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ELECTORAL COMPETITION WITH COSTLY ISSUE SELECTION

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In this work I analyze the effect of electoral uncertainty on issue trespassing. I build a model of political competition between two candidates in which each candidate decides how much effort to spend in order to increase her competence on each of the two issues. It is assumed that there are two groups of voters, each believing that one of the two issues is more salient. Each candidate is strong on one issue (so the costs of increasing the competence on that issue are lower), and weak on the other issue. I also assume that there is electoral uncertainty: the voters receive a valence shock in favor of one of the two candidates. I show that the effect of electoral uncertainty is conditional upon the payoffs to the candidates with respect to their vote shares. Electoral uncertainty results in more issue trespassing (when candidates focus more on the strong issues of their opponents) only if winning the election by a large margin confers additional benefits relative to winning by a narrow margin, and there are no benefits from losing by a narrow margin relative to losing by a wide margin. I also show that the competition on both issues is the strongest if the voter valuation of these issues is homogeneous, when more information on voter preferences is available to the candidates, and when the costs of competing on either strong or weak issues are lower.

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1 Introduction

1.1 Political competition through issue selection

Political competition has traditionally been studied as a contest between two or more candidates, each choosing a policy program from a one or multidimensional set of policy alternatives, over which the voters have single-peaked or similarly defined preferences (Downs, 1957). Recently, literature has given attention to an alternative viewpoint, that parties or candidates have a fixed set of competencies (or valences) on a set of political issues, and that they have limited resources to persuade the voters that one or the other issue is important. The main decision made by a candidate or party is the choice of an issue (or a set of issues) on which to focus the campaign rhethoric.

A policy issue on which all voters agree that is better to have more than less competence is a valence issue (Stokes 1962). One empirical work that supports the assertion that the valence issues may be more important than the Downsian spatial issues is a study of British general elections in 2005 by Whiteley et.al. (2005). The authors examined the set of issues relevant to that campaign and divided it into two groups. First, there were positional issues: Britain's relationship with the EU, taxation vs. public spending, and combating crime vs. rights of the accused. Second, these were valence issues, such as the party's ability to handle the economy, immigration problems, or fight crime. The authors argued that a voter's preferences toward each candidate is determined by both how close the stated positions of the candidate and her political are to the voter's position on that issue, and by the salience that the individual attaches to each of the valence issues, with the valence issues being more important. For one thing, Whiteley and co-authors found that the Labor

party's perceived abilities to handle economic and other problems were among the most important predictors of vote choice. At the same time, the candidate position on spatial or positional issues was a less important (albeit significant) predictor.

This account of voting in elections is consistent with a large body of political communications literature. Political parties have different reputations handling different policy issues — in ther words, different parties "own" different issues (Petrocik 1996). In a typical election camaign, a large part of the political rhetoric of parties (or the candidates representing them) focuses on reasserting their reputation on the issues that they own. It was indicated that the candidates should try to frame the political debate by emphasising the issues that their party owns (Riker 1993). Party's ownership of an issue can be eroded by poor policy performance. Arceneaux (2008), using both experimental and empirical data, argues that poor policy performance increases the effectiveness of issue trespassing by the opponent. Both empirical (Belanger and Meguid, 2005) and experimental (Ansolabehere and Iyengar, 1994) evidence suggests that it is more efficient to campaign on one's own issues, rather then those of the opponent.

Campaigning on issues that are historically associated with another party is known as issue trespassing or issue convergence. Several sources of issue covergence were identified. For example, Damore (2004, 2005) investigated which issues were addressed in advertisements produced by Presidential candidates in 1976-1996 campaigns. Issue convergence was defined as a response of one candidate to an issue adressed in another candidate's advertisement.

First, it was found that the candidates were more likely to converge on the issues that the voters considered salent. Moreover, convergence was more likely on issues owned by the Republican party, such as defense, taxes, government spending, or

crime. Finally, a candidiate was found to be more likely to trespass on an opponent's issue if he was trailing in the polls.

