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OPTIMAL POLICY WITHOUT EXPROPRIATION

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We argue that the result of dynamic inconsistency of optimal policy is based on an unrealistic assumption that expropriations of property rights or defaults of government debt may be optimal. If we pose an exogenous constraint, which forbids all implicit forms of government debt defaults or expropriations, we get that optimal policy is always dynamically consistent. We demonstrate this result in the Chamley-Judd's framework of an optimal capital taxation problem. We show that under the No Implied Default Condition optimal policy starts with a revision of consumption and labour taxes, afterwards all taxes are about constant and the capital income tax is about zero. The No Implied Default Condition, which guarantees property rights and dynamic consistency, requires that household's wealth be considered as predetermined in the same units as the household's objective function.

Key words: Consistency, Optimal taxation, Optimal monetary policy, No Implied Default.
JEL classification: E61, H21, E52.

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Мы считаем, что результат динамической несогласованности оптимальной политики основан на малореалистичной гипотезе, согласно которой экспроприация прав собственности или дефолт государственного долга могут быть оптимальными. Мы ставим экзогенное ограничение, которое запрещает все неявные формы дефолта государственного долга и экспроприации, и приходим к выводу, что оптимальная политика при этом ограничении всегда динамически согласована. Мы демонстрируем этот результат в рамках модели Шамлей – Джада оптимального налогообложения капитала. Показано, что при условии отсутствия неявного дефолта переход на оптимальную политику сопряжен с пересмотром налогов на потребление и на труд, после чего все налоги устанавливаются на приблизительно постоянном уровне, а налог на капитал – на нулевом уровне. Условие отсутствия неявного дефолта, гарантирующее права собственности и динамическую согласованность, требует, чтобы богатство домашних хозяйств, измеренное в единицах полезности, рассматривалось в качестве предопределенной переменной.

Ключевые слова: Согласованность, оптимальное налогообложение, оптимальная монетарная политика.

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One of the most amazed results in macroeconomics is the result of Kydland and Prescott (1977) of dynamic inconsistency of optimal policy: maximization of households' welfare with respect to policy requires that households be mistaken. This is optimal to announce low inflation, low capital income and consumption taxation and never realize these promises. In equilibrium rational households don't believe to erroneous announcements, this is why design for optimal policy involves into consideration reputation and commitments.

There is a reason to doubt in optimality of an inconsistent plan. According to the methodology, which was used to get the inconsistency result, government debt default and expropriation of capital are also properties of optimal policy. Nevertheless, economists generally don't believe that this is really the solution of optimal policy problems. Conventional wisdom and historical experience make it clear that default and expropriations are harmful for economic development. Thus, dynamic inconsistency models omit some important features of the real world, which may be crucial for the Kydland and Prescott result.

The central result of this paper is that the only reason for dynamic inconsistency is possibility of implied default or expropriation of property rights at the beginning of the "optimal" plan. That is, if we agree that expropriations and defaults are not optimal, then optimal policy is always dynamically consistent. We also precise in the paper what means "implied default" and what are optimal dynamically consistent fiscal and monetary policies without expropriation.

This argument makes us to be in some doubt about the theory of dynamic inconsistency of optimal policy. In fact, if we disagree with the idea that default and expropriation are suboptimal, then the optimum is to expropriate all property, default all government debts, and use capital income to finance government spending instead of taxes. There is no place for dynamic inconsistency: we get the first-best allocation and there is no reasons to improve it. In the opposite case, if we believe that default and expropriation are harmful, if we think that authorities should ensure property rights and debt payments, we get that optimal policy is dynamically consistent. Even if we believe that expropriations and default should take a place "at some extent" , there is no reason to believe that the dynamic inconsistency theory properly determines this extent and the way that implied default and expropriation are implemented. Consequently, even if dynamic inconsistency problem exists, it should be analyzed in a more appropriate way than we can find in contemporary literature.

In order to exclude all forms of implied default and expropriation, all we need is to choose appropriately prices, in which authorities should guarantee households' wealth. For example, if the government guarantees the nominal value of its debt, hyperinflation could destroy its real value. If the real value of the household's wealth is guaranteed, the government could tax at 100% the return on wealth, which would be equivalent to an expropriation. In both cases the definitions of property rights are not complete because they accept as legal different indirect forms of expropriation; this is why these definitions lead to a time-inconsistent bias in optimal dynamic plans.

The primal approach to optimal taxation, developed by Ramsey (1927), Atkinson & Stiglitz (1980), Lucas and Stockey (1983) Chari & Kehoe (1998), and others, helps to find appropriate prices. The essence of this approach is to find the optimal allocation, just as the social planer does, but adding the "implementability constraint" on the set of allocations, which ensures that the found allocation may be implemented in a decentralized economy without lump-sum taxes. The household wealth appears only in this additionally constraint. So, its value is well-defined only if it is measured in the same units as the implementability constraint, i.e. in units of the utility function; otherwise, this value is not determined at all.

The price of wealth in terms of utility is given by the co-state variable of the household's problem. If we multiply the co-state variable by the nominal value of the household wealth, we get the utilitarian value of the wealth that should be considered as predetermined in the problem of optimal dynamic policy. The solution to a problem posed in such a way is dynamically consistent.

Optimal policy under *No Implied Default Condition* has following properties. Optimal capital income tax is about zero and Friedman rule holds from the beginning of the period of planning. Consumption and labour taxes are about constant, but adjusted at some special way at the moment when a fiscal or monetary reform takes place. There is no good or bad news for households: they don't want to revise their previous decisions when a reform is announced.

We demonstrate our result in framework of optimal capital taxation problem; see Chamley (1985), Fisher (1980), and Judd (1985). Nevertheless, we believe that this is applicable to any general equilibrium model, because principles of optimal taxation depend neither on production nor on the way, the prices are adjusted.

