Extending the Condorcet Jury Theorem to a general dependent jury

Bezalel Peleg and Shmuel Zamir

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Abstract

We investigate necessary and sufficient conditions for the existence of Bayesian-Nash equilibria that satisfy the *Condorcet Jury Theorem* (*CJT*). In the Bayesian game G_n among n jurors, we allow for arbitrary distribution on the types of jurors. In particular, any kind of dependency is possible. If each juror i has a "constant strategy", σ^i (that is, a strategy that is independent of the size $n \geq i$ of the jury), such that $\sigma = (\sigma^1, \sigma^2, \ldots, \sigma^n \ldots)$ satisfies the *CJT*, then by McLennan (1998) there exists a Bayesian-Nash equilibrium that also satisfies the *CJT*. We translate the *CJT* condition on sequences of constant strategies into the following problem:

(**) For a given sequence of binary random variables $X = (X^1, X^2, ..., X^n, ...)$ with joint distribution P, does the distribution P satisfy the asymptotic part of the *CJT* ?

We provide sufficient conditions and two general (distinct) necessary conditions for (**). We give a complete solution to this problem when X is a sequence of exchangeable binary random variables.