1.2 Theoretic analysis of issue and valence competition

There is a number of theoretical works that look at the campaign choices with respect to valence and the choice of campaign issues. Groseclose (2001), Zakharov (2009) and Ashworth and Bueno de Mesquita (2009) look at spatial policy competition in which office-motivated candidates may have different valences[1]. In the first work, the valences are exogenous, and no equilibrium exists[2]; in the last two works, the valences are endogenous and can be increased at a cost; in equilibrium candidates choose different policy platforms.

In Amoris and Puy (2007), there are two candidates who have fixed policy positions on two policy issues. While the positions themselves are fixed, the candidates can spend time to "campaign" or talk about the issue (the total time available to both candidates is fixed). The voters have generalized Eucledian preferences over candidate positions, with the salience of each issue being a function of the time that both candidates spend campaigning on that issue. The authors have shown that,

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[2]Serra (2010) looks at policy-motivated parties. In Herrera, Levine, and Martinelli (2008), campaign spending is used to increase turnout instead of valence.

[2]Aragones and Xefteris (2012) derive a mixed-startegy equilibrium. Groseclose (2008) looks a the social choice problem with a valence dimension. Berger, Munger, and Porhoff (2000) argue that an equilibrium with fixed valences may be possible when advantaged candidate moves first. Ansolabehere and Snyder (2000) derive the conditions for an equilibrium with fixed valence and a multidimensional policy space. Aragones and Palfrey (2004) conducted a laboratory experiment for a spatial competition game with fixed valences.

depending on the fixed candidate positions, two types of equilibria may arise - issue specialization, when each candidate chooses to allocate all her time to a separate issue, and issue engagement, when both candidates campaign on both issues. Colomer and Llavador (2011) assume that the each candidate can choose one issue of many to campaign on, and only one issue (depending on the issues that the candidates campaign on) is decisive to the voters. Morelli and van Weelden (2011) allow the candidates to have policy preferences, which are privately observable; the choice of an issue on which to campaign serves as a credible signal of the candidate's type. Duggan and Martinelli (2011) develop a model of media slant: the candidates have fixed policy positions in two-dimensional space, but the voters perceive the competition as one-dimensional, and the pro-challenger and pro-incumbent media can affect how the policy platforms are projected into a one-dimensional space.

Egorov (2012) studies a model of election with two candidates and two valence dimensions, assuming that the competencies of the candidates on each issue are unobservable random variables. Each candidate makes a binary choice of an issue on which to campaign (with the challenger choosing first). The amount of information that is revealed to the voters depends on the choice of the candidates: If both choose to campaign on the same issue, then more information on that issue is revealed. It was shown that while the challenger always campaigns on the issue at which he is most competent, the incumbent's response depends on how much information is revealed if candidates campaign on diffrerent issues. The author, however, assumes that all voters are identical with respect to their preferences on different issues. A similar setting was considered earlier by Polborn and Yi (2006), who assumed that the candidate perfectly informs voters on her competence if he decides to campaign on an issue, and that "neg-

ative campaigning" – when a candidates reveals information on the competence of another candidate — is also possible.[3]

Aragones, Castanheira, and Giani (2012) develop a model that is very similar to the one analyzed in this paper. There are two candidates and three essies, and the election game is two-stage. At the first stage, each candidate chooses how much resources to spend building his competence on each of the three issues (with the candidates having different cost functions for the issues). At the second stage, each candidate decides how to divide her campaigning time between the three issues. The salience of each voter on each issue depends both on her ex ante salience (which are different for different voters) and on the time that both candidates spend campaigning on the issue. The authors have shown that the candidates may engage in issue stealing, when a candidate campaigns on an issue on which he did not have an ex ante comparative advantage.

1.3 Payoffs to politicians and votes

Existing literature generally assumed that the candidates were either winner-take-all or risk-neutral with respect to the number of votes received in the election. At the same time, one cannot argue that this is always true with respect to the payoffs of the political actors in real elections, due to a number of factors — both internalized and induced by the electoral system.