Three links to previous results on time consistency problem should be mentioned. First, Lucas and Stockey (1983), M. Persson, T. Persson and Svensson (1987) have shown that the inconsistency problem can be resolved if the government uses an appropriate debt management. In terms of our papers, these authors propose to use such a debt structure that the utilitarian value of the debt be independent of the policy. In this case, some implicit forms of default became explicit. Nevertheless, some other implicit forms of defaults, such as introduction of new taxes, remain available. Besides, this solution requires an unrealistic debt structure, and involves hyperinflation, expropriation and default in the beginning of the optimal plan.

Secondly, we should mention the Woodford (1999) "timeless perspective" proposal. He proposes to commit that the government chooses a "*pattern of behavior to which it would have wished to commit itself to at a date fare in the past*". At one extreme, if this date of reference coincides with the date the commit is announced, then the resulting allocation is the same one as under the policy, proposed by Lucas and Stockey (1983), M. Persson, T. Persson and Svensson (1987). At the other extreme, if this date of reference tends to minus infinity, then the allocation may be the same as under the policy, proposed in this paper.

Nevertheless, Woodford does not propose any reason to take the date of referents, which tends to minus infinity. In contrary, if we don't take into account the expropriation problem, the date of reference should coincide with the date of commitment announcement; it follows from the maximization of the government objective with respect to the date of reference.

Even if the government decides to take a contra intuitive decision and chooses the date of reference be equal to minus infinity, in the general case the Woodford policy does not coincide with ours. Consider following example. Let the date of reference be the 1 January 1901, and the 1 January 1951 the government introduces a new consumption tax. This tax reform includes a partial expropriation, because the real value of the household wealth decreases. In 1901, it would have wished to exclude all expropriations in 1950. Thus, the Woodford's commitment assumes that today, say, the 1 January 2001, the government would be obliged to compensate this implicit expropriation and the return on the implicitly expropriated wealth for 50 years. There is no evidence that the cost of such a decision would be lower that the expected gains. Moreover, if the reference date is the 1 January 901, then this commitment is not implementable at all.

In contrary to Woodford, our proposal is to exclude only new implicit expropriations. Our policy coincides with the Woodford's one only if (i) the date of reference is taken to be minus infinity and (ii) there have not ever been implicit or explicit expropriations before.

Thirdly, there are many papers on reputation, which start with the famous Barro and Gordon (1983) result. The key advantage of our paper is that it allows to separate analysis of reputation, which includes implicit default and inconsistency issues, from design of optimal policy. With our solution, it is not necessary to lose reputation in order to switch to an optimal policy. Besides, their approach does not really imply the optimal allocation.

The rest of the paper is organized as follows. In section 1 we give all assumptions on the macroeconomic equilibrium; we just take a standard neoclassical framework. Section 2 derives the set of allocations that may be implemented in a decentralized economy; a particular attention is devoted to the implementability constraint, which justifies our approach. In section 3 we state the modified Ramsey problem, solve it, and give a formal argument why the solution is time-consistent. In section 4 we give some results on optimal policies. There are some examples in section 5; section 6 concludes.

1. Model

The representative household maximizes utility, which depends on consumption c, labor l, and real money balances m

$$\max_{[c,l,m]} \int_0^\infty e^{-\rho t} u(c,l,m)\, dt \tag{1}$$

The producer price of the final good is the numeraire. The real wealth A consists of capital K, government debt B, and money. Its accumulation is given by

$$\dot{A} = r(A-m) + wl - (1+\tau_c)c - \pi m \tag{2}$$

Where r and w are the after-tax equilibrium real rate of return and the real wage, τ_c is the consumption tax, and π is the inflation rate. A_0 is given and the intertemporal budget constraint holds.

The co-state variable for equation (2) is γ. The first-order conditions for the household problem are

$$u_c = (1+\tau_c)\gamma \tag{3a}$$

$$u_l = -w\gamma \tag{3b}$$

$$u_m = (r+\pi)\gamma \tag{3c}$$

$$\dot{\gamma} = (\rho - r)\gamma \tag{3d}$$

Production is not of a particular importance in problems of optimal taxation; see Judd (1999) for discussions. We suppose perfectly competitive markets and constant returns to scale at the individual level, what implies that there is no profit. Externalities are possible, and the production function may explicitly depend on time t. The social production function net of depreciation is given by

$$y = F(K,l,t) \tag{4}$$

The government collects taxes to supply an exogenous amount of public good g. Its budget constraint has the following form

$$\dot{B} = rB + g - \tau_c c - \dot{m} - \pi m - \left[F(K,l,t) - rK - wl\right] \tag{5}$$

The market clearing requires

$$\dot{K} = y - c - g \tag{6}$$

2. Attainable allocation set

The set of allocations, which may be realized in the decentralized economy, is given by the resource and implementability constraints: the former guarantees that the firm's behavior is consistent with the decentralized equilibrium, and the later that the household's behavior is consistent with this equilibrium; government's behavior is consistent with the equilibrium by Walras' law.

This section derives the two constraints, finds an appropriate measure for household's wealth, and proves that these constraints in fact describe the attainable allocation set. With respect to the literature, we prefer to consider separately the roles of these constraints, to make our argument on consistency more evident.

2.1. Allocations, consistent with the firms' behavior

The set of allocations, which are attainable for the social planner (which finds the first-best allocation), is given by the resource constraint. This constraint may be found by substitution of the production function (4) into market clearing condition[1] (6) (K_0 is given, and the intertemporal constraint on the dynamics of K holds):

$$\dot{K} = F(K,l,t) - g - c \tag{7}$$

This constraint guarantees that the considered allocation is placed on the production possibility frontier.

Let $\hat{r}$ and $\hat{w}$ be the before-tax interest rate and wage. Remember that the price of the produced good is the numeraire. According to lemma 1, the resource constraint ensures that the considered allocation is consistent with the firm's behavior in the decentralized economy, but not necessary with the household's behavior..

