-state to engage in process innovation. This result holds in general equilibrium because, in an economy in which every firm exports, the increased incentives to innovate resulting from the increase in profits that come from a reduction in marginal trade costs are exactly offset by an increase in the cost of the research good necessary for innovation. This offsetting change in the cost of the research good is required by the zero-profit condition associated with product innovation. As a result, the optimal process innovation decision of all firms is unchanged. We also discuss the impact of changes in the marginal cost of trade on product innovation in the steady-state of our model.

We state our main result in the following proposition

*Proposition 1:* Consider two different world economies, each consisting of two countries as described above, with no fixed costs of trade ( $n_x = 0$ ). Let the marginal cost of trade be  $D \geq 1$  in the first world economy and  $D' \neq D$  in the second world economy. Then, the process innovation decisions of firms in a steady-state in both economies are identical in that  $p(z) = p'(z)$  for all productivities  $z$ .

*Proof:* First observe that if all firms export, then from (18) and (19), variable profits of home firms are directly proportional to  $\exp(z)$  in that  $\Pi(z) = \Pi \exp(z)$ , where  $\Pi = \Pi_d + \Pi_x$ . The analogous result is also true for firms in the foreign country. In a steady-state, all equilibrium prices and value functions are constant and the interest rate is constant at  $1/\beta$ . Hence, if we impose steady-state and divide the firms' Bellman equation given in (9) by the steady-state price of the research good  $W_m$ , we can re-write this Bellman equation as

$$w(z) = \max_{p \in [0,1]} \frac{\Pi}{W_m} \exp(z) - H(z, p) + (1 - \delta)\beta [pw(z + s) + (1 - p)w(z - s)]. \quad (22)$$

Standard arguments give that this Bellman equation has a unique solution  $w(z)$  corresponding to any fixed value of  $\Pi/W_m$ . In addition, the solution  $w(z)$  is strictly increasing in  $\Pi/W_m$ . We can regard the optimal choice of  $p(z)$  implied by this Bellman equation as the optimal process innovation decision corresponding to a fixed ratio  $\Pi/W_m$  of variable profits to innovation costs.<sup>6</sup>

Similarly, if we impose steady-state and divide the zero-profit condition (10) by the steady-state price of the research good  $W_m$ , we can rewrite this condition as

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<sup>6</sup>To ensure that this value function  $w(z)$  is well defined, we need that for all  $p \in [0, 1]$ , either (i)  $\beta(1 - \delta)[p \exp(s) + (1 - p) \exp(-s)] < 1$  or (ii)  $\Pi/W_m - c(p) < 0$ . The first condition requires that the expected growth of firm profits is less than the interest rate. If this is not satisfied, the second condition requires that the static profits of choosing such a high growth rate be negative.

$$n_e = \beta \int w(z) dG. \quad (23)$$

We see from this scaled version of the free entry condition that the ratio of the discounted expected value of profits from starting a new firm to the price of the research good must be fixed at  $n_e$  in any equilibrium in which there is entry. In a steady-state, the ratio of the discounted expected value of profits from starting a new firm to the price of the research good is strictly increasing in  $\Pi/W_m$ . Thus, since there is entry in a steady-state, there is a unique value of  $\Pi/W_m$  consistent with equilibrium. The equilibrium process innovation decisions of home firms in steady-state are thus independent of the parameter  $D$ . Clearly, the analogous results hold for foreign firms. Q.E.D.

Our proof of Proposition 1 relies on our assumption that all innovation activities use the same research good. If different inputs were required for product and process innovation, then a change in trade costs might affect the relative price of the inputs into these activities and thus affect equilibrium process innovation. Our proof of Proposition 1 also relies on our steady-state assumption. In a transition to steady-state following a change in trade costs, the interest rate and the ratio  $\Pi_t/W_{mt}$  can vary over time and hence process innovation decisions can also vary. Note, however, that this result does not depend on the functional form for  $H(z, p)$ . Note also that our proposition also holds if countries are asymmetric in terms of all the parameters of the model including the marginal trade costs  $D$  and  $D^*$  — changes in any one country's marginal trade cost leaves process innovation unchanged in the steady-state.

What is critical for our result here is that changes in model parameters, such as the marginal cost of trade, change equilibrium variable profits in a manner that is weakly separable with firm productivity (as indexed by  $z$ ) — that is, these changes affect profits  $\Pi(z) = \Pi \exp(z)$  only by changing the scalar  $\Pi$ . Because of this weak separability in firm productivity, when the cost  $W_m$  of the research

good adjusts to ensure that the zero-profit condition for product innovation holds, the ratio of the returns to process innovation to the cost of the research good, as measured by  $pw(z+s) + (1-p)w(z-s)$ , is left unchanged *for all*  $z$  and hence the equilibrium process innovation decisions are left unchanged. Given this intuition, it is clear that in our model firm level process innovation decisions are also unaffected if a country moves from autarky to free trade, or by changes in tariffs or tax rates on firm profits, revenues, or factor use that alter the variable profit function in the same weakly separable manner with  $z$ .

This logic implies that Proposition 1 would also hold in a two-sector model in which the aggregate outputs of each sector are imperfect substitutes and firms face separate entry condition of the form (23). Specifically, assume that firms in each sector face no fixed cost of trade but face different marginal costs of trade across the two sectors. Consider the impact in steady state of a reduction in marginal trade costs in one of the two sectors. A simple extension of the proof of Proposition 1 implies that in steady state, process innovation in the two sectors does not vary with the parameter  $D$ . Here, we can think of (22) as the Bellman equation for firms in each sector, with one  $\Pi$  for each sector. Using the same logic as before, in steady state the ratio  $\Pi/W_m$  in each sector remains unchanged.

Our result in Proposition 1 that, in the absence of fixed costs of trade, a reduction in the marginal cost of trade has no impact on steady-state equilibrium process innovation implies that adjustment comes entirely through changes in relative prices and product innovation. Hence, in the absence of fixed costs of exporting, our model has steady-state implications for the impact of changes in marginal trade costs on product innovation closely related to several existing results in the literature on the interaction of trade and innovation, building on the work of Grossman and Helpman (1989, 91), Romer (1986, 90), Rivera-Batiz and Romer (1991 a, b), Coe and Helpman (1995) and others. Our result implies that our model has the same qualitative implications for product innovation in the steady-state as

versions of these other models in which the process for productivity in ongoing firms is fixed exogenously.

Many of these previous papers considered the impact of changes in the marginal costs of trade on product innovation in the presence of spillovers. One can introduce similar spillovers from aggregate cumulative process and product innovation to the costs of product innovation in our model in a straightforward way that preserves our result in Proposition 1. To be concrete, consider versions of our model in which the productivity of the technology for producing the research good is given by  $AF(X, L_m)$  where  $A$  is some measure of aggregate, cumulated innovation. Examples of such spillovers include the stock of domestically produced varieties ( $A = \int dM(z)$ ), or some weighted average of the stock of domestically produced and imported varieties ( $A = \int dM(z) + \mu \int dM^*(z)$ ), or the aggregate productivity of domestic firms ( $A = \int \exp(z)^{1/(\rho-1)} dM(z)$ ), or a similarly weighted average of the aggregate productivities of domestic and foreign firms ( $A = \int \exp(z)^{1/(\rho-1)} dM(z) + \mu \int \exp(z)^{1/(\rho-1)} dM^*(z)$ ). Clearly, our Proposition 1 can easily be extended to cover each of these examples because the specific form of the technology for producing the research good did not enter into our proof.

We finish this section with a characterization of the impact of changes in the marginal costs of trade on product innovation. For tractability, we focus our attention on equilibria that are symmetric, as defined in Section 3. We also assume, for the remaining of the paper, that the production function for the research good takes the form:

$$F(X, L_m) = X^{1-\lambda} L_m^\lambda . \quad (24)$$

Here  $\lambda$  is the share of labor in the production of the research good. Note that if we set  $\lambda = 0$ , then our model has endogenous growth stemming from product innovation.

We now state our second proposition that describes a condition on the model parameters under which a decline in marginal trade costs leads to an increase in *product innovation* across steady states.

*Proposition 2:* Consider two different world economies, each consisting of two *symmetric* countries as described above, with no fixed costs of trade ( $n_x = 0$  for all firms). Let the marginal cost of trade be  $D \geq 1$  in the first world economy and  $D' < D$  in the second world economy. If  $\lambda = 1$ , then the measure of entering firms in a steady state is equal in the two world economies. If  $\lambda < 1$ , then the measure of entering firms in the second world economy is larger than that in the first world economy ( $M'_e > M_e$ ) if and only if  $\rho + \lambda > 2$ .

*Proof:* See Appendix 2.

The proof uses the following logic. We know from Proposition 1 that the ratio of profits to the price of the research good,  $\Pi/W_m$ , must remain unchanged across the two world economies. Product innovation responds to ensure that this is the case. The precise response depends on the parameters  $\lambda$  and  $\rho$ . Start with the case in which  $\lambda = 1$ . In this case,  $W_m = W$ . Since, holding the number of firms fixed,  $W$  changes one for one with  $\Pi$  when  $D$  falls, there is no need for product innovation to respond since  $\Pi/W_m$  remains unchanged. When  $\lambda < 1$ ,  $W_m = W^\lambda$  — we choose  $P$  as the numeraire. Hence, holding the number of firms fixed,  $\Pi/W_m$  rises when  $D$  falls. Changes in product innovation restore the equilibrium. Whether this requires an increase or decrease in product innovation is determined by the elasticity of demand  $\rho$  relative to  $\lambda$ .

In the next two sections, we examine the response of process and product innovation to a decline in marginal trade costs when not all firms export, due to the presence of positive fixed export costs. We first present analytical results in a symmetric steady state. We then present numerical results in a calibrated version of our model.

## 5 Trade and innovation when not all firms export

In this section we present analytical results on the response of process innovation to a reduction in the costs of international trade

when not all firms export. To obtain these results we focus on a symmetric steady state (see characterization in Appendix 1). We obtain our results in two steps. We first solve for the impact of a reduction in trade costs on the profits of firms that export and firms that do not export. We then solve for the impact of these changes in profits on process innovation decisions.

In our model with a fixed cost of exporting, not all firms export, and hence, a change in international trade costs in equilibrium reallocates variable profits from non-exporters to exporters in a manner similar to that described in Melitz (2003). To solve for this equilibrium reallocation of variable profits, we use the analog to the Bellman equation (22) describing the value of an existing firm together with the free-entry condition (23). With a fixed cost of exporting, the Bellman equation for  $w(z)$  is now given by:

$$w(z) = \max_{p \in [0,1]} \Pi_d/W_m \exp(z) + \\ + \max \{ \Pi_d/W_m D^{1-\rho} \exp(z) - n_x, 0 \} - h \exp(z) c(p) + \\ + (1 - \delta)\beta [pw(z + s) + (1 - p)w(z - s)]. \quad (25)$$

Here, the first term denotes static variable profits from the firm's domestic operations, and the second term denotes the profits from exporting (with symmetric countries  $\Pi_x = \Pi_d D^{1-\rho}$ ). The free entry condition (23) is unchanged. Clearly, this value function  $w(z)$  is decreasing in both  $D$  and  $n_x$ , and increasing in  $\Pi_d/W_m$ .

We use this Bellman equation (25) and free-entry condition (23) to determine the impact of changes in trade costs on variable profits in equilibrium. Consider first the impact of a decline in the marginal cost of trade,  $D$ , on variable profits (rescaled by the cost of the research good)  $\Pi_d/W_m$ . Since this raises  $D^{1-\rho}$ ,  $\Pi_d/W_m$  has to fall in equilibrium to restore the free entry condition. Note as well that  $\frac{\Pi_d}{W_m} D^{1-\rho}$  must rise if the free entry condition is to be satisfied. Hence, for firms that do not switch export status, the profits of exporters — proportional to  $\Pi_d/W_m (1 + D^{1-\rho})$ , must rise relative

to the profits of non exporters. Likewise, now consider the impact of a decline in the fixed cost of exporting  $n_x$  on variable profits  $\Pi_d/W_m$ . Because  $w(z)$  is decreasing in  $n_x$ , such a reduction in this fixed trade cost must lead to a reduction in variable profits  $\Pi_d/W_m$  to restore the free entry condition. This implies that the variable profits of all firms that do not switch export status must fall proportionally. In both cases, the export threshold falls so that some firms that previously did not export now start to export.