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[3]Bhattacharya (2011) looks at information revelation by candidates through positive and negative advertisement. Callander and Wilkie (2007) explicitly model lying in political campaigns, when candidates are policy-motivated but can exert effort in order to publically display a policy preference that is different from the true one. Kartik and McAfee (2007) analyze a setting where candidates can be of two types — nonstrategic with high valence and a fixed policy position, and policy-motivated strategic, who can attempt to emulate the strategic candidate by choosing a similar policy position.

There are at least three reasons why one might depart from the classical assumptions.[4]   First, a large margin of victory may induce a very large payoff. Simpser (2013) argues that in autrhoritarian regimes, a large margin of victory may serve to deter potential opposition to the regime, inducing a larger payoff than a narrow victory.

Second, large margin of victory is needed (especially in countries without a prolonged tradition of a democratic power transfer) in order to initiate large-scale economic or political reform. The consequences may be dire for their initiator, unless he or she relies on the support of a large majority. A telling example is Chilean president Salvador Allende, who won the 1970 Presidential elections on top of a 36.63% plurality (with the runner-up receiving 35.29%); the broad socialist reforms that he authored provoked a coup in which he ultimately lost his life.

Third, a loser who lost by a narrow margin may receive an additional reward — a "consolation prize" (Hojman (2004)). Finally, floor requirement, quotient formula, and district magnitude all affect the translation of votes into seats (Lijphart, 1990; Gallagher, 1992) even in "proportional representation" electoral systems. Coalition-building concerns further complicate the payoff functions of political parties (Snyder, Ting, and Ansolabehere, 2005; Laver and Shepsle, 1996; Schofield and Sened, 2006, among others).

2   The model

As the framework for the analysis, I use the probabilistic voting model (Hinich, Ledyard, and Ordeshook, 1972, Hinich 1977,1978, Banks and Duggan, 2005). In such a model it is assumed that each voter receives a random utility shock with respect to each

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[4]See Zakharov (2012) for a more complete review.

candidate. I assume that the utility of each voter with respect to each candidate has both a voter-specific and a candidate-specific component. The strategy of each candidate is the vector of competencies of that candidate with respect to a number of policy issues. Suppose that there are J candidates and K policy issues. Denote by x_j be a vector of length K of issue competencies for candidate j, and let x be the $K \times J$ vector of all issue competencies. Let there be a continuum of voters belonging to L groups of weights $(w_1, \ldots, w_L)$, with $w_l \geq 0$ and $\sum w_l = 1$. For any voter s belonging to group l, let the utility of voting for candidate j is equal to

$$u_{sj} = x_j \cdot \alpha_l + \epsilon_{sj} + \delta_j, \tag{1}$$

where α_l are the saliences of issues $1, \ldots, K$ to the voters belonging to group l, ϵ_{sj} is the random utility shock to voter s with respect to candidate j, and δ_j is the random utility shock to all voters with respect to candidate j. We assume that $\epsilon_s = (\epsilon_{sj})_{j \in J}$ are drawn independently across all voters from distribution $F_\epsilon(\cdot)$ that has a nonzero, differentiable density $f_\epsilon(\cdot)$. Similarly, $\delta = (\delta_j)_{j \in J}$ is drawn from distribution $F_\delta(\cdot)$ (we do not necessarily assume that F_δ is continuous).

Fix δ. For voter s in group l, let

$$P_{lj}(x, \delta) = P(u_{sj} = \max_{k \in J} u_{sk} | \delta) \tag{2}$$

be the probability that voter s votes for candidate j, given a common utility shock δ. As all voters in group l are identical up to the realization of ϵ_s, this also denotes the fraction of voters in group l that vote for candidate j (for that reason we used the l subscript instead of the s subscript in (2). Denote by

$$V_j(x, \delta) = \sum_l w_l P_{jl} \tag{3}$$

the vote share of Candidate j in state δ. Let $u_j : [0, 1] \to \mathbf{R}$ be a differentiable function that denotes the benefit that Candidate j

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gets depending on her vote share. Let $c_j : [0, \infty)^K \to \mathbf{R}$ be the cost to Candidate i of bearing issue competencies. The expected payoff of Candidate j will then be

$$u_j(x) = \int V_j(x, \delta) dF_\delta - c_j(x_j). \tag{4}$$

I then proceed to analyze equilibria in game with payoffs (4).