Lemma 1. *Equation (7) is the resource constraint for the considered problem. In other words, (i) any allocation $(c(t), l(t): t \in [0,\infty))$ that may be implemented in the decentralized economy, satisfies equation (7), and (ii) if an allocation $(c(t), l(t): t \in [0,\infty))$ satisfies equation (7), then we can find the dynamics of the producer prices $(\hat{r}, \hat{w})$ under which the firms will choose an input-output vector such that the equilibrium market condition will be satisfied.*

Proof. (*i*) Resource constraint (7) is obtained from equations (4) and (6) that hold in equilibrium, so it is satisfied for any equilibrium allocation.

[1] To be precise, the two constrains, which are considered in this section, should be formulated as inequalities. From Diamond-Mirrlees (1971) principle of production efficiency it is clear, that in optimum they are satisfied as equalities.

(ii) If we know the dynamics of $c(t)$, $l(t)$ and $g(t)$, then from equation (7) and initial conditions we can calculate the dynamics of K, which gives the dynamics of output $y = c + g + \dot{K}$. Knowing the dynamics of y, K, and l, from the firms' first-order conditions we get the prices $(\hat{r}, w)$ under which the firms choose the considered allocation. For example, if there are no externalities, then $\hat{r} = F_K$ and $\hat{w} = F_l$.

2.2. Allocations, consistent with the households' behavior

If lump-sum taxes were available, then the resource constraint would be the only constraint on the considered allocation set. Otherwise, it arises a problem of revenue redistribution between the government and the households. This redistribution is possible only for a restricted allocation set, which is given by the implementability constraint.

Figure 1 gives an example of a resource allocation that may not be implemented in a two-good economy without lump-sum taxes even if there is no production possibility frontier, i.e. when the firms can produce with zero costs. The considered point is not attainable because the household will never chose this allocation, whatever are the consumer prices.

For example, price vector $\vec{a}_1$ satisfies the household's budget constraint, but the household will not choose the considered allocation under this price vector for the given indifference curve. Price vector $\vec{a}_2$ satisfies first-order conditions but is not consistent with the household's budget constraint. So, *the implementability constraint requires* that for a considered allocation there exists a vector of consumer prices that satisfies simultaneously the household's budget constraint and its first-order conditions.

To get the implementability constraint in form of an equation, one should substitute the household's first-order conditions into its budget constraint. The intuition of this approach is the following: if the obtained equation is satisfied for a given allocation, then this allocation may be substituted into the household's first-order conditions to determine the price vector that satisfies both its first-order conditions and its budget constraint; so such a price vector in fact exists[2].

In our framework, substitution of the first-order conditions (3) into the budget constraint (2) gives the following implementability constraint (see the proof of lemma 2 in appendix 1 for details):

$$\int_0^\infty e^{-\rho t} \left(u_c c + u_l l + u_m m\right) dt = a_0 \tag{8}$$

Where a measures the household's wealth in terms of utility:

[2] The proof of lemma 2 in Annex 1 clarifies this point.

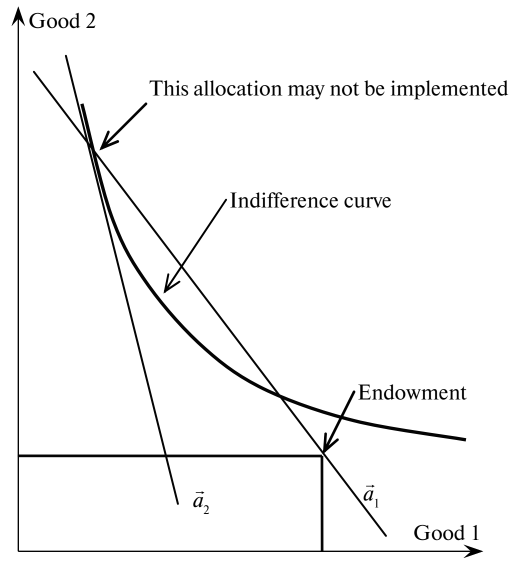

Figure 1. An example of an allocation that may not be implemented even in an economy without production possibility frontier

$$a(t) = \gamma(t) A(t) \tag{9}$$

The right-hand side of equation (8) is discussed in the next subsection.

Let's introduce the nominal interest rate, $R(t) = r(t) + \pi(t)$. According to lemma 2, the implementability constraint ensures that the considered allocation is consistent with the household's behavior in the decentralized economy, but not necessary with the firms's behavior..

Lemma 2. *Equation (8) is the implementability constraint for the household's problem given by equations (1) and (2). In other words, (i) any allocation* $(c(t), l(t), m(t) : t \in [0, \infty))$ *that may be implemented in the decentralized economy, satisfies equation (8), and (ii) if an allocation* $(c(t), l(t), m(t) : t \in [0, \infty))$ *satisfies equation (8), then for any given strictly positive dynamics of one of the consumer prices* $(r(t), \tau_c(t), w(t), R(t) : t \in [0, \infty))$, *there exists a dynamics of the other prices such that households will choose the considered allocation.*

Remark 1. *If for a good* i $(i = c, l, m)$ *there exists a point in time* t *such that* $u_i(c(t), l(t), m(t)) = 0$ *then the dynamics of its price should be considered as en-*

dogenous; it may be the case, for example, if we consider the Friedman monetary policy[3]*. This point is clarified in appendix 1.*

Proof. See appendix 1.

2.3. Measure of household's wealth

Let's consider two complementary interpretations of the implementability constraint. The first one follows directly from the way we get it: one can think that it is the household budget constraint, where the utility is chosen as the numeraire. The second interpretation involves Walras' law: from this law it follows that if the government budget constraint and equilibrium market conditions are satisfied, then the household's budget constraint is also satisfied[4]. Thereafter, the implementability constraint may be thought as the government budget constraint where market clearing conditions have been substituted, and utility chosen as the numeraire.