Note that the magnitude of the decline in  $\Pi_d/W_m$  in response to a decline in international trade costs is determined in large part by the distribution  $G$  of productivities of newly entering firms. Consider a decline in marginal trade costs  $D$ . If newly created firms tend to be small non-exporters, then free entry requires that the discounted expected value of profits of these firms remain roughly constant. In this case,  $\Pi_d/W_m$  remains roughly constant and the profits of for large exporting firms,  $\Pi_d/W_m (1 + D^{1-\rho})$  rises by roughly the change in  $(1 + D^{1-\rho})$ . In contrast, if newly created firms tend to be large exporting firms, then free entry requires that  $\Pi_d/W_m (1 + D^{1-\rho})$  remains roughly constant, and  $\Pi_d/W_m$  falls.

We now examine the impact of these equilibrium changes in variable profits on the level of process innovation. To do so, we solve for the process innovation decisions  $p(z)$  in (25) as a function of variable profits  $\Pi_d/W_m$  and the other parameters of the model. The Bellman equation (25) is a standard problem of valuing the profits of the firm together with an option: the option to start exporting. We use our functional form for the process innovation cost function  $H(z, p) = h \exp(z)c(p)$  to obtain the following lemma, proved in the appendix, regarding the shape of the value function  $w(z)$  and the process innovation decision  $p(z)$ .

*Lemma 1:* The value function  $w(z)$  that solves (25) has the form  $w(z) = A(z) \exp(z)$  with  $\lim_{z \rightarrow \infty} A(z) = A_x$  and  $\lim_{z \rightarrow -\infty} A(z) = A_d$ , and the optimal  $p(z)$  has  $\lim_{z \rightarrow \infty} p(z) = \bar{p}_x$  and  $\lim_{z \rightarrow -\infty} p(z) = \bar{p}_d$  where  $A_d$  and  $\bar{p}_d$  solve



$$A_d = \frac{\Pi_d/W_m - hc(\bar{p}_d)}{1 - (1 - \delta)\beta [\bar{p}_d \exp(s) + (1 - \bar{p}_d) \exp(-s)]}, \quad (26)$$

$$hc'(\bar{p}_d) = (1 - \delta)\beta A_d [\exp(s) - \exp(-s)], \quad (27)$$

and  $A_x$  and  $\bar{p}_x$  solve these two equations with the term  $\Pi_d/W_m$  in (26) replaced with  $\Pi_d/W_m (1 + D^{1-\rho})$ . These solutions have  $A_x > A_d$  and  $\bar{p}_x > \bar{p}_d$ . Moreover,  $A_d, A_x$  and  $\bar{p}_d$  and  $\bar{p}_x$  are increasing in  $\Pi_d/W_m$ , while  $A_x$  and  $\bar{p}_x$  are decreasing in  $D$ .

*Proof:* See Appendix 3.

This lemma implies that for very small firms, the process innovation decision  $p(z)$  is constant at  $\bar{p}_d$ . These firms do not export and all grow at the constant rate  $[\bar{p}_d \exp(s) + (1 - \bar{p}_d) \exp(-s)]$  in expectation. Likewise, for very large firms, the process innovation decision  $p(z)$  is constant at  $\bar{p}_x$ . These firms do export and all grow at the constant rate  $[\bar{p}_x \exp(s) + (1 - \bar{p}_x) \exp(-s)]$  in expectation. The intuition for how  $A_i$  and  $\bar{p}_i$  change with changes in profits is then straightforward. If variable profits  $\Pi_d/W_m \exp(z)$  or  $\Pi_d/W_m (1 + D^{1-\rho}) \exp(z)$  rise, this raises the spread between the value of a firm that successfully innovates to  $z + s$  relative to the same firm that fails to innovate and falls to  $z - s$ . This increased spread in profits raises the incentives to engage in process innovation.

Note that the responsiveness of very large and very small firms' process innovation decisions  $\bar{p}_x$  and  $\bar{p}_d$  to changes in variable profits and marginal trade costs is determined by the curvature of the innovation cost function as indexed by  $c''(p)/c'(p)$ . In particular, because the process innovation choice is optimal,  $\partial A_i / \partial \bar{p}_i = 0$ , and hence the change in steady state process innovation with a change in profits is given by

$$\begin{aligned} d\bar{p}_d &= \frac{c'(p_d)}{c''(\bar{p}_d)} \frac{d(\Pi_d/W_m)}{\Pi_d/W_m - hc(\bar{p}_d)} \text{ and} \\ d\bar{p}_x &= \frac{c'(p_x)}{c''(\bar{p}_x)} \frac{d(\Pi_d(1+D^{1-\rho})/W_m)}{\Pi_d(1+D^{1-\rho})/W_m - hc(\bar{p}_x)}. \end{aligned} \quad (28)$$

If  $c''(\cdot)/c'(\cdot)$  is very large, then process innovation decisions and firm growth rates are not very responsive to changes in profits, while if this curvature is small, then innovation decisions and firm growth rates are very responsive to changes in profits. By a similar argument, this curvature of the innovation cost function  $c''(\cdot)/c'(\cdot)$  also controls the difference in the process innovation decisions and implied growth rates of very large firms ( $\bar{p}_x$ ) and very small firms ( $\bar{p}_d$ ) in a steady-state.

With this lemma, we have the following results regarding the impact of changes in trade costs on the process innovation decisions of very large firms (exporters) and very small firms (non-exporters). A reduction in the marginal costs of trade  $D$  leads to a reduction in the process innovation of very small firms and an increase in the process innovation of very large firms relative to very small firms. This result follows directly from the fact that a reduction in the marginal costs of trade reduces  $\Pi_d/W_m$  and increases  $\Pi_d/W_m (1 + D^{1-\rho})$  relative to  $\Pi_d/W_m$ . The extent of reallocation of process innovation from non-exporters to exporters depends in part on the size distribution of newly created firms. If newly created firms are small, then  $\Pi_d/W_m (1 + D^{1-\rho})$  increases and process innovation in very large firms rises in absolute terms while  $\Pi_d/W_m$  remains roughly constant leaving process innovation in small firms roughly unchanged. Conversely, if newly created firms are large exporting firms, then profits  $\Pi_d/W_m (1 + D^{1-\rho})$  and process innovation in these firms remains roughly unchanged, while for small firms profits and process innovation falls.

In contrast, a reduction in the fixed costs of trade, by lowering the equilibrium level of variable profits  $\Pi_d/W_m$ , leads to a reduction in process innovation in both very large and very small firms (note that these firms do not switch export status). Similar arguments give that a decline in the entry cost  $n_e$  results in a decline in process innovation for both very large and very small firms.

We have shown analytical results for the impact of a reduction in the costs of international trade on process innovation in a symmetric

steady state when not all firms export. In our quantitative work, we are interested in the implications of a reduction in trade costs for product innovation and for the transition dynamics of trade and output and for welfare. To work out these implications, we solve the model numerically. We also use our quantitative model below to examine the implications of a reduction in trade costs in an asymmetric two-country world economy.

## 6 Implications for Firm Growth

Recall that in our model, each firms' domestic employment, sales, and variable profits are all directly proportional to  $\exp(z)$ . Thus, our model of process innovation within firms has sharp implications for patterns of firm growth. Specifically, since a firm with current size determined by  $\exp(z)$  has an expected growth rate conditional on survival of  $\exp(s)p(z) + \exp(-s)(1 - p(z))$ , the relationship between firm size and firm growth implied by our model is given by the equilibrium process innovation decisions  $p(z)$ . We discuss this relationship between process innovation and firm growth in this section.

The observation known as Gibrat's Law — that firm growth rates are independent of firm size, at least for large enough firms — is a robust empirical finding.<sup>7</sup> Following Luttmer (2006), we can show that if all firms were to exogenously choose a constant value of  $p(z) = \bar{p}$  for process innovation, then our model would reproduce Gibrat's Law. In proving Lemma 1, we showed that in our model very large firms endogenously choose a constant value of  $p(z) = \bar{p}_x$ , so it is straightforward to show that our model replicates Gibrat's Law, at least for large firms.

This finding that our model can replicate Gibrat's Law depends on our parametric assumption that the process innovation cost function has the form  $H(z, p) = h \exp(z)c(p)$ . In Lemma 1, in proving

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<sup>7</sup>For references on Gibrat's law, see Sutton (1997) and Caves (1998).

that process innovation is constant for very large firms, we use the assumption that the costs of innovation scale with firm size (through  $\exp(z)$ ) in the same way as do the incentives to innovate. Specifically, dividing both sides of (21) by the price of the research good  $W_{mt}$  gives that in steady-state, the first order condition governing process innovation is given by

$$\frac{\partial}{\partial p} H(z, p) = (1 - \delta)\beta [w(z + s) - w(z - s)] \quad (29)$$

With our parametric assumption on  $H$ , since  $w(z) = A_x \exp(z)$  for large firms, this optimality condition reduces to (27) with constant  $p(z) = \bar{p}_x$ .

We refer to the term  $(w(z + s) - w(z - s)) / \exp(z)$  in (29) as the *scaled incentive to innovate* as it is directly proportional to the ratio of the returns to increasing the probability of advancing the productivity index from  $z$  to  $z + s$  to the scaling cost  $\exp(z)$  of doing so for a firm with current productivity index  $z$ . The results in Lemma 1 imply that in a symmetric steady-state, the scaled incentive to innovate is given by  $A_d (\exp(s) - \exp(-s))$  for very small firms and  $A_x (\exp(s) - \exp(-s))$  for very large firms.

Our model fails to reproduce Gibrat's Law for large firms if we chose a common alternative scaling assumption for the innovation investment cost function. In particular, assume that the costs of innovation scale with the productivity rather than the size of the firm so that  $H(z, p) = h \exp(z)^{1/(\rho-1)} c(p)$ . This scaling assumption follows naturally if one assumes that productivity of the firm  $\exp(z)^{1/(\rho-1)}$  is modeled as a capital stock to be accumulated within the firm as in the Griliches knowledge capital model (see, for example, Griliches and Klette 2000). This assumption, however, implies that firm growth is increasing in firm size. This is because the profits that provide the incentive to innovate are directly proportional to firm size,  $\exp(z)$ . But, with this alternative innovation cost function, the cost of innovation grows more slowly than firm size as long as the elasticity of demand  $\rho > 2$ . Hence, this alternative specification

of innovation investment costs cannot be consistent with Gibrat's Law over a wide range of firm sizes for sufficiently high demand elasticities.

Our model also fails to reproduce Gibrat's Law over the full range of firm sizes because, as we showed in Lemma 1, very large firms export and hence have a higher incentive to engage in process innovation relative to their domestic employment (which is proportional to  $\exp(z)$ ) than very small firms. In particular, in Lemma 1, we showed that the constant process innovation decision for large firms  $\bar{p}_x$  is larger than the constant process innovation for very small firms  $\bar{p}_d$ . Our model can generate the same constant process innovation decision for very small and very large firms if we assume that process innovation costs scale with total employment within the firm rather than with  $\exp(z)$ . With this assumption, in a symmetric steady-state process innovation costs would have the form  $H(z, p) = h \exp(z)c(p)$  for very small firms and  $H(z, p) = h \exp(z)c(p) (1 + D^{1-\rho})$  for very large firms. We show below in our quantitative section that with this assumption, there is very little reallocation of process innovation in response to a decline in the marginal costs of trade.

As we have discussed, our model implies different process innovation decisions and hence different growth rates for very small and very large firms, as well as a reallocation of process innovation, and hence firm growth rates, from small non-exporting firms to large exporting firms, in response to a decline in the marginal costs of trade. From (28), we see that in a symmetric steady-state, the magnitude of these effects on the process innovation decision depends in an important way on the curvature of the innovation cost function as measured by  $c''(p)/c'(p)$ . In our quantitative work below, we assume that this cost function has the form  $\exp(bp)$  so that this curvature is indexed by the parameter  $b$ . If this parameter  $b$  is high (low), so that this curvature is high (low), then from (28) we have that the steady-state differences in growth rates of very small and very large

firms as well as the reallocation of process innovation across firms in response to a change in trade costs are quantitatively small (large).

In our quantitative work, we consider alternative values of  $b$  ranging from very large ( $b = 3000$ ) in which the process innovation decisions of firms are highly inelastic and hence effectively constant as in the model of Luttmer (2006) to lower values of  $b$  ( $b = 30$  and  $b = 10$ ), in which process innovation decisions are elastic and vary substantially across firms in different size classes in steady-state, and in which the change in the steady-state firm size distribution following a change in trade costs is quite large.