3 The two-candidate, symmetric case

Suppose that there are two candidates and two gropus of voters of equal size. Let x_{jk} be the amount of competence of Candidate k on issue $k = 1, 2$. Let the utility of voter i given the election of Candidate j be

$$v_{ij} = \alpha_{i1} x_{j1} + \alpha_{i2} x_{j2} + \epsilon_j \tag{5}$$

where α_{ik} is the salience of issue k to voter i, and e_j is the valence of Candidate j. Suppose that there are two states, 1 and 2, occuring with equal probability. In state 1, Candidate 1 receives a positive valence shock $\epsilon_1 = \delta$, in state 2 — a negative shock $\epsilon_1 = -\delta$. The valence of Candidate 2 is zero in both states. Let the fraction of voters from group i that votes for party 1 be a function of the utility difference $v_{i1} - v_{i2}$:

$$P_{i1} = P(\alpha_{i1}(x_{11} - x_{21}) + \alpha_{i2}(x_{12} - x_{22}) + \delta) \tag{6}$$

in state 1 and

$$P_{i2} = P(\alpha_{i1}(x_{11} - x_{21}) + \alpha_{i2}(x_{12} - x_{22}) - \delta) \tag{7}$$

in state 2. Let $u(2x)$ be the candidate utility of getting a share of the total vote equal to x. Assume that $u(\cdot)$ is a twice differentiable function with $u' > 0$. Let $P(\cdot)$ be a twice differentiable

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function with $P(x) = 1 - P(x)$ and $P''(x) < 0$ for $x > 0$. In addition, we assume that raising competence on each issue is costly for the candidate. Let $e_{jk}c(x_{jk})$ be the cost of competence for Candidate j on issue k, where $c'(0) = 0$, $c' \geq 0$, and $c'' > 0$.

The expected utilities for the two candidates then become

$$
\begin{aligned}
U_1 &= u(P_{11} + P_{21}) + u(P_{12} + P_{22}) - e_{11}c(x_{11}) - \\
&\quad - e_{12}c(x_{12}) & (8) \\
U_2 &= u(2 - P_{11} - P_{21}) + u(2 - P_{12} - P_{22}) - e_{21}c(x_{21}) - \\
&\quad - e_{22}c(x_{22}). & (9)
\end{aligned}
$$

Assume that $\alpha_{11} = \alpha_{22} = \alpha$ and $\alpha_{12} = \alpha_{21} = 1 - \alpha$, with $\alpha > \frac{1}{2}$. Let $e_{11} = e_{22} = e_1$ and $e_{12} = e_{21} = e_2$, with $e_2 > e_1$. So we assume that the candidates and voters are symmetric. Voter 1 believes that Issue 1 is more important that Issue 2, Voter 2 believes that Issue 2 is more important, Candidate 1 is more competent at Issue 1, and Candidate 2 is more competent on Issue 2. The following statement is true.

Theorem 1   1. Let $u'' \geq 0$ and $u''' \leq 0$. Then there exists $\bar{\delta} > 0$ such that for all $\delta < \bar{\delta}$, there exist $x_1 > x_2$ such that $x_{11} = x_{22} = x_1$ and $x_{12} = x_{21} = x_2$ are a local Nash equilibrium in game with payoffs (8), (9).

2. Let x_1, x_2 be a local Nash equilibrium in game with payoffs (8), (9). Let $u'(1 + P_1 - P_2) < u'(1 - P_1 + P_2)$, where $P_1 = P((2\alpha - 1)(x_1 - x_2) + \delta)$ and $P_2 = P((2\alpha - 1)(x_1 - x_2) - \delta)$. Then $x_1 > x_2$.