From these interpretations it follows that if the resource constraint is satisfied, the implementability constraint gives the frontier between the government's and household's budgets. And the numeraire for this frontier is the utility.

What differentiates our research from previous ones, is the right-hand side of implementability constraint (8): we argue that if the frontier between the households' and government budgets over the interval $t \in (0, \infty)$ is measured in terms of utility, then the right-hand side of (8), which determines what part of the wealth at the point $t = 0$ belongs to the households, and what part belongs to the government, should be also measured in terms of utility; otherwise, these parts are not determined at all.

For example, the literature on optimal capital taxation supposes that predetermined is the real value of wealth, which is given by A. Then, substitution of (9) into (8) gives:

$$\int_0^\infty e^{-\rho t}\left(u_c c + u_l l + u_m m\right) dt = \gamma_0 A_0 \qquad (10)$$

The variable γ_0 gives the price of wealth in terms of utility, and if we don't pose any artificial condition, discussed in the introduction, then the government can freely choose this price, simultaneously choosing what part of the initial wealth belongs to the government, and what part belongs to the households. For example, a decrease in γ_0 may imply permanent rise of the consumption tax, or temporary intensive capital taxation; see equation (3a). In this case, the right-hand side of equation (10) is not determined at all, and this equation is not really a constraint on the considered allocation set.

The literature on monetary policy assumes that predetermined is the nominal value of the household's wealth. Let $\hat{A}$ be the nominal wealth, and P be the price level. Then (10) takes the form

$$\int_0^\infty e^{-\rho t}\left(u_c c + u_l l + u_m m\right) dt = \frac{\gamma_0}{P_0}\hat{A}_0 \qquad (11)$$

We can see from (11) that the nominal measure of wealth gives a new undesirable degree of freedom for discretion with respect to the real measure of wealth.

Thereafter, we measure the household wealth in the implementability constraint (8) in terms of utility; we put the variable a_0 in the right-hand side of this constraint. Its value depends on future fiscal policy (just as any value measured in other prices than the numeraire), which the government is permitted to revise. The condition that we pose is that any policy revision should not lead to a damage for wealth holders; the value of a should be considered as predetermined.

We call the condition that a_0 does not decrease in result of policy revision as the "*No implied default condition*".

Note, that some substitutes for the no implied default condition are always introduced in researches, which develop optimal policies. For example, the condition that all taxes are constant and consumption tax is zero, which is usually introduced to ensure dynamic consistency in a closed economy, in fact guarantees that the marginal positive effect of wealth redistribution from households to the government in result of a possible fiscal reform is equal to the marginal loss of economic efficiency net of this effect; clearly, this condition leads to a loss of economic efficiency. Another example is the condition that the capital tax is bounded at 100 percent and that the consumption tax is zero. This condition ensures that the physical capital will not be entirely expropriated in result of a fiscal reform, and something will rest to the wealth holders; but only until the next fiscal reform, when property rights will be again revised.

This paper gives an exact necessary and sufficient condition that should be satisfied to ensure both dynamic consistency and a secure economic environment. We can choose between policies that are more transparent but less efficient, and more efficient but less transparent; however, all considered policies should satisfy the no implied default condition. Otherwise, nobody will buy property, nobody will hold money or government debt, and nobody will invest.

This finding allows to describe the set of allocations that are in fact attainable in a decentralized economy, and to find the best allocation from this set.

[3] Assumption of exogenous dynamics of $r(t)$ or $R(t)$ is possible in the model but not always realistic. It may lead, for example, to infinite growth of $\tau_c(t)$.

[4] Reader can verify that equation (2) follows from equations (4), (5), and (6).

2.4. Allocations, which may be implemented

According to theorem 1, the resource and implementability constraints exactly describe the set of allocations, which may be implemented in a decentralized economy without lump-sum taxes.

Let's τ_l and τ_K denote the labor and capital taxes.

Theorem 1. *The implementability (8) and resource (7) constraints exactly describe the set of allocations that may be implemented in a decentralized economy. In other words, (i) these two constraints are satisfied for any allocation* $(c(t), l(t), m(t) : t \in [0,\infty))$ *that may be implemented in a decentralized economy, and (ii) if these constraints are satisfied for a given allocation* $(c(t), l(t), m(t) : t \in [0,\infty))$, *then for given dynamics of any one tax* $(\tau_c(t), \tau_l(t), \tau_K(t) : t \in [0,\infty))$ *there exists a dynamics of the other taxes and an inflation rate such that the considered allocation will be implemented.*

Remark 2. *Note that the same allocation is attainable by the means of consumption tax and capital tax. The former implies taxation of a final good and the later – of an intermediate good. The equivalence of allocations seems to be inconsistent with the Diamond-Mirrlees (1971) principle of production efficiency, which asserts that intermediate goods taxation is inefficient because it puts the economy inside the production possibility frontier.*

In fact, the Diamond-Mirrlees principle is not applicable for the capital tax: one of its necessary conditions is not satisfied. This principle assumes that there exists a common input for the production of intermediate and final goods. There is no of such an input in the considered model; for example, production in different periods requires labor of different periods, and not the same labor. This is why the economy rests on the production possibility frontier even if physical capital is taxed. Formally, the production possibility frontier is given by equation (7), and this constraint is satisfied, whether the capital is taxed or not. In fact, capital taxation is just a special form of taxation of labor and consumption.

Proof. (i) The first part of the theorem directly follows from Lemma 1 and Lemma 2.