## 7 Quantitative Analysis

We now present a quantitative version of our model that is consistent with many features of the data on firm size dynamics (both in terms of employment and export status), and the firm size distribution in the U.S. economy. We then use that model to examine the quantitative impact of a change in the costs of international trade on output and the volume of trade in both the short and the long run, as well as on the product and process innovative activity of individual firms. We also use the model to compute the welfare implications of this reallocation of innovative activities.

Our quantitative model extends the model described above in two ways. First, we assume that production requires  $n_f$  units of the research good during each period  $t$  as a fixed cost of production. This fixed cost is denominated in terms of the research good, and gives rise to endogenous exit. In particular, only firms with a productivity index  $z$  above a certain threshold (that can change over time) decide to operate every period, while those with productivity index below this threshold choose to exit and receive a value of zero. We find below that consideration of endogenous exit has a significant effect on the process innovation decisions of small firms. This arises from the option to exit. Given that the value of exit

is independent of  $z$ , the scaled incentive to innovate is smaller for these firms relative to others that have high values of  $z$  and hence are far from exiting.

Second, we assume that the fixed cost of exporting is random and i.i.d. over time for each firm. Each period, the firm draws a random fixed cost of exporting  $n_x$  from a distribution  $G_x$  that is lognormal with mean  $\bar{n}_x$  and variance  $\sigma_{n_x}^2$  ( $\sigma_{n_x} = 0$ , corresponds to a constant fixed cost of exporting). This model generates heterogeneity in export behavior for firms with equal  $z$  but different fixed cost of exporting. The variable  $x_t(z) \in [0, 1]$  now indicates the fraction of home firms with productivity index  $z$  that export any output at all at time  $t$ . With  $\sigma_{n_x} > 0$ , so that the fixed costs of exporting are random, firms switch status from exporting to not exporting and vice-versa more frequently the larger is  $\sigma_{n_x}$ . We use data on the fraction of employment in firms that switch status from exporting to not exporting, and vice versa, in our calibration. Further details of the extended model are provided in Appendix 1. It is straightforward to see that Propositions 1 and 2 presented in Section 4 also hold in this extended model.

### Calibration

We now discuss how the model is parameterized to reproduce a number of salient features of U.S. data on firms dynamics, the firm size distribution, and international trade. We choose time periods equal to two months so there are six time periods per year.<sup>8</sup> We parameterize the distribution  $G$  of productivity draws of entrants to be normal so that for entering firms,  $z \sim N(0, \sigma_e)$ . We parameterize the distribution of fixed costs of exporting so that  $\log(n_x) \sim N(\log(\bar{n}_x) - \sigma_{n_x}^2/2, \sigma_{n_x})$  — the mean of  $n_x$  is  $\bar{n}_x$ .

The parameters of the model that we must choose then are the steady-state interest rate given by  $1/\beta$ , the total number of workers  $L$ , the parameters governing the variance of employment growth

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<sup>8</sup>As we reduce the period length, we keep the entry period of new firms at one year. Otherwise, the allocations would change significantly as the cost of waiting for a new draw (and hence the entry threshold  $\bar{z}$ ) would decline.

for surviving firms  $s$ , the exogenous exit rate of firms  $\delta$ , the fixed costs of operation  $n_f$  and entry  $n_e$ , the parameters of the innovation cost function  $h$  and  $b$ , the parameter of the distribution of initial productivity draws for firms  $\sigma_e$ , the parameters of the distribution of fixed costs of exporting  $\bar{n}_x$  and  $\sigma_{nx}$ , and the export intensity of exporting firms  $D^{1-\rho}/(1 + D^{1-\rho})$ . We also need to choose the elasticity of substitution  $\rho$ , and the share of labor in the production of research goods  $\lambda$ . In our model, the distribution of employment across firms in a symmetric steady-state depends on the elasticity parameter  $\rho$  only through the trade intensity for firms that do export given by  $D^{1-\rho}/(1 + D^{1-\rho})$ . Since our calibration procedure is based on employment data, we choose the trade intensity  $D^{1-\rho}/(1 + D^{1-\rho})$  as a parameter, and hence our steady-state calibration is invariant to the choice of  $\rho$ . For similar reasons, our steady-state calibration is also invariant to the choice of  $\lambda$ .

We now describe how all parameters are chosen. We set  $\beta$  so that the steady-state interest rate (annualized) is 5%. We normalize  $L = 1$ . Now consider the parameters shaping the law of motion of firm productivity  $z$  ( $s$ ,  $\delta$ ,  $n_e$ ,  $n_f$ ,  $\sigma_e$ ,  $\bar{n}_x$ ,  $\sigma_{nx}$ ,  $D^{1-\rho}$ ,  $h$  and  $b$ ). We choose  $s$  so that the standard deviation of the growth rate of employment of large firms in the model is 25% on an annualized basis. This figure is in the range of those for US publicly-traded firms reported in Davis et. al. (2006).<sup>9</sup> We choose the exogenous death rate  $\delta$  so that the model's annual employment-weighted death rate of large firms is 0.55%, consistent with the corresponding one for large firms in the US data.<sup>10</sup> Note that in our model, over the course of one year, large firms do not choose to exit endogenously because they have productivity far away from the threshold productivity for

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<sup>9</sup>We abstract from the trend in employment growth rate volatility discussed in Davis et. al. (2006) and pick a number that roughly matches the average for the period 1980-2001.

<sup>10</sup>This is the 1997-2002 average employment-based failure rate of US firms larger than 500 employees, computed using data reported by the Statistics of U.S. Businesses and Nonemployer Statistics.



exit. Hence  $\delta$  determines the annual exit rate of these firms directly. We normalize  $n_e = 1$ , and set  $n_f = 0.1$ .<sup>11</sup>

Corresponding to each value of  $b$  (3000, 30 and 10), we choose the parameters  $\sigma_e, \bar{n}_x, \sigma_{nx}, D^{1-\rho}$ , and  $h$  to match the following five observations. First, the fraction of employment by entering firms of size under 500 is 90% of all employment in entering firms.<sup>12</sup> Note that firm sizes in terms of number of employees in the model are simply a normalization. We choose this normalization so that the median firm in the employment-based size distribution is of size 500. In other words, 50% of total employment in the model is accounted for by firms of size under 500.<sup>13</sup> Second, the fraction of total employment accounted for by exporting firms is 40%, and third, the fraction of exports in gross output is 7.5%.<sup>14</sup> We abstract from variation in trade costs across sectors. Fourth, we match patterns of firms switching from exporters to non-exporters, and vice-versa, over time. We simulate the steady state of the model for seven years to compare the model's predictions with those in the US between 1993 and 2000 reported in Bernard et. al. (2005). The employment-based switching rate is defined as the average of the year seven employment of non-exporters that become exporters, and the year-one employment of exporters that become non-exporters, as a fraction of total exporter's employment. Using the data in Bernard et. al.

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<sup>11</sup>The statistics that we report are invariant to proportional changes in all three fixed costs and  $h$ .

<sup>12</sup>This fraction is the 1999-2003 average calculated using US firms birth data, computed using data reported by the Statistics of U.S. Businesses and Nonemployer Statistics.

<sup>13</sup>This is the size of the median firm in the US firm employment-based size distribution on average in the period 1999-2003, as reported by the Statistics of U.S. Businesses and Nonemployer Statistics.

<sup>14</sup>Bernard et. al. (2005) report that the fraction of total US employment (excluding a few sectors such as agriculture, education, and public services) accounted for by exporters is 36.3% in 1993 and 39.4% in 2000. The average of exports and imports to gross output for the comparable set of sectors is roughly 7.5% in the U.S. in 2000. The steady state of our model abstracts from trends in trade costs that would lead to changes in trade volumes over time.

(2005) we compute this seven year switching rate in US data to be 12%.

For the fifth observation, we match the shape of the right tail of the firm size distribution in the U.S. Specifically, consider representing the right tail of the distribution of employment across firms in the U.S. data with a function that maps the logarithm of the number of employees  $\log(l)$  into the logarithm of the fraction of total employment employed in firms with this employment or larger. We plot this function in Panel B, Figure 1, for firms of size 5 to 10000. If the distribution of employment across firms actually was Pareto, this functions would be a straight line. As is evident in this figure, this function is close to linear, with the slope becoming steeper for larger firms. In calibrating the model with inelastic process innovation (fixed  $p$  for all firms) we set the model parameters so that the model matches the slope coefficient for firms within a certain size range. This calibration procedure is similar to that used by Luttmer (2006). Concretely, we target a slope coefficient of  $-0.2$  for firms ranging between 1000 and 5000 employees, as in Panel B, Figure 1. The calibrated model then implies a value of  $\bar{p}_x$  for large firms from (26) and (27). As we lower  $b$ , we adjust the model parameters to keep  $\bar{p}_x$  constant and thus keep the dynamics of large firms unchanged.

Table 1 summarizes the observations used in the calibration, as well as the resulting parameter values, for each level of  $b$ . Recall that by calibrating the model to data on firm size, we did not need to take a stand on the values of  $\rho$  and  $\lambda$ . The aggregate implications of changes in trade costs are, however, affected by the values of  $\rho$  and  $\lambda$ . In our benchmark parameterization we set  $\rho = 5$ , and  $\lambda = 0.5$ .<sup>15</sup>

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<sup>15</sup>Our choice of  $\rho = 5$  roughly coincides with the average elasticity of substitution for US imports of differentiated 4-digit goods estimated in Broda and Weinstein (2006) for the period 1990-2001. Due to the scarcity of information on the value of  $\lambda$ , we perform sensitivity analysis for a wide range of  $\lambda$  between 0.25 and 0.95.

We now discuss some steady state implications of our calibrated model. We start with the parametrization that assumes inelastic process innovation. We then discuss how some of these additional implications of the model change as we decrease the curvature parameter  $b$  on the innovation cost function, and hence increase the elasticity of process innovation.

**Steady state implications: Inelastic process innovation**

The benchmark calibration with high  $b$  ( $b = 3000$ ) gives  $p(z) = \bar{p}$  constant for all  $z$  (see Panel A in Figure 1). Panel C of Figures 1 displays the one-year growth rate of continuing firms as a function of the log of firm size. In it, we observe that for small firms, the model generates growth rates for continuing firms that are decreasing in size. This is consistent with Gibrat’s law: small surviving firms grow faster than large firms due to selection, and among larger firms growth is unrelated to size.<sup>16</sup>

Panel B of Figure 1 shows that the employment-based size distribution implied by the model resembles that in the U.S. for firms ranging between 5 and 10000 employees (recall that our calibration procedure targets the slope in the data for firms between 1000 and 5000 employees).<sup>17</sup>

We also assess the benchmark model’s implications for the size distribution and growth dynamics of exporters, in relation to the data reported in Bernard et. al. (2005). Panel D in Figure 1

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<sup>16</sup>Note that selection only operates for very small firms. Extending the model to allow for cross-firm variation in fixed costs would generate a smoothly decreasing conditional growth schedule.

<sup>17</sup>Note that if the size distribution were Pareto, then the slope of the right tail distribution based on the number of firms (this function maps firm employment into the logarithm of the fraction of firms with this or higher employment) would be equal to the slope of the employment-based right tail coefficient plus one. Both the data and the model show a size distribution that departs from a Pareto distribution in the sense that the slope of the employment-based size distribution is steeper than that of the firm-based size distribution. The distribution of entrants  $G$  plays the key role in the model in shaping this departure from a Pareto distribution.

displays the exporters' employment size distribution of the model and the US data in 2000. The size distribution generated by the model is only slightly more concentrated than that in the data. This suggests that the concentration of exporters is not that striking in light of the concentration of firms in the overall economy.<sup>18</sup>

### **Steady state implications: Elastic process innovation**

We now re-calibrate our model using lower values for the innovation cost curvature,  $b$ . Specifically, we consider  $b = 30$ , and  $b = 10$ . The resulting parameter values are presented in columns 2 and 3 of Table 1.

In Panel A of Figures 2 and 3 we plot the innovation intensity  $p(z)$  and the fraction of exporting firms  $x(z)$  for operating firms  $z$ . As we lower  $b$ , firms that export (those with  $x(z) > 0$ ) choose a higher investment in innovation than non-exporters (those with  $x(z) = 0$ ).