The first part of this theorem staes that, if the level of common voter utility shocks is small relative to due utility that the voters receive from the candidates, then we should expect a local equilibrium to exist in the elections with two symmetric groups of voters, and symmetric cost functions of the candidates. If

the level of uncertainty is large, then equilibrium existence is not guaranteed even in the simplest case. The second part of the theorem states that, under certain conditions, each candidate will invest more effort in the issue that he is relatively more competent in. This should happen if $u'(1 + P_1 - P_2)$ — the additional utility that a candidate receives from an extra vote in her best-case scenario (i.e. when voters receive a positive utility shock for that candidate) is smaller than her worst-case marginal utility $u'(1 - P_1 + P_2)$. Thus we should expect the candidates to campaign predominantly on their strong issues if the value of a victory by a large majority relative to a victory by a narrow majority is smaller than the value of a narrow defeat relative to a defeat by a large margin. This should happen if there are consolation prizes to strong runner-ups (such as the ability to attract funds for subsequent elections), but no additional benefits to strong winners.

In the next theorem we derive the comparative statics of equilibrium competence levels, relative to the cost of acquiring competencies e_1 and e_2, the importance of the more preferred issue relative to the less preferred issue, and the magnitude of the uncertainty δ. Our results apply to the case when the candidates invest more in the issues in which they are more competent. We also assume that the second-order utility effects for the candidates are not large enough.

Theorem 2 Suppose that $c'''(\cdot) \leq 0$. There exists $\bar{\delta} > 0$, such that for all $\delta < \bar{\delta}$, for all $x_1 > x_2$ such that $x_{11} = x_{22} = x_1$ and $x_{12} = x_{21} = x_2$ are a local Nash equilibrium in game with payoffs (8), (9). Let $u''(1 - P_1 + P_2) = u''(1 + P_1 - P_2) = 0$. Then we have

1. $\frac{\partial x_1}{\partial \alpha} < 0$, $\frac{\partial x_2}{\partial \alpha} < 0$ $\frac{\partial x_1}{\partial e_2} < 0$, $\frac{\partial x_2}{\partial e_1} > 0$, and $\frac{\partial x_2}{\partial e_2} < 0$.

2. The sign of $\frac{\partial x_2}{\partial \delta}$ is equal to the sign of $u_1' - u_2'$.

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3. Let $\tilde{P}(u, \beta)$ be a twice continuously differentiable function such that $\tilde{P}((2\alpha - 1)(x_1 - x_2), \beta) = P((2\alpha - 1)(x_1 - x_2))$ and $\tilde{P}_u((2\alpha - 1)(x_1 - x_2), \beta) = P'((2\alpha - 1)(x_1 - x_2))$ for all β, and $\lim_{\beta \to 0} \tilde{P}_{uu}((2\alpha - 1)(x_1 - x_2), \beta) = 0$. Then there exists $\bar{\beta} > 0$ such that for all $0 < \beta < \bar{\beta}$, we have $\frac{\partial x_1}{\partial e_1} < 0$, and sign of $\frac{\partial x_1}{\partial \delta}$ is equal to the sign of $u_2' - u_1'$.

The first result is rather surprising. As Issue 1 becomes more important than Issue 2 to the voters from the first group (and, symmetrically, Issue 2 becomes more important to voters of the second group), then both candidates reduce the amoung of campaigning on both issues. The greatest total amount of campaign effort is achieved when both issues are equally important to the two groups of voters.

The second result is that if it becomes more expensive for both candidates to campaign on either the issues in which they are competent, or the issues on which they are not competent, then, once again, both candidates will campaign less on both issues.

The effect of uncertainty depends on the utilities of the candidates with respect to their vote shares. If there are significant consolation prizes to runner-ups and no significant benefits to the large-margin winners, then the candidates will invest more effort in the issues in which they are competent, and less effort in the issues in which they are less competent. Such tactic will allow each candidate to secure a larger share of the her core group (those voters who put more value on that candidate's strong issue) in the worst-case scenario, when that candidate receives a negative valence shock. If, on the other hand, the utility function for tthe candidates is convex — so there are large benefits to winning by a large margin, but no benefits to losing by a small margin — then the effect will be opposite: there will be issue trespassing, as the candidates will invest more effort in the issues

in which they are less competent, and less effort in the issues in which they are more competent.