(ii) From Lemma 1 we can find the price vector $(\hat{r}, \hat{w})$ under which firms' first-order conditions and equilibrium market conditions will be satisfied. Taking the dynamics of one tax as exogenous, say, the dynamics of the labor tax τ_l, we can find the dynamics of the after-tax wage from $w = (1-\tau_l)\hat{w}$. Then from Lemma 2 we get he dynamics of $(r(t), \tau_c(t), w(t), R(t) : t \in [0,\infty))$ under which the household's budget constraint and its first-order conditions are satisfied. The government budget constraint is satisfied by Walras' law. The tax rates that lead to the considered allocation may be found from the difference between the consumer and producer prices, for example, $r = (1-\tau_K)\hat{r}$.

3. The modified Ramsey problem

The government maximizes the utility of the representative agent under condition that the allocation may be implemented in a decentralized economy, i.e. under constraints (7) and (8).

$$\max_{[c,l,m]} \int_0^\infty e^{-\rho t} u(c,l,m)\,dt \tag{12a}$$

$$\dot{a} = \rho a - u_c c - u_l l - u_m m \tag{12b}$$

$$\dot{K} = F(K,l,t) - c - g \tag{12c}$$

K_0 is given, a_0 should be found from the solution of the household problem under existing (may be non-optimal) policy; this point is clarified in examples. Intertemporal constraints on dynamics of K and a hold.

The co-state variable for the implementability constraint is λ (negative), and for the resource constraint μ (positive). First-order conditions are

$$u_c\left[1-\lambda(1+H_c)\right] = \mu \tag{13a}$$

$$u_l\left[1-\lambda(1+H_l)\right] = -\mu F_l \tag{13b}$$

$$u_m - \lambda\left(u_{mc}c + u_{ml}l + u_{mm}m + u_m\right) = 0 \tag{13c}$$

$$\dot{\lambda} = 0 \tag{13d}$$

$$\dot{\mu} = \mu(\rho - F_K) \tag{13e}$$

Where the term H_i, if exists, is given by

$$H_i = \frac{u_{ic}}{u_i}c + \frac{u_{il}}{u_i}l + \frac{u_{im}}{u_i}m \tag{14}$$

The term H_i is a measure of the excess tax burden related to a particular form of taxation. It plays the same role as the inverse elasticity of demand in microeconomic analysis of the deadweight loss of taxation; see Atkinson and Stiglitz (1980). A possible interpretation of $(-\lambda)$ is the marginal excess burden of taxation measured in terms of utility.

The first-order conditions (13) are standard for a static Ramsey problem. We need a special form for the first-order condition (13c) because the term H_m doesn't exist under the Friedman policy that we would like to take into consideration.

What differentiates (13) from a solution to a traditional dynamic Ramsey problem, is that we don't need a special form of the first-order conditions for the initial point of time. In previous papers, the variable a_0 in the implementability constraint (8) had been substituted by equation (9), and γ in (9) had been found from (3a). This is why consumption at the initial point of time in the implementability constraint appeared asymmetrically to consumption over other periods of time, and it's why we needed a special form of the first-order conditions for this point.

From (12) we already see, that the solution to the problem restated in such a way is dynamically consistent: all state variables are in fact *state* variables, which don't include forward-looking terms. If a formal argument is required, the consistency may be shown, for example, by comparison of the solutions, obtained in two alternative ways: using the Pontriagin and Bellman principles. The Pontriagin principle maximizes the discounted value of the objective function and may be dynamically inconsistent. The Bellman principle recognizes that in the future, there will be chosen a plan, which will be optimal for that period, what resolves the consistency problem. From the fact that these two solutions are equivalent, it follows that there is no short-run bias, and the optimal plan is dynamically consistent.

4. Optimal policy

Optimal tax rates may be found in the following order. First, solve the household problem under the existing (may be non-optimal) policy to find the initial value of the co-state variable for accumulation equation (2) $\hat{\gamma}_0$, which determines a_0. Second, find the optimal tax rates using first-order conditions (13), taking the consumption tax at the initial point of time $\tau_c(0)$ as a parameter, and get a function $\gamma_0(\tau_c(0))$. Third, get $\tau_c(0)$ from the no implied default condition, i.e. from the condition that $\gamma_0(\tau_c(0)) = \hat{\gamma}_0$. This procedure is clarified in examples, and this section demonstrates only the second step of the solution.

4.1. Fiscal policy

According to theorem 1, the optimal allocation may be achieved by an infinite number of policies. Thereafter, one tax rate should be chosen exogenously. We consider two fiscal policies: the first one is found under assumption that the consumption tax is constant, the second supposes that the capital tax is zero. The initial value of the consumption tax is chosen in such a way that the capital tax is bounded. The

both policies are equivalent under homogenous preferences or under assumption that the economy is always on the balanced growth path.

To find the optimal capital tax rate, it is useful to introduce a composite multiplier, which is analogous to the Judd's (1999) one. Let's define

$$\Lambda = \frac{\gamma}{\mu} \tag{15}$$

On one hand, by taking the logarithmic derivative of (15) and substituting (3d) and (13e) into the obtained equation, we show that the dynamics of this multiplier determines the optimal capital tax:

$$\frac{\dot{\Lambda}}{\Lambda} = F_K - r \tag{16}$$

On the other hand, from first-order conditions (3a) and (13a) we get its dynamics

$$\Lambda = \frac{1}{(1+\tau_c)\left[1-\lambda(1+H_c)\right]} \tag{17}$$

From (16) and (17) we see that the capital and consumption taxes are substitutable. The optimal capital taxation literature concentrates the attention on the case where τ_c is constant. Then, from (16) and (17), we see that

$$F_K - r = \frac{\lambda \dot{H}_c}{1-\lambda(1+H_c)} \tag{18}$$

From (18) it follows that if H_c is constant, then the optimal capital tax net of externalities is zero.