Panel C of Figures 2 and 3 again displays the one-year growth rate of continuing firms as a function of the log of firm size. Recall that in the model with inelastic process innovation ( $b = 3000$ ), consistent with Gibrat's law, the growth rate is positive for small firms due to selection, and is roughly zero for larger firms. As we lower  $b$ , the slope of the conditional growth rate schedule is the result of a tension between selection (negative slope) and increasing innovation intensity (positive slope). Panel C in Figures 2 and 3 shows that the second force strengthens as we lower  $b$ . Under  $b = 30$ , conditional growth rates become increasing in size, but these differences in growth rates across sizes are still quite small. Under  $b = 10$ , the second force is very strong, leading to growth rates of large firms that are over 10 percentage points higher than the growth rates of

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<sup>18</sup>Bernard et. al. (2005) report that the exporters' concentration of export values is significantly more concentrated than that of employment. This suggest that large exporters also have a higher trade intensity. Our model abstracts from these considerations.

small firms. Very low levels of  $b$  thus deliver a clear violation of Gibrat's law.

Panel B in Figures 2 and 3 compares the employment-based size distribution produced by the model with US data. As we lower  $b$ , our calibration procedure of fixing  $\bar{p}_x$  implies that the slope of the firm size distribution for large firms remains in line with the data. But the model generates a divergence from the data for smaller firms. In particular, a lower innovation intensity for small firms and non-exporters shows up as a flatter slope of the distribution for these firms relative to large firms. We quantify this change in the slope of the firm size distribution as the ratio of the employment-based slope coefficient for firms of size 100 – 1000 to that of firms of size 5000 – 10,000. This ratio is equal to 0.74 under  $b = 3000$ , 0.86 under  $b = 30$ , and 1.38 under  $b = 10$ . This flattening of the slope of the distribution under  $b = 10$  is counterfactual relative to US data (see Panel B).

As a further check on our model, we also consider our model's implications for variation in process innovation intensity with firm size for different values of the parameter  $b$ . We find that our model implies that there is at most a very small systematic relationship between research intensity and firm size for large firms even when  $b = 10$ . Specifically, consider the slope coefficient of a regression of the logarithm of each firms' innovative investment (as measured by  $H(z, p)$ ) on its size as measured by sales. When we run this regression in our model using only firms of size larger than 500 (these firms account for 50% of total employment), this coefficient ranges from 1.004 when  $b = 30$  to 1.06 when  $b = 10$ . These results are roughly consistent with the data surveyed in Cohen (1995) and Cohen and Klepper (1996). Thus it does not appear that direct data on research intensity across firms of different sizes is that useful for disciplining the choice of the parameter  $b$  in our model. Differences in research intensity across firms do appear if smaller firms are included in this regression. In particular, if all firms of size larger than 5 are included, the regression coefficient ranges from 1.03 when  $b = 30$  to

1.54 when  $b = 10$ . However, it is difficult to compare our model to the data in this case because data on the research intensity of smaller firms are typically not available.

Overall, we see that our model, in steady state, reproduces quantitatively many salient features of the US data on firm dynamics and export behavior if the curvature parameter  $b$  is sufficiently high so that process innovation decisions are not too elastic.

## 8 Aggregate implications of a decline in marginal trade costs

We now study the aggregate implications of a trade liberalization, defined as a reduction in marginal trade costs  $D$  leading to an increase in the trade intensity of firms that export. We choose to lower  $D$  so that the export intensity of exporters,  $D^{1-\rho} / (1 + D^{1-\rho})$ , increases by a factor of 1.15. Using this procedure, the resulting change in steady state exports/GDP is invariant to the elasticity parameter  $\rho$ .<sup>19</sup> Results are reported in Tables 2, 3 and in Figure 4.

To understand the aggregate implications of a change in trade costs, we can express aggregate output in a symmetric equilibrium as the product of an average productivity index  $\tilde{z}_t$ , the measure of operating firms  $N_t$ , and production employment  $L - L_{mt}$  (see Appendix)

$$Y_t = (\tilde{z}_t)^{1/(\rho-1)} (N_t)^{1/(\rho-1)} (L - L_{mt}). \quad (30)$$

Here  $\tilde{z}_t$  is the average productivity index  $\exp(z) (1 + x(z) D^{1-\rho})$  across operating firms, evaluated using the distribution  $M_t(z) / N_t$ .

A reduction in marginal trade costs  $D$  has two static effects on output both of which affect average productivity  $\tilde{z}_t$  : one by improving the technology of shipping goods abroad, and the other by

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<sup>19</sup>In our benchmark calibration, the initial export intensity of exporters is 18.75%. With  $\rho = 5$ , our experiment amounts to reducing  $D$  from 1.44 to 1.38.

changing firms' exit and export decisions  $x_t(z)$ . It also has two dynamic effects on output: one by changing firms' process innovation decisions  $p_t(z)$  — which shape the distribution  $M_t(z)/N_t$  over which the average productivity index  $\tilde{z}_t$  is calculated — and the other by changing the level of product innovation — which changes  $N_t$ .

### Steady State

Consider the model with inelastic process innovation ( $b = 3000$ ). In Row 1, Column 1 of Table 2, we see that steady state exports/GDP increases by a factor of 1.22 (from 7.5% to 9.1%). This is larger than the change in export intensity of firms, for two reasons. First, exporters are larger than non-exporters, so a change in their export intensity has a more than proportional impact on exports/GDP. Second, the reduction in trade costs induces more firms to pay the fixed cost of exporting.

Now consider the model with elastic process innovation ( $b = 30$  and  $b = 10$ ). The results are reported in Row 1, Columns 2 and 3 of Table 2. Now exports/GDP increases by a factor of 1.4 when  $b = 30$  (from 7.5% to 10.5%), and 2.71 when  $b = 10$  (from 7.5% to 20.1%). The model with elastic process innovation amplifies the impact of the reduction in the trade costs on trade volumes because those firms that export increase their investments in process innovation and those that do not export decrease their investments in process innovation. These shifts in process innovation, by shifting the distribution of firm productivities  $M_t(z)/N_t$ , imply that the average productivity index across firms  $\tilde{z}$  increases by a factor of 1.02 when  $b = 3000$ , 1.14 when  $b = 30$ , and 9.1 when  $b = 10$ .

Consider now the implications of this reallocation of process innovative activity on the steady state distribution of firm size. When process innovation is inelastic, the distribution of firm sizes changes very little. For example, if we consider the concentration of production as measured by the fraction of employment in firms with greater than 5000 employees, this rises from 32.4% to 33.1% when

process innovation is inelastic. In contrast, this measure of concentration rises from 32% to 39% when  $b = 30$ , and from 27% to 91% when  $b = 10$ . Hence, our model predicts that there are substantial changes in the size distribution of firms in those cases in which there are also substantial long-run changes in the response of trade to a reduction in marginal trade costs. These changes in the firm size distribution might be used to discipline the choice of  $b$ . In particular, very low values of  $b$  lead to implausibly large shifts in the firm size distribution in response to a relatively small increase in export intensity.

The impact of a decline in marginal trade costs on product innovation are very different from those discussed in Section 4 when all firms export. Recall from Proposition 2 that if  $\rho + \lambda > 2$ , then the measure of firms increases when all firms export. In contrast, when not all firms export, the measure of firms falls. Row 5 of Table 2 reports that the measure of firms falls by a factor 0.99 when  $b = 3000$ , 0.88 when  $b = 30$ , and 0.11 when  $b = 10$ . To understand this change in the measure of firms, we can write an expression for variable profits from the firm's domestic operations  $\Pi_d/W_m$  (see Appendix) as:

$$\Pi_d/W_m = \bar{\kappa} (\tilde{z}N)^{(2-\rho-\lambda)/(\rho-1)} (L - L_m) , \quad (31)$$

where  $\bar{\kappa}$  is a constant determined by  $\rho$  and  $\lambda$ . A decline in marginal trade costs leads to a small decrease in  $\Pi_d/W_m$  to satisfy the free-entry condition, and also leads to a large increase in average productivity  $\tilde{z}$  (as discussed in Section 5). This small decline in  $\Pi_d/W_m$  and large increase in  $\tilde{z}$  are determined from the Bellman equation (25) and the free entry condition (23). Production employment  $L - L_m$  increases only slightly, so (31) requires that  $N$  must fall substantially to offset the large increase in  $\tilde{z}$  when process innovation is elastic. Hence, a reduction in the marginal costs of trade results, in equilibrium, in a reallocation of innovative activity into innovation within ongoing firms and away from the creation of new firms.



Row 2 of Table 2 reports the steady state increase in final output  $Y$  resulting from the decline in trade costs. The increase in output is 0.09% when  $b = 3000$ , 0.32% when  $b = 30$ , and 1.94% when  $b = 10$ . Hence, elastic process innovation also magnifies the output increase from a decline in variable trade costs. Using (30), we can break this change in final output into an increase in average productivity  $\tilde{z}$  that is partly offset by a decline in the measure of operating firms  $N$  (both of which get larger as we lower  $b$ ), and a small change in production employment ( $L - L_m$  falls slightly when  $b = 3000$  and increases slightly when  $b = 10$ ).

Finally, Row 3 of Table 2 displays the increase in final consumption in response to a decline in trade costs. Consumption increases by a factor of 1.001 when  $b = 3000$ , 1.005 when  $b = 30$ , and 1.033 when  $b = 10$ . The change in consumption differs from the change in output due to the reallocation of resources between production of research and final consumption.

### Transition dynamics

Figure 4 displays the path of exports/GDP (Panel A) and consumption (Panel B) in the transition to the new steady state, for each value of  $b$ . While the transition is almost immediate in the economy with inelastic process innovation, it takes many years when process innovation is elastic. The change in the innovation intensity of exporters increases their expected growth, but this higher growth takes time to materialize as higher levels of exports. In the transition, the distribution of firm level productivities  $M_t(z)/N_t$  slowly shifts, leading to a gradual increase in the export share and consumption.

The model with elastic process innovation is capable of producing substantial differences in the short- and long-run response of trade volumes to a trade liberalization. The contemporaneous increase in exports/GDP as a fraction of the overall increase across steady states, reported in Row 7, Columns 1-3 of Table 2, is 99% under  $b = 3000$ , 54% under  $b = 30$ , and 14% under  $b = 10$ .

## Welfare

We compute welfare as the equivalent variation in consumption that keeps the representative household indifferent between the economy with high and low trade costs, taking into account the transition dynamics. Row 8 of Table 2 reports the welfare gains from a reduction in the marginal cost of trade. In our benchmark model with  $b = 3000$ , the welfare gains are 0.36%. These welfare gains correspond to the welfare gains one would find in a calibrated version of Melitz' (2003) model.

We do not take a stand on whether these welfare gains are large or small. Instead, we focus on the question of how consideration of elastic process innovation affects these welfare gains. We have seen that elastic process innovation has a large impact on our model's implications for trade costs on trade volumes, output, and consumption. In contrast, we see in Row 8 of Table 2 that the welfare gains are only slightly higher when we consider elastic process innovation.

Consideration of elastic process innovation does not substantially alter the welfare implications of a reduction in international marginal trade costs, for two reasons. First, investing in process innovation is costly in that resources are drawn out of alternative activities. Second, the productivity gains from this investment come only slowly.

## Sensitivity Analysis

Table 3 reports some sensitivity analysis with respect to key parameters of the model. For comparison, Columns 1-3 reports the results for our benchmark model (also reported in Table 2).

We first consider a calibration of the model with a higher initial share of exports in GDP (15% instead of 7.5%) to see if greater exposure to trade affects our results. These results are presented in columns 4-6 of Table 3. As in our benchmark experiments, we reduce  $D$  so that the export intensity of exporters rises by a factor of 1.15. The welfare gains are roughly double than those under our benchmark calibration, but as in our benchmark model, the

variation in welfare gains across levels of  $b$  is still small in comparison to the change in trade volumes, output, and consumption.

Next, we report results for a lower elasticity of substitution ( $\rho = 2$  instead of  $\rho = 5$ ) — columns 7-9 of Table 3. Recall that the elasticity parameter  $\rho$  does not affect our model’s implications for the change steady-state process innovation in response to a given change in firms’ steady-state export intensity  $D^{1-\rho}/(1 + D^{1-\rho})$ . Output gains are larger the lower is  $\rho$  (i.e.: from (30), a given change in aggregate productivity index  $\tilde{z}$  translates into a larger change in final output), and these translate into welfare gains that are 4 to 5 times larger. Even though the differences in welfare gains across values of  $b$  are higher (1.6% for  $b = 3000$  and 2.2% for  $b = 10$ ), these differences across values of  $b$  are small relative to the differences in the change in steady state consumption across values of  $b$ .