Proving some of these results requires an assumption that, a change in one candidate's valence will have the same effect on a voter's probability of voting for that candidate, regardless of which candidate receives a positive utility shock. This additional assumption is required for two of the eight comparative statics results obtained above.

The following theorem is concerned with the comparative statics relative to the changes in the candidate utility functions.

Theorem 3 Suppose that $c'''(\cdot) \leq 0$. There exists $\bar{\delta} > 0$, such that for all $\delta < \bar{\delta}$, for all $x_1 > x_2$ such that $x_{11} = x_{22} = x_1$ and $x_{12} = x_{21} = x_2$ are a local Nash equilibrium in game with payoffs (8), (9), and for any family of functions $\tilde{u}(x, \beta_1, \beta_2)$ such that

- For all $x \in (1 + P_1 - P_2 - \epsilon, 1 + P_1 - P_2 + \epsilon) \cup (1 - P_1 + P_2 - \epsilon, 1 - P_1 + P_2 + \epsilon)$ and for all β_1, β_2 we have $\frac{\partial^2 \tilde{u}}{\partial x^2} = 0$ for some $\epsilon > 0$.

- For $x = x_1$ and $x = x_2$, we have $\frac{\partial \tilde{u}}{\partial x} = u'(x)$.

- For $x = 1 + P_1 - P_2$, we have $\frac{\partial \tilde{u}}{\partial x \partial \beta_1} > 0$ and $\frac{\partial \tilde{u}}{\partial x \partial \beta_2} = 0$.

- For $x = 1 - P_1 + P_2$, we have $\frac{\partial \tilde{u}}{\partial x \partial \beta_2} > 0$ and $\frac{\partial \tilde{u}}{\partial x \partial \beta_1} = 0$.

we have the following:

1. $\frac{\partial x_1}{\partial \beta_1} > 0$ and $\frac{\partial x_2}{\partial \beta_1} > 0$.

2. Let $\tilde{P}(u, \beta)$ be a twice continuously differentiable function such that $\tilde{P}((2\alpha - 1)(x_1 - x_2), \beta) = P((2\alpha - 1)(x_1 - x_2))$ and $\tilde{P}_u((2\alpha - 1)(x_1 - x_2), \beta) = P'((2\alpha - 1)(x_1 - x_2))$ for all β, and $\lim_{\beta \to 0} \tilde{P}_{uu}((2\alpha - 1)(x_1 - x_2), \beta) = 0$. Then there exists $\bar{\beta} > 0$ such that for all $0 < \beta < \bar{\beta}$, we have $\frac{\partial x_1}{\partial \beta_2} > 0$, $\frac{\partial x_2}{\partial \beta_2} > 0$, $\frac{\partial x_1}{\partial \beta_1} - \frac{\partial x_1}{\partial \beta_2} > 0$, and $\frac{\partial x_2}{\partial \beta_1} - \frac{\partial x_2}{\partial \beta_2} < 0$.

The first result is that when marginal payoff from an extra vote increases, either in the worst-case or in the best-case scenario for each candidate. It then follows that both candidates will exert greater effort campaigning on both issues. This is not surprising: if there is an extra benefit from getting an additional vote in either of the two scenarios, and the marginal cost of competence is unchanged, then there will be more campaigning in either issue. The second result concerns the effect of a change in the utility function when the marginal utility of an extra vote is increased in the best-case scenario, and decreased in the worst-case scenario. In that case, the candidates will campaign more on their strong issues, and less on their weak issues.