There are two special cases. The first is the case when preferences are homogenous in consumption. It implies that H_c is constant; thereafter, the capital tax net of externalities is zero. For example, if the instantaneous utility function is of the following form

$$u(c,l,m) = \frac{c^{1-\theta}}{1-\theta} + v(l,m) \tag{19}$$

Then $H_c = -\theta$, and $r = F_K$.

The second case is the balanced growth path, where γ and μ grow at the same rate. Then from (15) we see that Λ is constant, and capital is not taxed.

These two cases are not too different: the balanced growth path is possible only if preferences are isoelastic in consumption for the *realized* allocation; for example, if the instantaneous utility takes the form (19); or if the rate of growth on the balanced growth path is zero. Formally, from (15) and (17) we see that γ and μ grow at the same rate if and only if H_c is constant.

The result that under isoelastic preferences for the realized allocation the capital tax in the long run is zero is already known in the literature, see Chamley (1986) and Judd (1985). According to these authors, the optimal capital tax rate is 100% in the short run and close to zero afterwards. A new result is that under isoelastic preferences for the realized allocation and well-defined property rights the capital tax is zero even in the short run.

Our approach allows to consider another optimal policy: instead of hypothesis that consumption tax is constant, let's suppose that the capital tax net of externalities is zero, $F_K = r$, and find the optimal consumption tax[5].

Taking into consideration (16) and (17), we can compare the optimal consumption tax rates at dates 0, t_1 and t_2:

$$\frac{\left[\tau_c(t_1)-\tau_c(0)\right]/\tau_c(t_1)}{\left[\tau_c(t_2)-\tau_c(0)\right]/\tau_c(t_2)}=\frac{H_c(t_1)-H_c(0)}{H_c(t_2)-H_c(0)} \tag{20}$$

Equation (20) is standard for the microeconomic analysis of taxation, see Atkinson and Stiglitz (1980). Thereafter, we interpret this equation in the following way: *if all tax rates at the microeconomic level are chosen optimally, then the optimal capital tax net of externalities is always zero, whatever are the preferences.*

To find the optimal labor tax, use the first-order conditions of the Ramsey problem (13a), (13b) and of the household's one (3a) and (3b):

$$\frac{w}{F_l}=\frac{(1+\tau_c)\left[1-\lambda(1+H_c)\right]}{\left[1-\lambda(1+H_l)\right]} \tag{21}$$

Note, that equation (21) may be rewritten in the same form as (20).

From (21) we see that the labor tax is constant if the preferences are homogenous in c and l for the realized allocation, for example, if the economy is on the balanced growth path.

4.2. Monetary policy

The first-order condition for the monetary policy (13c) is satisfied at a "saturation point", where the first and all second derivatives of the utility with respect to money u_{im} are zero. From (3c) we see that this point corresponds to the Friedman monetary rule. Nevertheless, in general, it's not clear whether the second order conditions are satisfied at this point. If the Friedman rule holds then the optimal nominal interest rate is zero, if not then it is implied by

$$\lambda=\frac{1}{1+H_m} \tag{22}$$

The solution (22) may be optimal, but its existence is not always evident, and in some papers it is skipped over. For example, if the preferences are homogenous, then H_m is constant, and if $H_m < -1$, then the inflationary tax allows to collect taxes with constant marginal excess tax burden. In this case, two solutions are possible: either the marginal excess tax burden $|\lambda|$ is small enough, and the Friedman rule holds, or the marginal excess burden is uniquely determined by (22).

However, we don't believe that the solution (22) has an economic sense: in models, which precise better the role of money (cash-in-advance, shopping time), under realistic assumptions the Friedman rule holds; see Chari and Kehoe (1996) for a discussion.

A new result with respect to the literature is that under well-defined property rights, the Friedman rule holds even in the short run[6].

5. Examples

In this section we give examples, which demonstrate the nature of the solution. We consider an example that explains the meaningful of the no implied default condition, and find an optimal time-consistent fiscal plan in the framework of the Barro (1990) model.

5.1. No implied default condition

Let's assume that there is no externalities and the production doesn't depend on time, thereafter the economy converges to a steady state. The dynamics of the system is given by two differential equations: the first one is the resource constraint (7) and the second one is the equation, which gives the consumption dynamics, and which is implicitly given by the household's first-order conditions (3), production function (4), and macroeconomic policy. There is a fiscal reform that is announced

[5] There are no first-order conditions for the consumption tax if we don't introduce the no implied default condition: it should be taken as large as possible. It explains why this policy hasn't been considered in previous papers.

[6] In the paper of Chari and Kehoe (1996) the Friedman rules holds both in the short and in the long run, but this result is based on an unrealistic hypothesis that the initial household's wealth is zero.

in advance, which consists in a switch from taxation of capital to taxation of consumption and leisure. All tax rates are supposed to be constant.

The traditional optimal policy analysis supposes that there is no consumption tax. Thereafter, only labor is taxed after the reform. The dynamics of the economy is depicted in the figure 2. Initially the economy converges from point A to the steady state B along the initial stable brunch of the system. When the economy riches point C, the reform is announced. The consumption immediately jumps down to point D, to an unstable trajectory that leads the economy to point E on the new stable brunch. Point E will be achieved exactly at the time when the reform will take place. Afterwards, the economy will converge to the new steady state F.

Note, that if the consumption decreases in result of an announce of a fiscal reform, then the price of wealth γ increases. And an increase in the initial value of the household's wealth, measured in terms of utility, destimulates labor, which leads to decrease in the steady-state values of c and K.