We now report results when new firms are larger, and hence more likely to be exporters — columns 10-12 of Table 3. As we discussed in Section 5, the size distribution of newly entering firms affects our model’s implications for the impact of a reduction in marginal trade costs on the absolute levels of process innovation in small and large firms. We change the parameters of our benchmark model so that the fraction of employment by entering firms of size under 500 is 72% of employment in entering firms instead of 90% in our benchmark calibration — this requires increasing the variance  $\sigma_e$  of productivities of entering firms. In rows 4 and 5 of Table 3, we see that there is a smaller reallocation of process innovation as measured by the increase in average productivity  $\tilde{z}$ , and a smaller reduction in product innovation as measured by the number of firms. To understand the smaller response of process and product innovation recall that, as discussed above, free entry requires that  $\Pi_d/W_m$  must fall by more when entering firms are more likely to export. As a result, profits of exporters rise by less and hence so does their process innovation. This effect is what generates a smaller increase in average productivity. From (31), we then get the smaller decline

in the number of firms. Note that the results for welfare are largely unchanged relative to those in our benchmark parameterization.

We then consider the role of the share of labor in the production of research goods ( $\lambda = 0.25$  and  $\lambda = 0.95$ ) — columns 13-18 of Table 3. This parameter is known to affect the change in product innovation in response to a given change in firm profits. Note that, for low levels of  $b$ , the decline in the measure of firms is slightly larger under  $\lambda = 0.25$ . This is because the value of entering firms (which tend to be small non-exporters) falls substantially for low  $b$ , so labor is reallocated from research to production and final goods are reallocated from research to consumption. This results stands in contrast to the model with full export participation, in which the increase in the measure of firms is decreasing in  $\lambda$  (e.g.: the number of firms is constant under  $\lambda = 1$ ). Note, however, that quantitatively these differences are small. In particular, the steady state changes in consumption and welfare do not vary substantially with  $\lambda$ .

We then consider a version of our model in which process innovation costs  $H(z, p)$  scale up with export intensity — columns 19-21 of Table 3. That is,  $H(z, p) = h \exp(z) \exp(bp)$  for non-exporters and  $H(z, p) = h \exp(z) \exp(bp) (1 + D^{1-\rho})$  for exporters. As we discussed in Section 6, under this specification, the growth rate differences between exporters and large non-exporters in a steady state are small even for a low value of  $b$ . The aggregate results in columns 19-21 of Table 3 reveal that this alternative specification generates a reduction in the impact of the decline in trade costs on trade volumes and consumption relative to our benchmark model in which process innovation costs scale with  $\exp(z)$ . This is because, for exporters, the scaled-incentives to innovate fall relative to the cost of innovation, so they cut their level of process innovation.

Finally, we consider a version of the model in which exporters grow faster than non-exporters due to the presence of externalities and spillovers from export activities — columns 22-24 of Table 3. We refer to this model as ‘learning-by-exporting’. In this model, differences in growth across firms are assumed rather than being the

outcome of firms' optimal choice of innovation investment. Moreover, process innovation costs do not appear in the resource constraint (13). We use this case to demonstrate that our assumption that firms' investments in process innovation are costly are critical in generating our results for welfare. We set the growth rate of exporters and non-exporters at the steady state levels  $\bar{p}_d$  and  $\bar{p}_x$  that we solved for in the Lemma of Section 5 (both before and after the reduction in trade costs). We consider the same experiment of increasing the trade intensity of exporters by a factor of 1.15. The increase in steady state consumption and welfare is significantly larger in the model with 'learning-by-exporting'. For example, if we assume that exporters and non-exporters grow at the rates of the benchmark model under  $b = 10$ , consumption and welfare increase by 12% and 2.2%, respectively (they increase by 3.2% and 0.4%, respectively, in the model with process innovation). When firms 'learn-by-exporting', increases in their growth rates do not take up research goods, so employment and final good can be reallocated towards production and final consumption, respectively. It is only in this case that the welfare gains from a reduction in marginal trade costs vary substantially depending on the assumptions regarding the impact of exporting on firm growth.

We conclude from this sensitivity analysis that, even when reallocation in process innovation leads to very large steady state changes in export volumes, output, consumption, and substantial changes in the firm size distribution, the dynamic gains of trade are not much larger than those in a model with inelastic process innovation. The dynamic gains can be substantial when firms 'learn-by-exporting' without requiring research resources, particularly when this learning effect is large.

### **Other experiments**

We now examine the response of our model economy to three additional aggregate experiments. First, we re-examine the response to a decline in marginal trade costs under the optimal allocations

of our model. Second, we consider a reduction in marginal trade costs in an asymmetric two country world economy (i.e.: small open economy). Third, we consider a reduction in the fixed costs of exporting.

*Optimal allocations*

We examine process an product innovation in the planning problem that maximizes welfare in the two countries. The equilibrium allocations are potentially suboptimal due to the presence of markups in the prices set by the intermediate good producing firms, which can distort product innovation. It is straightforward to show that, if  $\lambda = 1$ , the allocations of the planning problem coincide with the equilibrium allocations both in and out of steady state. If  $\lambda < 1$ , then the measure of firms and the level of consumption are higher in the planning problem because the monopoly distortion alters the value of entry relative to the cost of entry.

In our benchmark parameterization with  $\lambda = 0.5$ , changes in the allocations and welfare gains from reducing the level of marginal trade costs as discussed above are very similar in the planning problem and equilibrium allocations. The welfare gains in the planning problem are 0.37% with  $b = 3000$ , 0.4% with  $b = 30$ , and 0.44% with  $b = 10$ .

*Small open economy*

We now consider a version of our model with asymmetric countries. In particular we assume that the home country is very small relative to the foreign country, and aggregate variables in the foreign country are held constant. In particular, we assume that home firms face a foreign demand  $a_t^*(z) = p_{at}^*(z)^{-\rho}$ , where we normalize  $Y_t^* = P_t^* = 1$ . We abstract from considerations of product variety for imported goods by assuming the presence of a single imported intermediate good that is available to the home country at a price equal to  $D^*$ , where  $D$  can differ from  $D^*$ . We also abstract from dynamic considerations in the current account by assuming trade balance every period.

We consider a reduction in marginal trade costs such that, when  $b = 3000$ , exports/GDP in the home country increases by the same factor of 1.22 as in the symmetric two-country model (implying also very similar output and welfare gains). For lower levels of  $b$ , for the same change in  $D$ , GDP rises in the home country relative to the rest of the world, and the exporter's export intensity increases by less than in the symmetric model. This implies that the reallocation of profits and process innovation from non-exporters to exporters is smaller compared to the symmetric two-country world. Across steady states, exports/GDP rises by a factor of 0.27 if  $b = 30$  and by a factor of 1.34 if  $b = 10$  (it increases by a factor of 1.4 and 2.71, respectively, in the symmetric economy). Taking into account the transition dynamics, the welfare gains are still very similar for values of  $b$  ranging between 3000 and 10.

#### *Reduction in fixed costs of exporting*

We also consider the aggregate implications of a reduction in the average fixed cost of exporting,  $\bar{n}_x$ . We choose the decline in  $\bar{n}_x$  so that the increase in exports/GDP on impact roughly matches the one from reducing  $D$  in the benchmark experiments discussed above (i.e. a factor of 1.22) This requires lowering  $n_x$  by roughly 60%.

The decline in  $\bar{n}_x$  leads to an increase in export participation, and new exporters increase their level of process innovation investment. This increase in innovation activities by new exporters is partly offset by a decline in process innovation by continuing exporters and non-exporters as discussed in Section 5.

Overall, this reallocation of process innovation activities leads to an increase in the fraction of research goods used in process innovation. Thus, the model with elastic process innovation amplifies the impact of the reduction in the fixed trade costs on trade volumes. Comparing across steady states, exports/GDP increases by a factor of 1.22 under  $b = 3000$ , 1.28 under  $b = 30$ , and 1.37 under  $b = 10$ . However, taking into account transition dynamics, the equivalent variation in consumption is very similar in the model with inelas-

tic and elastic process innovation. These welfare gains are also very close to those obtained from a reduction in marginal trade costs.

## 9 Concluding remarks

In this paper we build a model of the impact of international trade on firms' process and product innovation decisions. We show that the extent of firm's export participation has an important influence on the impact of changes in trade costs on process and product innovation. We show in a quantitative version of our model that, in response to a decline in international trade costs, changes in process and product innovation largely offset. We also find that the dynamic welfare gains from trade when process innovation is elastic are not substantially larger than those gains when it is not. This is true despite the fact that elastic process innovation leads to very large dynamic responses of exports and the firm size distribution.

Our model has abstracted from three important considerations. First, we have assumed constant elasticity of demand. This assumption implies that changes in trade cost have no impact on firms' markups and that there is no strategic interaction in firms' affecting process innovation decisions. Ericson and Pakes (1995), and Aghion et. al. (2003) consider models of process innovation with strategic interactions.

Second, in making our model quantitative, we have assumed that all firms are single-product firms. In doing so, we have abstracted from the effects that a reduction in trade costs might have on product innovation by incumbent firms. Consideration of process and product innovation in models with multi-product firms would be an important extension of this paper (see Klette and Kortum 2004, Luttmer 2007, and Bernard, Redding and Schott 2007 for models of multi-product firms).

Finally, we have also abstracted from spillover effects that might lead to endogenous growth. Since we have found that a decline



in trade costs can lead to a substantial reallocation of process and product innovation, a model with spillovers favoring one type of innovation over the other might predict larger welfare gains when process innovation is elastic.

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## 10 Appendices

### 10.1 Appendix 1: Equilibrium Characterization

We first describe the modifications to the model in Section 2 with the two extensions introduced in Section 6. Next, we state the aggregate equilibrium conditions of the model. Then, we specialize the equilibrium conditions to the symmetric case. Finally, we specialize these conditions to a steady state, and sketch an algorithm to solve for allocations and prices.

#### Extending the basic model of Section 2

We first introduce random exporting costs and fixed operating costs into the basic model of Section 2.

Let  $\xi_t(z, n_x) \in \{0, 1\}$  be an indicator of the export decision of home firms with productivity  $z$  and fixed cost  $n_x$  (it is 1 if the firm exports and 0 otherwise). Let  $x_t(z) = \int \xi_t(z, n_x) dG_x$  be the fraction of home firms with productivity index  $z$  that export any output at all. Define  $\xi_t^*$ ,  $x_t^*$  in the same manner. Then feasibility in production for intermediate good  $z$  in the home country requires that

$$a_t(z) + x_t(z)Da_t^*(z) = y_t(z)$$

and that  $n_f + \int \xi_t(z, n_x)n_x dG_x$  units of the research good be used to pay fixed costs of operation and exporting.

Static profits, exclusive of fixed costs of production, are given by

$$\begin{aligned} \Pi_t(z) = & \max_{l(n_x), p_a, p_a^*, a, a^*, \xi(n_x) \in [0, 1]} p_a a + \int \xi(n_x) dG_x p_a^* a^* - \\ & - W_t \int l(n_x) dG_x - W_{mt} \int \xi(n_x) n_x dG_x \quad (32) \end{aligned}$$

subject to

$$a + \xi(n_x)Da^* = \exp(z)^{\frac{1}{\rho-1}} l(n_x)$$

and demand functions

$$a = \left( \frac{p_a}{P_t} \right)^{-\rho} Y_t \text{ and } a^* = \left( \frac{p_a^*}{P_t^*} \right)^{-\rho} Y_t^*.$$

Note that it is not necessary to index prices  $p_a$ ,  $p_a^*$  or quantities  $a$  and  $a^*$  by  $n_x$ . Clearly the optimal choices of these variables does not depend of the level of the fixed cost of exports.

The expected discounted present value of profits satisfies the Bellman equation

$$V_t(z) = \max [0, V_t^o(z)] \quad (33)$$

where

$$V_t^o(z) = \max_{p \in [0,1]} \Pi_t(z) - W_{mt}n_f - W_{mt}h \exp(z) c(p) + \quad (34)$$

$$(1 - \delta) \frac{Q_{t+1}}{Q_t} [pV_{t+1}(z + s) + (1 - p)V_{t+1}(z - s)].$$

Note here that  $V_t^0(z)$  is the value of choosing to operate the firm in period  $t$ . The firm exits if this value falls below zero. Since  $V_t^0(z)$  is strictly increasing in  $z$ , it is clear that at each date  $t$ , the decision of firms to operate (33) follows a cutoff rule with firms with productivity above a cutoff  $\bar{z}_t$  choosing to operate and firms with productivity below that cutoff exiting. Note that if  $n_f = 0$ , then  $V_t^o(z) = V_t(z)$  and  $\bar{z}_t = -\infty$ .

The feasibility constraint on labor in the home country is given by

$$\int \int l_t(z, n_x) dG_x dM_t(z) + L_{mt} = L \quad (35)$$

and likewise in the foreign country.

The feasibility constraint on the research good in the home country is

$$M_{et}n_e + \int \left( n_f + \int \xi_t(z, n_x) n_x dG_x + h \exp(z) c(p_t(z)) \right) dM_t(z) =$$

$$= F(X_t, L_{mt}), \quad (36)$$

and likewise in the foreign country.

The evolution of the measure of operating firms  $M_t(z)$  over time, taking into account endogenous exit, is given by

$$\begin{aligned}
M_{t+1}(z') &= (G(z') - G(\bar{z}_{t+1}))M_{et} + & (37) \\
&(1 - \delta) \int_{\bar{z}_{t+1}-s}^{z'-s} p_t(z) dM_t(z) + \\
&(1 - \delta) \int_{\bar{z}_{t+1}+s}^{z'+s} (1 - p_t(z)) dM_t(z)
\end{aligned}$$

for  $z' \geq \bar{z}_{t+1}$ , and  $M_{t+1}(z') = 0$  for  $z' < \bar{z}_{t+1}$ . The evolution of  $M_t^*(z)$  for foreign firms is defined analogously.

### Aggregate equilibrium conditions

It is convenient to define four indices of aggregate productivity across firms. For home firms these are

$$Z_{at} = \int \exp(z) dM_t(z) \text{ and } Z_{a^*t} = \int \exp(z) x_t(z) dM_t(z) \quad (38)$$

where the first of these is an index of productivity aggregated across all operating firms (recall that  $M_t$  takes into account endogenous exit), and the second is an index of productivity across all firms that export. Likewise, for the foreign firms we have

$$Z_{b^*t}^* = \int \exp(z) dM_t^*(z) \text{ and } Z_{bt}^* = \int \exp(z) x_t^*(z) dM_t^*(z). \quad (39)$$

The definition of the price index in the home country (5) and the analogous for the foreign country imply

$$1 = \frac{\rho}{\rho - 1} \left[ \left( \frac{W_t}{P_t} \right)^{1-\rho} Z_{at} + \left( D \frac{P_t^*}{P_t} \frac{W_t}{P_t^*} \right)^{1-\rho} Z_{bt}^* \right]^{1/(1-\rho)}, \quad (40)$$

$$1 = \frac{\rho}{\rho - 1} \left[ \left( \frac{W_t^*}{P_t^*} \right)^{1-\rho} Z_{b^*t}^* + \left( D \frac{P_t}{P_t^*} \frac{W_t}{P_t} \right)^{1-\rho} Z_{a^*t} \right]^{1/(1-\rho)}. \quad (41)$$

Labor market clearing requires that

$$\left(\frac{\rho-1}{\rho}\right)^\rho \left(\frac{W_t}{P_t}\right)^{-\rho} Y_t \left[ Z_{at} + \left(\frac{P_t}{P_t^*}\right)^{-\rho} D^{1-\rho} Z_{a^*t} \frac{Y_t^*}{Y_t} \right] + L_{mt} = L \quad (42)$$

$$\left(\frac{\rho-1}{\rho}\right)^\rho \left(\frac{W_t^*}{P_t^*}\right)^{-\rho} Y_t^* \left[ D^{1-\rho} \left(\frac{P_t^*}{P_t}\right)^{-\rho} Z_{bt}^* \frac{Y_t}{Y_t^*} + Z_{b^*t}^* \right] + L_{mt}^* = L^*. \quad (43)$$

The first order conditions for the production of the research good implies

$$\frac{F_2(X_t, L_{mt})}{F_1(X_t, L_{mt})} = \frac{W_t}{P_t} \quad (44)$$

in the home country, and the analogous expression in the foreign country.

Note that with our assumption of log-utility and common intertemporal prices  $Q_t$  across countries, the ratio of expenditure across countries  $P_t^* C_t^* / P_t C_t$  is constant over time with that constant determined by the initial distribution of assets  $\bar{W}$  and  $\bar{W}^*$ . More precisely, we have

$$P_t C_t = \beta^t \frac{Q_0}{Q_t} P_0 C_0 \quad \text{and} \quad P_t^* C_t^* = \beta^t \frac{Q_0}{Q_t} P_0^* C_0^*. \quad (45)$$

Hence, given expenditure in period 0 and measures  $M_{t-1}(z)$ ,  $M_{t-1}^*(z)$ , the aggregates  $\{Z_{at}, Z_{a^*t}, Z_{bt}^*, Z_{b^*t}^*, L_{mt}, L_{mt}^*, X_t, X_t^*, Y_t, Y_t^*, C_t, C_t^*, W_t/P_t, W_t^*/P_t^*, P_t, \text{ and } P_t^*\}$  are the solution to these eight equations (40)-(45), the feasibility constraints for the final good (11) in each country, and the free entry condition (10) in each country, with the exit and export thresholds obtained from the solution of (33) and (34), the measures of firms  $M_t(z)$  and  $M_t^*(z)$  from (37), and the aggregate productivity indices from (38) and (39).



Firm entry  $M_{et}$  in each country is solved for using the feasibility constraints for the research good (36).

### Symmetric equilibrium

From (18) and (19), home variable profits are given by  $\Pi_{dt} \exp(z)$  for domestic sales and  $\Pi_{dt} D^{1-\rho} \exp(z)$  for export sales, where

$$\Pi_{dt} = \frac{(W_t)^{1-\rho} (P_t)^\rho Y_t}{\rho^\rho (\rho - 1)^{1-\rho}}. \quad (46)$$

Total static profits (32) are given by

$$\Pi_t(z) = \Pi_{dt} \exp(z) + \int \max \{ \Pi_{dt} D^{1-\rho} \exp(z) - W_{mt} n_x, 0 \} dG_x. \quad (47)$$

Using conditions (40) and (42), aggregate output and the real wage are given by

$$Y_t = [Z_{at} + D^{1-\rho} Z_{a^*t}]^{1/(\rho-1)} (L - L_{mt}) , \quad (48)$$

and

$$\frac{W_t}{P_t} = \frac{\rho - 1}{\rho} \frac{Y_t}{L - L_{mt}}. \quad (49)$$

### Symmetric Steady State

We now describe how one can solve for the steady state allocations and prices in the quantitative model. We normalize the aggregate price index  $P$  in each country to one.

1. Given  $\frac{\Pi_d}{W_m}$ , solve for the Bellman equations described by (33) and (34) rescaled by the price of the research good  $W_m$

$$w(z) = \max [0, w^o(z)] \quad (50)$$

$$\begin{aligned}
w^o(z) &= \max_{p \in [0,1], \xi(n_x)} \frac{\Pi_d}{W_m} \exp(z) - n_f + \\
&+ \int \max \left\{ \frac{\Pi_d}{W_m} D^{1-\rho} \exp(z) - n_x, 0 \right\} dG_x - h \exp(z) c(p) + \\
&+ (1 - \delta) \beta [pw(z + s) + (1 - p)w(z - s)], \quad (51)
\end{aligned}$$

and obtain the exit threshold  $\bar{z}$  defined by  $w^o(\bar{z}) = 0$ , and firms' export and process innovations decisions. Note that in the absence of fixed operating costs,  $w^o(z) = w(z)$  and  $\bar{z} = -\infty$ . Note also that in steady state, the discount factor is given by  $\beta$ . Note also that, given our binomial process for  $z$ , we can solve the value functions in a discrete grid given by  $\{z_{low}, z_{low} + s, z_{low} + 2s, \dots, z_{high} - s, z_{high}\}$ , where  $z_{low}$  and  $z_{high}$  are the low and high truncation of the grid, that are sufficiently large so that we can impose that  $z$  always belongs to the grid.

2. Solve for  $\frac{\Pi_d}{W_m}$  that satisfies the free entry condition

$$n_e = \beta \int w(z) dG \quad (52)$$

3. Compute the productivity measure normalized by the measure of entering firms,  $M(z)/M_e$ , using the steady state version of the law of motion for  $M$ , (37), and using  $\bar{z}$  and  $p(z)$  from 1.
4. Compute the aggregate productivity indices normalized by the number of entering firms,  $\tilde{z}_a = Z_a/M_e$  and  $\tilde{z}_{a^*} = Z_{a^*}/M_e$ :

$$\tilde{z}_a = \int \exp(z) \frac{dM(z)}{M_e} \quad \text{and} \quad \tilde{z}_{a^*} = \int \exp(z) x(z) \frac{dM(z)}{M_e}. \quad (53)$$

Note that  $\tilde{z}$ , defined in words in Section 8, is given by  $\tilde{z} = \tilde{z}_a + D^{1-\rho} \tilde{z}_{a^*}$ .

5. Solve for the remaining 6 aggregate unknowns,  $(W, Y, C, X, L_m, M_e)$  that satisfy the following 6 equations:

$$Y = [M_e (\tilde{z}_a + \tilde{z}_{a^*} D^{1-\rho})]^{1/(\rho-1)} (L - L_m) \quad (54)$$

$$W = \frac{\rho - 1}{\rho} \frac{Y}{L - L_m} \quad (55)$$

$$C + X = Y \quad (56)$$

$$\frac{\lambda}{1 - \lambda} \frac{X}{L_m} = W \quad (57)$$

$$\Pi_d / W_m = \kappa (W)^{1-\rho-\lambda} Y \quad (58)$$

$$\Upsilon M_e = (L_m)^\lambda (X)^{1-\lambda}, \quad (59)$$

where  $1/\kappa = \rho^\rho (\rho - 1)^{1-\rho} (\lambda)^\lambda (1 - \lambda)^{1-\lambda}$ , and  $\Upsilon$  is given by

$$\Upsilon = n_e + \int [n_f + x(z) n_x + h \exp(z) c(p(z))] \frac{dM(z)}{M_e}.$$

Expressions (54) and (55) are the steady state versions of the expressions for output (48) and the real wage (49). Expression (56) is the resource constraint for the final good. Expression (57) combines the first order condition for the production of the research good (44) and the functional forms for  $F$  given by (24). Expression (58) uses the definition of variable profits (46) and  $W_m = [\lambda^\lambda (1 - \lambda)^{1-\lambda}]^{-1} W^\lambda$  (recall that we choose  $P$  as the numeraire). Finally, expression (59) is the steady state version of the resource constraint for the research good, given by (13).

## 10.2 Appendix 2: Proof of Proposition 2

We use the steady state characterization of Appendix 1. When all firms export,  $\tilde{z}_a = \tilde{z}_{a^*}$ . We also have  $\Pi/W_m = \Pi_d (1 + D^{1-\rho}) / W_m$ .

First, given the result in Proposition 1 that process innovation decisions of firms (and exit thresholds in the model with  $n_f > 0$ ) are unchanged with  $D$  in a steady state where all firms export,  $\tilde{z}_a$ ,  $\Pi/W_m$ , and  $\Upsilon$  are also unchanged with  $D$ .

Second,  $L_m$  does not change with  $D$ . Starting from (58),

$$\begin{aligned}
\frac{\Pi}{W_m} &= \frac{\kappa (W)^{1-\rho} Y (1 + D^{1-\rho})}{(W)^\lambda} \\
&= \frac{\kappa \left(\frac{\rho}{\rho-1}\right)^{\rho-1} Y}{(W)^\lambda M_e \tilde{z}_a} \\
&= \frac{\kappa \left(\frac{\rho}{\rho-1}\right)^{\rho-1} \left(\frac{\lambda}{1-\lambda}\right)^{1-\lambda} \Upsilon Y}{W L_m \tilde{z}_a} \\
&= \kappa \left(\frac{\rho}{\rho-1}\right)^\rho \left(\frac{\lambda}{1-\lambda}\right)^{1-\lambda} \frac{\Upsilon (L - L_m)}{\tilde{z}_a L_m},
\end{aligned}$$

where the first step combines (54) and (55), the second step combines (57) and (59), and the third step uses (55). Clearly, given that  $\Pi/W_m$  is unchanged with  $D$ ,  $L_m$  is unchanged too.