An optimal switch to the no capital taxation, satisfying the no implied default condition, is depicted in the figure 3. The economy starts at point A, and the reform is announced at point C. The phase diagram will not change until the reform will take place, and the no implied default condition requires that the co-state variable does not jump at the point C. Thereafter, before the reform, the economy continue to grow along the initial stable brunch. At the moment the reform takes place (point D), the consumption tax changes discontinuously, what leads to a jump in

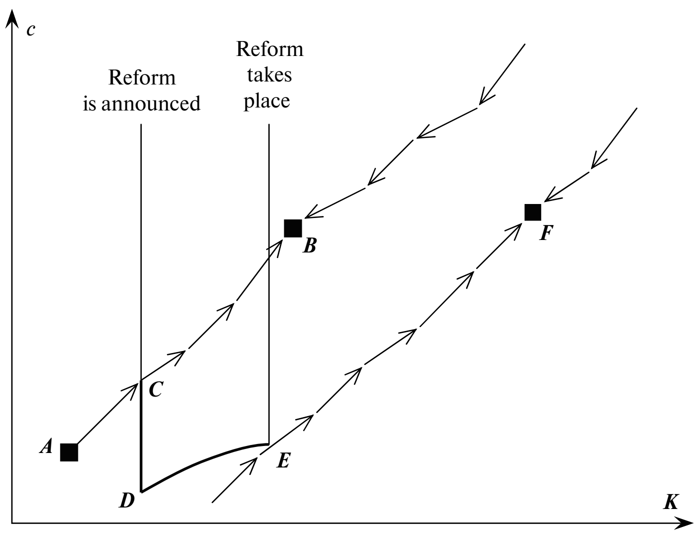

Figure 2. Traditional analysis of a switch to zero capital taxation

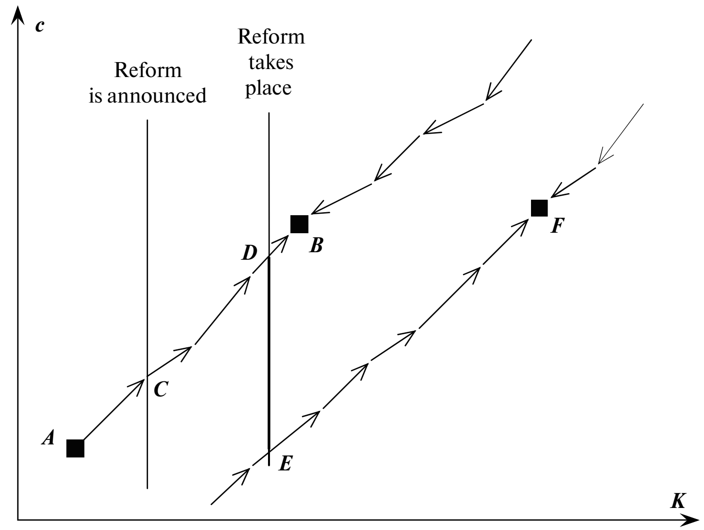

Figure 3. Optimal switch to zero capital taxation

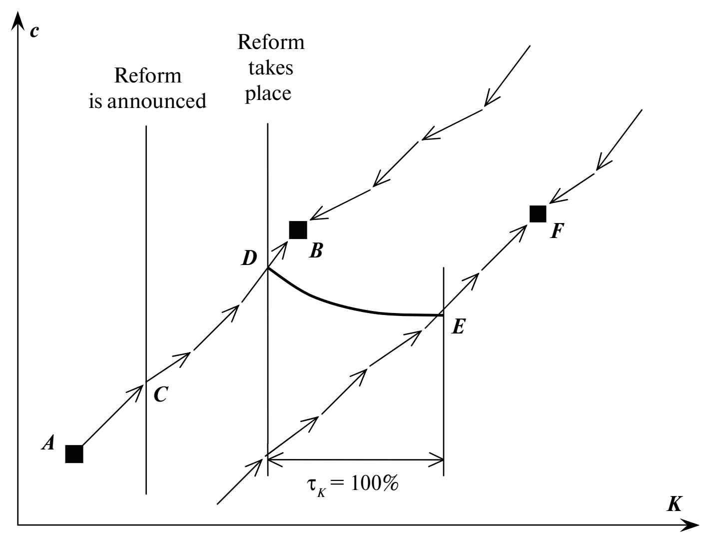

Figure 4. Optimal switch to zero capital taxation when the consumtion tax is zero

consumption from D to E. The consumption tax should be changed in the way to avoid any jump in γ.

If the consumption tax is not available, then the optimal allocation should be implemented through the capital tax, and if capital taxation is bounded (for example, at 100%), then the dynamics of the consumption will be smooth; this case is depicted in the figure 4, the dynamics of the economy is $A \to C \to D \to E \to F$. During the transitional period between points D and E, the return on capital is taxed at 100%, and the after-tax real interest rate is zero.

The no implied default condition may be reformulated as the condition that the government doesn't generate bad news: an announcement of a fiscal reform doesn't influence the economic development until the reform takes place. Thereafter, there is no desire to revise previous decisions in result of an announcement of a new policy.

5.2. Optimal fiscal policy in the Barro model

Consider an economy that switches from the policy suggested by Barro (1990) to an optimal one. We generalize the Barro model, supposing that labor supply is endogenous. Otherwise, the solution to the optimal taxation problem doesn't really imply the second-best allocation; see in Atkinson and Stiglitz (1980) a discussion on the Musgrave wrong-result of uniform taxation.

The household's problem is given by

$$\max_{[c,l]} \int_0^\infty e^{-\rho t} \frac{1}{\sigma} \left(c^\theta (1-l)^{1-\theta} \right)^\sigma dt \tag{23a}$$

$$\dot{A} = rA + wl - (1+\tau_c)c \tag{23b}$$

$$\sigma < 1, \theta \in (0,1)$$

We change the production (4) to make government expenditure endogenous:

$$F(K,l,g,t) = K^\alpha (gl)^{1-\alpha} \tag{24}$$

First-order conditions (13) do not change, but a new condition appears:

$$F_g = 1 \tag{25}$$

Equation (25) is an application of the Diamond-Mirrlees principle of production efficiency: the government expenditure plays the role of an intermediate good; this is why it is supplied as under the first-best. Equations (24) and (25) indicate that the ratio of government expenditure to GDP is constant and equals to $(1-\alpha)$. The same government expenditure level had been suggested by Barro.