Third, we now have an expression for the number of entering firms  $M_e$ . Starting again from (58),

$$\begin{aligned}
\frac{\Pi}{W_m} &= \frac{\kappa (W)^{1-\rho} Y (1 + D^{1-\rho})}{(W)^\lambda} = \frac{\kappa \left(\frac{\rho}{\rho-1}\right)^{\rho-1} Y}{(W)^\lambda M_e \tilde{z}_a} = \\
&= \frac{\kappa \left(\frac{\rho}{\rho-1}\right)^{\lambda+\rho-1} Y^{1-\lambda}}{M_e \tilde{z}_a} (L - L_m)^\lambda = \\
&= \kappa \left(\frac{\rho}{\rho-1}\right)^{\lambda+\rho-1} (M_e \tilde{z}_a)^{(2-\lambda-\rho)/(\rho-1)} (1 + D^{1-\rho})^{\frac{1-\lambda}{\rho-1}} (L - L_m)
\end{aligned}$$

where we used (54) and (55).

From this expression we can see that if  $\lambda = 1$ ,  $M_e$  does not change with  $D$ . If  $\lambda < 1$ , given that  $\Pi/W_m$ ,  $\tilde{z}_a$ , and  $L_m$  are independent of  $D$ ,  $M_e$  is decreasing in  $D$  if and only if  $\rho + \lambda > 2$ .

### 10.3 Appendix 3: Proof of Lemma

The first part of the Lemma follows by construction. The term  $A_d \exp(z)$  with  $A_d$  given as above represents the expected discounted present value of variable profits of a firm that sets its process innovation decision  $p(z)$  to the constant  $\bar{p}_d$  and which never exports. Likewise, the term  $A_x \exp(z)$  with  $A_x$  given as above represents the expected discounted present value of variable profits of a firm that sets its process innovation decision  $p(z)$  to the constant  $\bar{p}_x$ , which always exports, and which is so large that the fixed cost of exporting  $n_x$  is a negligible portion of its variable profits.

For the second part of the Lemma, we show that  $A_d$  is increasing in  $\Pi_d/W_m$ . Differentiating (26):

$$\frac{\partial A_d}{\partial (\Pi_d/W_m)} = \frac{1}{1 - (1 - \delta)\beta [\bar{p}_d \exp(s) + (1 - \bar{p}_d) \exp(-s)]} + \frac{\partial A_d}{\partial \bar{p}_d} \frac{\partial \bar{p}_d}{\partial \Pi_d/W_m} > 0,$$

where we used the fact that process innovation choice is optimal,  $\partial A_d / \partial \bar{p}_d = 0$ , and  $(1 - \delta)\beta [\bar{p}_d \exp(s) + (1 - \bar{p}_d) \exp(-s)] < 1$  to guarantee that the discounted value of profits is finite. This same logic can be used to show that  $A_x > A_d$ ,  $A_x$  is increasing in  $\Pi_d/W_m$ , and  $A_x$  is decreasing in  $D$ .

For the third part of the Lemma, we show that  $\bar{p}_d$  is increasing in  $A_d$ . Differentiating (27):

$$\frac{\partial \bar{p}_d}{\partial A_d} \frac{\partial A_d}{\partial \Pi_d/W_m} = \frac{(1 - \delta)}{hc''(\bar{p}_d)} \beta [\exp(s) - \exp(-s)] \frac{\partial A_d}{\partial \Pi_d/W_m} > 0,$$

where we used  $\frac{\partial A_d}{\partial \Pi_d / W_m} > 0$ , and the assumption that  $c''(\bar{p}_d) > 0$ . The same logic is used to show that  $\bar{p}_x$  is increasing in  $\Pi_d / W_m$  and decreasing in  $D$ .

## 10.4 Appendix 4: Solving transition dynamics

Here we sketch how one can solve for the transition dynamics of our quantitative model in response to a one time change in trade costs. We consider the case of symmetric countries.

We first solve for the steady state value functions  $V(z)$ , labor  $l(z, n_x)$  and export  $\xi(z, n_x)$  decisions, exit threshold  $\bar{z}$ , process innovation decisions  $p(z)$ , measure of firms  $M(z)$ , aggregate allocations  $\{Y, C, X, L_m, M_e\}$ , prices  $\{W, W_m\}$  and profits  $\Pi_d$  before and after the change in international trade costs, as described in Appendix 1.

We denote by  $t = 1$  the period in which the trade cost changes to its new value. We assume that the aggregate allocations and prices converge to the new steady state in  $T$  periods, where  $T$  is sufficiently high so that the resulting allocations are very insensitive to increasing  $T$ . We normalize the price of the final good  $P_t$  to 1 every period. Then we implement the following iterative procedure:

1. Guess a sequence of variable domestic profits, research good prices, and interest rates  $\{\Pi_{dt}, W_{mt}, Q_{t+1}/Q_t\}_{t=1}^{T-1}$ .
2. Solve for the value functions  $V_t(z)$  and  $V_t^o(z)$  from (33) and (34), with  $\Pi_t(z)$  given by (47), and going backwards from  $t = T - 1$  to  $t = 1$ . Note that in order to compute the value function at  $t = T - 1$ , we are using the new steady state value functions to compute the expected future values. Solving these value functions, we obtain the firms' policy functions  $l_t(z, n_x)$ ,  $\xi_t(z, n_x)$ ,  $x_t(z)$ ,  $\bar{z}_t$ , and  $p_t(z)$ .

3. Obtain a new guess for the research good prices  $\{W_{mt}\}_{t=1}^T$  using the free entry condition (10), and a sequence of wages as  $W_t = \left[ \lambda^\lambda (1 - \lambda)^{1-\lambda} W_{mt} \right]^{1/\lambda}$ .
4. For  $t = 1$  to  $T - 1$ , moving forward:
  - Solve for  $M_t(z)$  using (37), given  $M_{t-1}(z)$ ,  $M_{et-1}$ , and using the firms' policy functions  $\bar{z}_t$ ,  $p_t(z)$  from 2. Note that at  $t = 1$ ,  $M_{t-1}(z)$  and  $M_{et-1}$  are given from the initial steady state.
  - Compute  $Z_{at}$ ,  $Z_{a^*t}$  from (38), and  $L - L_{mt}$  from (35), using the firms' policy functions  $l_t(z, n_x)$ ,  $x_t(z)$  from 2. and the updated measures  $M_t(z)$ .
  - Compute  $X_t = (1 - \lambda) / \lambda L_{mt} W_t$ .
  - Solve for  $M_{et}$  from (36).
  - Compute  $Y_t$  and  $C_t$  using (48) and (11).
5. Using the new values of  $\{Y_t, C_t, W_t\}_{t=1}^T$ , update guesses for  $\Pi_{dt}$  using (46), and  $Q_{t+1}/Q_t = \beta C_t / C_{t+1}$ .
6. If the initial and updated guesses for  $\{\Pi_{dt}, W_{mt}, Q_{t+1}/Q_t\}$  are not sufficiently close, repeat 1. — 5.

A similar logic can be used to solve for the transition dynamics in the asymmetric small open economy.

**Table 1.** Model Calibration

		US Data	1 $b = 3000$	2 $b = 30$	3 $b = 10$
Calibrated parameters					
1	$S$ , annualized		0,25	1,25	2,25
2	$\delta$ , annualized		0,0055	0,0055	0,0055
3	$h$ (or employment-based right-tail coefficient of large firms)		4,66E+40 (-0.25)	4,01E-02 (-0.25)	3,57E-01 (-0.25)
4	$\sigma_e$ , annualized		2,3	1,8	0,7
5	$D^{1-\rho}$		0,231	0,231	0,231
6	$\bar{n}_x$		30	7,9	0,625
7	$\sigma_{nx}$ , annualized		0,6	0,5	0,3
Targets					
8	Employment growth rate of large firms, annual standard deviation	25%	25%	25%	25%
9	Annual employment-based exit rate, firms larger 500 employees	0,55%	0,55%	0,55%	0,55%
10	Employment-based right tail coefficient, firms of size 1000 to 5000	-0,2	-0,20	-0,20	-0,26
11	New firms, % employment of firms smaller than 500 employees	90%	90%	89%	86%
12	Exports / GDP	7,50%	7,5%	7,5%	7,4%
13	Employment share of exporters	40%	40%	40%	40%
14	Average switching rate, seven years	12%	12%	12%	12%



**Table 2.** Steady State Comparison, 15% increase in exporter's trade intensity

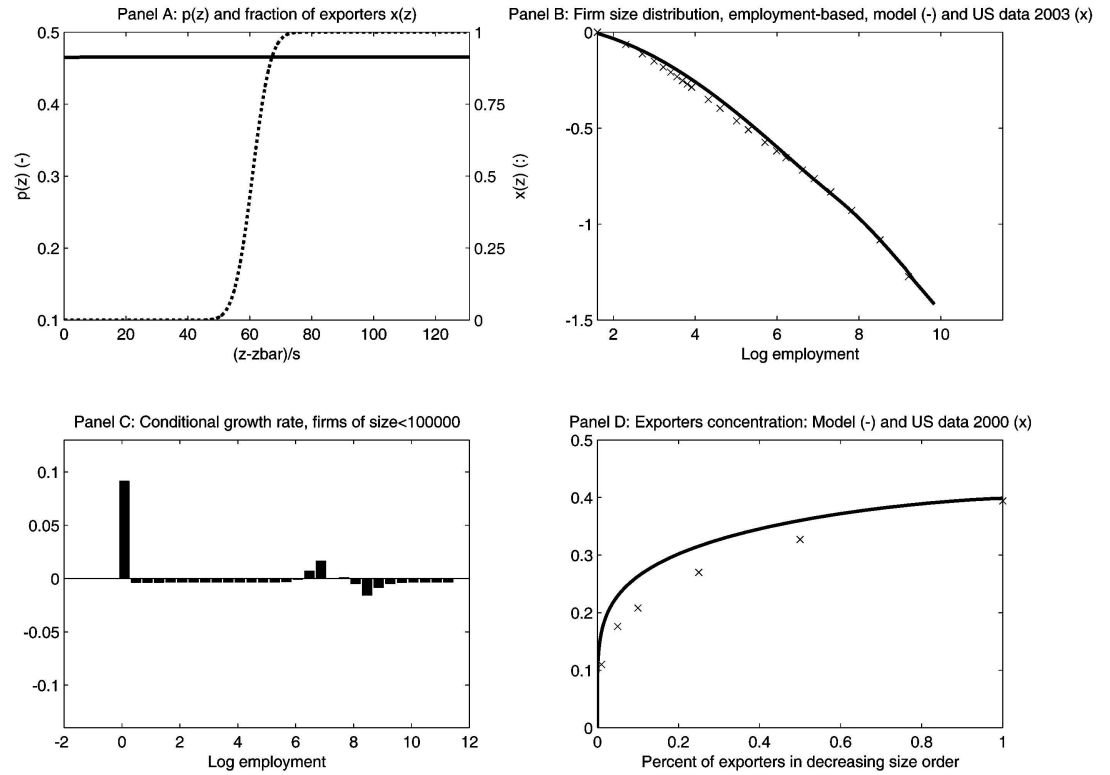
	$b$ , Curvature parameter of $C(p)$	Benchmark Model		
		1 $b = 3000$	2 $b = 30$	3 $b = 10$
1	Exports / GDP (ratio of new to old SS)	1,22	1,40	2,71
2	Output (ratio of new to old SS)	1,001	1,003	1,019
3	Consumption (ratio of new to old SS)	1,001	1,005	1,033
4	Average productivity index $\tilde{Z}$ (ratio of new to old SS)	1,02	1,14	9,07
5	Number of firms (ratio of new to old SS)	0,99	0,88	0,11
6	Production employment (ratio of new to old SS)	1,000	1,002	1,014
7	Exports / GDP, contemporaneous increase / SS increase	0,98	0,54	0,14
8	Welfare gain (equivalent variation)	0,36%	0,38%	0,42%

**Table 3.** Sensitivity Analysis, Steady state and transition dynamics,  
15% increase in exporter's trade intensity

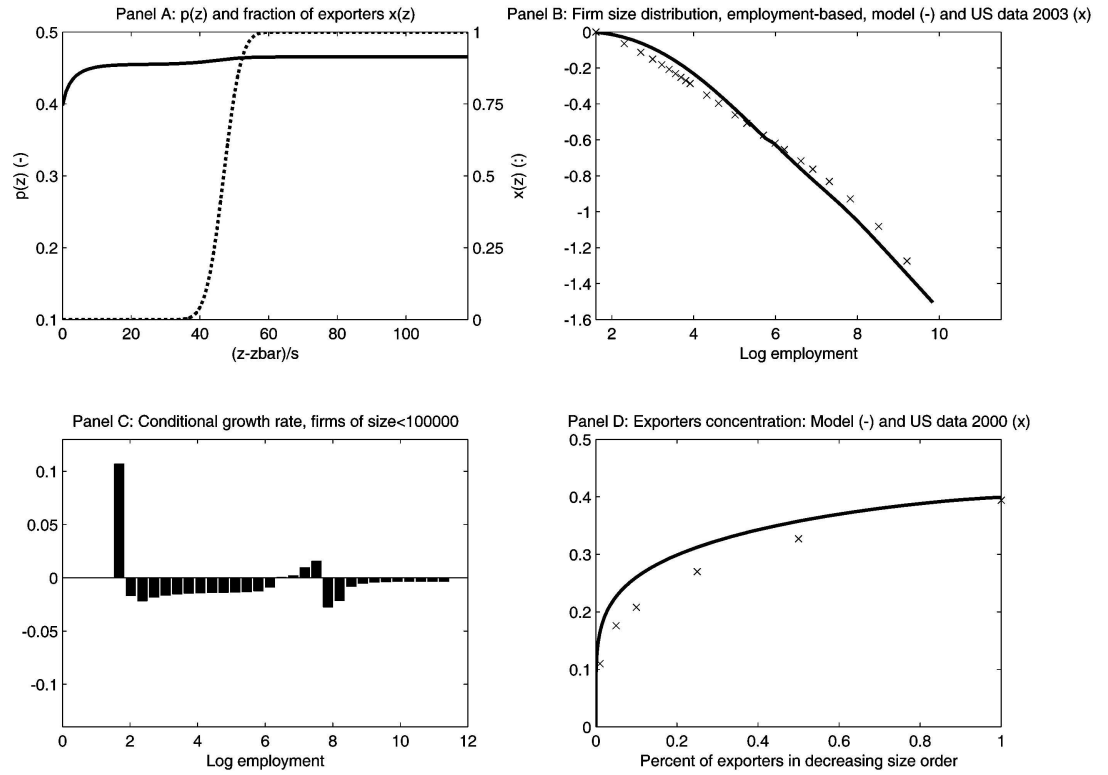
	$b$ , Curvature parameter of $C(p)$	Benchmark			Higher trade share exports / GDP = 0.15		
		1 $b = 3000$	2 $b = 30$	3 $b = 10$	4 $b = 3000$	5 $b = 30$	6 $b = 10$
1	Exports / GDP (ratio of new to old SS)	1,22	1,40	2,71	1,21	1,33	1,90
2	Output (ratio of new to old SS)	1,001	1,003	1,019	1,004	1,007	1,023
3	Consumption (ratio of new to old SS)	1,001	1,005	1,033	1,004	1,010	1,036
4	Average productivity index $\tilde{Z}$ (ratio of new to old SS)	1,02	1,14	9,07	1,04	1,25	60,67
5	Number of firms (ratio of new to old SS)	0,99	0,88	0,11	0,98	0,82	0,02
6	Production employment (ratio of new to old SS)	1,000	1,002	1,014	1,000	1,002	1,014
7	Exports / GDP, contemporaneous increase / SS increase	0,98	0,54	0,14	0,98	0,62	0,25
8	Welfare gain (equivalent variation)	0,36%	0,38%	0,42%	0,80%	0,83%	0,92%
	$b$ , Curvature parameter of $C(p)$	Lower substitution elasticity			Higher variance of entering firms		
		7 $b = 3000$	8 $b = 30$	9 $b = 10$	10 $b = 3000$	11 $b = 30$	12 $b = 10$
9	Exports / GDP (ratio of new to old SS)	1,22	1,40	2,71	1,20	1,33	2,44
10	Output (ratio of new to old SS)	1,006	1,024	1,152	1,002	1,003	1,009
11	Consumption (ratio of new to old SS)	1,006	1,028	1,195	1,002	1,004	1,014
12	Average productivity index $\tilde{Z}$ (ratio of new to old SS)	1,02	1,14	9,07	1,02	1,09	3,72
13	Number of firms (ratio of new to old SS)	0,99	0,89	0,12	0,99	0,92	0,27
14	Production employment (ratio of new to old SS)	1,000	1,005	1,044	1,000	1,001	1,005
15	Exports / GDP, contemporaneous increase / SS increase	0,99	0,55	0,14	0,99	0,59	0,14
16	Welfare gain (equivalent variation)	1,55%	1,81%	2,22%	0,37%	0,38%	0,40%

**Table 3.** Sensitivity Analysis, Steady state and transition dynamics, 15% increase in exporter's trade intensity (continued)

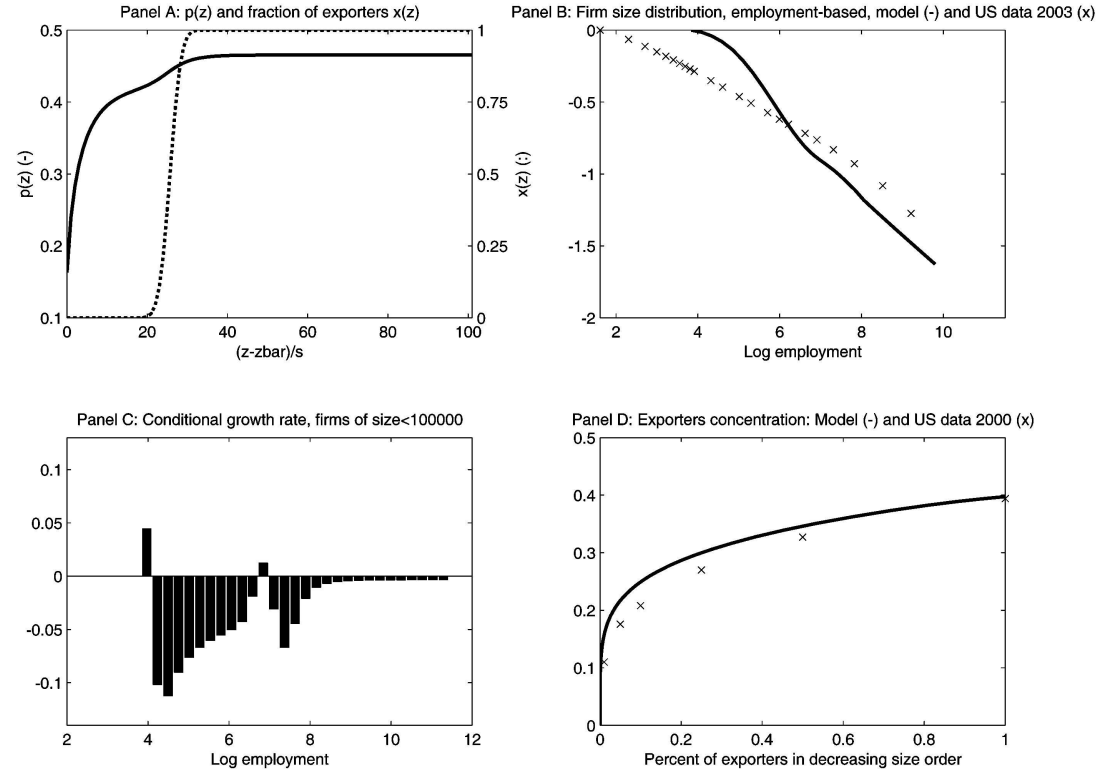
	$b$ , Curvature parameter of $C(p)$	Higher share of labor in innovation, $\lambda = 0.95$			Lower share of labor in innovation, $\lambda = 0.25$		
		13	14	15	16	17	18
		$b = 3000$	$b = 30$	$b = 10$	$b = 3000$	$b = 30$	$b = 10$
17	Exports / GDP (ratio of new to old SS)	1,22	1,40	2,71	1,22	1,40	2,71
18	Output (ratio of new to old SS)	1,001	1,005	1,032	1,001	1,002	1,011
19	Consumption (ratio of new to old SS)	1,001	1,005	1,033	1,001	1,005	1,032
20	Average productivity index $\tilde{Z}$ (ratio of new to old SS)	1,02	1,14	9,07	1,02	1,14	9,07
21	Number of firms (ratio of new to old SS)	0,985	0,880	0,114	0,986	0,879	0,112
22	Production employment (ratio of new to old SS)	1,000	1,003	1,024	1,000	1,001	1,007
23	Exports / GDP, contemporaneous increase / SS increase	0,98	0,54	0,14	0,98	0,54	0,14
24	Welfare gain (equivalent variation)	0,36%	0,37%	0,40%	0,35%	0,38%	0,42%
	$b$ , Curvature parameter of $C(p)$	Process innovation costs scaled by total employment			Learning by exporting		
		19	20	21	22	23	24
		$b = 3000$	$b = 30$	$b = 10$	$b = 3000$	$b = 30$	$b = 10$
25	Exports / GDP (ratio of new to old SS)	1,22	1,21	1,19	1,22	1,42	2,70
26	Output (ratio of new to old SS)	1,001	1,000	0,999	1,001	1,009	1,076
27	Consumption (ratio of new to old SS)	1,001	1,000	0,999	1,001	1,012	1,120
28	Average productivity index $\tilde{Z}$ (ratio of new to old SS)	1,02	1,01	0,98	1,02	1,16	8,61
29	Number of firms (ratio of new to old SS)	0,99	0,99	1,02	0,99	0,88	0,13
30	Production employment (ratio of new to old SS)	1,000	1,000	1,000	1,000	1,004	1,049
31	Exports / GDP, contemporaneous increase / SS increase	0,99	1,02	1,23	0,99	0,52	0,12
32	Welfare gain (equivalent variation)	0,35%	0,20%	0,07%	0,36%	0,78%	2,23%



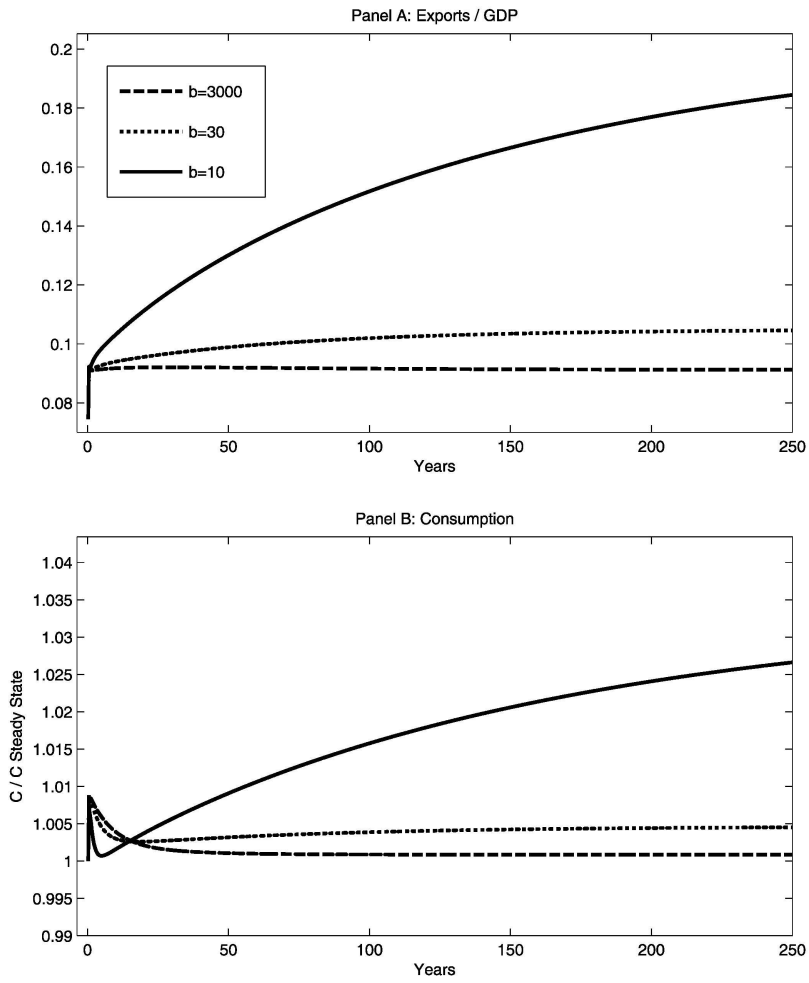
**Figure 1.** Steady State, Inelastic Process Innovation ( $b = 3000$ )



**Figure 2.** Steady State, Moderately Elastic Process Innovation ( $b = 30$ )



**Figure 3.** Steady State, Highly Elastic Process Innovation ( $b = 10$ )



**Figure 4.** Transition Dynamics from a Decline from Marginal Trade Costs

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