Barro had assumed that there is a unique tax on all income

$$\tau_K = \tau_l = (1-\alpha) \tag{26a}$$

$$\tau_c = 0 \tag{26b}$$

To find an optimal policy, we use the equations (16) – (21). Equation (17) takes the following form

$$\Lambda = \frac{1}{(1+\tau_c)\left[1-2\lambda\theta(\sigma-1)\right]} \tag{27}$$

Equations (27) and (16) imply that under constant consumption tax the optimal capital tax is always zero. The equation (21) becomes

$$\frac{1-\tau_l}{1+\tau_c} = \frac{1-2\lambda\theta(\sigma-1)}{1-\lambda\frac{2l-1}{1-l}} \tag{28}$$

The considered model has no transitional dynamics, so labor is constant; together with (28) it implies that the labor tax is also constant. If we knew λ, we could find the relationship between consumption and labor taxes. Otherwise, we can get it from the government budget constraint written in real terms:

$$\tau_c c + \frac{\tau_l}{1-\tau_l} wl = g + \frac{\rho - \theta\sigma r}{1-\theta\sigma} B \tag{29}$$

The right-hand side of the equation (29), which reflect the debt service, takes into account the rate of economic growth.

Equation (29) has two unknowns: the labor and consumption taxes. The second equation, including these terms, is the no implied default condition. Initial welfare A_0 is given, and, from (9) we conclude that the co-state variable for the equation (23b) should not change when we switch between the two policies. Let (c_B, l_B) be the allocation under the Barro policy, and (c_S, l_S) be the second-best allocation. Then the no-implied default condition requires that at the time of a switch between policies, the following equation holds:

$$\theta c_B^{\theta\sigma-1}(1-l_B)^{(1-\theta)\sigma} = \frac{\theta c_S^{\theta\sigma-1}(1-l_S)^{(1-\theta)\sigma}}{1+\tau_c} \tag{30}$$

Note, that the second-best allocation (c_S, l_S) depends itself on the consumption tax.

In result of this switch, under reasonable parameter values, current output and consumption decrease, but the rate of economic growth increases.

6. Conclusion

The inconsistency problem arises if wealth at the initial point of time is measured in other units than in future. The solution to a problem posed in such a way has no economic sense: consumption is predetermined instead of household's wealth, and lack of lump-sum taxes is not a constraint on the attainable allocation set. If we properly measure household's wealth, then the solution to the optimal policy problem is dynamically consistent.

Our solution seems to be realistic: if the economy is on the balanced growth path, then all optimal tax rates and the ratio of the government debt to GDP are constant; otherwise, the dynamics of these variables is independent of the choice of the initial point of time. This solution also seems to be in fact benevolent: announces of new policies are not "bad news", and do not lead to intention to revise previous decisions. Policies, which imply no capital taxation and the Friedman monetary rule, seem to be good policies.

There are many questions that remain open. For example, how the no implied default condition should be formulated if we analyze heterogenous agents or open economy? How should be taken into consideration the existence of durable goods (see fig. 3)? Whether the researches, which analyze how market imperfections influence the no capital taxation result, are consistent with our finding? How our paper corresponds to the new political economy literature? Nevertheless, we believe that we have found the last missing analytical tool, which finally will allow to implement the theory of optimal dynamic policy. And implementation of this theory will enhance both current economic activity and the rate of economic growth.

A. Appendix 1
Proof of the Lemma 2

To prove the first part of the lemma, we derive the implementability constraint (8) from equations that hold in equilibrium.

The solution to equation (2) that takes into account the intertemporal household budget constraint, is given by

$$\int_0^\infty e^{-\int_0^t r(\tau)d\tau}\left[(1+\tau_c)c+(r+\pi)m-wl\right]=A_0$$

The solution to the equation that gives the first-order condition (3d) is

$$\gamma=\gamma_0 e^{\rho t}e^{-\int_0^t r(\tau)d\tau}$$

Combining these two equations, we obtain

$$\int_0^\infty e^{-\rho t}\left[(1+\tau_c)\gamma c+(r+\pi)\gamma m-w\gamma l\right]=\gamma_0 A_0$$

Substitution of the first-order conditions (3a)-(3c) into this equation gives the implementability constraint (8)

So, if the equations (2) and (3) are satisfied then the equation (8) is also satisfied.

To prove the second part of the lemma, it should be shown how to find the dynamics of consumer prices under which the households choose the considered allocation. To be precise, suppose that exogenous is the dynamics of $w(t)$. Note, that the considered allocation gives the dynamics of u_c, u_l, and u_m.

From equations (3b) we evaluate the dynamics of γ.

$$\gamma=-\frac{u_l}{w}$$

Then from (3d) we find the real interest rate dynamics

$$r=\rho-\frac{\dot\gamma}{\gamma}$$

From equations (3a) and (3c) the dynamics of the consumption tax and of the inflation rate may be found.

$$\tau_c = \frac{u_c}{\gamma}-1$$

$$\pi = \frac{u_m}{\gamma}-r$$

Note that if the dynamics of the real interest rate is given exogenously then we need implementability constraint (8) to determine the initial value of γ. The dynamics of the nominal interest rate may also be taken as the exogenous, but only if we don't consider the Friedman monetary policy, i.e. if only $u_m(c(t),l(t),m(t))$ and $R(t)$ are positive for any t. Otherwise, we would not be able to find the dynamics of γ from the equation (3c); this is why we need the remark 1.

It's clear that prices, found in such a way, satisfy the household's first-order conditions. If we substitute these prices into the implementability constraint, we get an equation that coincides with the household's budget constraint; so, the budget constraint is also satisfied.

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