I then investigate the effect on the candidate strategies of the information regarding the preferences of the voters. The functions $P(\cdot)$ reflect how the voting outcomes of each voter group depend on the difference between the voter utilities. I derive the following result:

Theorem 4 Suppose that $c'''(\cdot) \leq 0$. There exists $\bar{\delta} > 0$, such that for all $\delta < \bar{\delta}$, for all $x_1 > x_2$ such that $x_{11} = x_{22} = x_1$ and $x_{12} = x_{21} = x_2$ are a local Nash equilibrium in game with payoffs (8), (9), with $u''(1 - P_1 + P_2) = u''(1 + P_12 - P_2)$, and for any function $\bar{P}(u, \beta, \gamma)$ such that $\bar{P}((2\alpha-1)(x_1-x_2), \beta, \gamma) = P((2\alpha-1)(x_1 - x_2))$, $\bar{P}_u((2\alpha - 1)(x_1 - x_2), \beta, \gamma) = \gamma P'((2\alpha - 1)(x_1 - x_2))$ for all β, and $\lim_{\beta \to 0} \bar{P}_{uu}((2\alpha-1)(x_1-x_2), \beta, \gamma) = 0$, the following is true:

1. $\frac{\partial x_1}{\partial \gamma}\big|_{\gamma=1} > 0$.

2. There exists $\bar{\beta} > 0$ such that for all $0 < \beta < \bar{\beta}$, we have $\frac{\partial x_2}{\partial \gamma}\big|_{\gamma=1} > 0$.

As the marginal probability of voting P' increases, the voters become more responsive to candidate issue competencies, and

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the candidate campaign efforts increase. On the other hand, if P' is low — so, the voter-specific utility shocks are large in magnitude compared to the deterministic part of voter utility — the candidates spend little effort to increase their issue competencies.[5]

4 Discussion

This work is an attempt to analyze issue-based political competition in the probabilistic voting framework. I assume that the voters choose between candidates based on how competent the candidates are on the two political issues, and that increasing one's issue competency is costly. I show that the effect of electoral uncertainty on candidate campaign efforts depends on how electoral results are translated into candidate payoffs. Issue treapasing occurs when there are no significant consolation prizes to strong runner-ups, and/or when there are significant benefits from winning the election by a large margin.

The model that is analyzed here is a simple one. First, I only look at two issues, two candidates, and two voters. Second, the candidates and the candidate cost functions are assumed to be symmetric (so there is no high-quality and low-quality candidates).

Finally, all analytic results of this work concern local Nash equilibria. I do not claim that a global Nash equilibrium exists as well (the existence of global Nash equilibria will require additional, nontrivial inequality conditions on the voter utility functions). However, should a global equilibrium exist, the equilibrium comparative statics derived in this paper obviously apply

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[5]That volatility of voter preferences affects equilibrium polities was shown in a number of works (Linbeck and Weibull, 1987, Schofield, 2006, Wittman, 1983).

to it as well. All of this points to the need to use numeric methods to obtain additional results.

Proofs

Proof of Theorem 1.

We have the following first-order conditions:

$$\frac{\partial U_1}{\partial x_{11}} = (\alpha_{11}P'_{11} + \alpha_{21}P'_{21})u'(P_{11} + P_{21}) +$$
$$+ \quad (\alpha_{11}P'_{12} + \alpha_{21}P'_{22})u'(P_{12} + P_{22}) - e_{11}c'(x_{11}) = 0$$
$$\frac{\partial U_1}{\partial x_{12}} = (\alpha_{12}P'_{11} + \alpha_{22}P'_{21})u'(P_{11} + P_{21}) +$$
$$+ \quad (\alpha_{12}P'_{12} + \alpha_{22}P'_{22})u'(P_{12} + P_{22}) - e_{12}c'(x_{12}) = 0$$
$$\frac{\partial U_2}{\partial x_{21}} = (\alpha_{11}P'_{11} + \alpha_{21}P'_{21})u'(2 - P_{11} - P_{21}) +$$
$$+ \quad (\alpha_{11}P'_{12} + \alpha_{21}P'_{22})u'(2 - P_{12} - P_{22}) - e_{21}c'(x_{21}) = 0$$
$$\frac{\partial U_2}{\partial x_{22}} = (\alpha_{12}P'_{11} + \alpha_{22}P'_{21})u'(2 - P_{11} - P_{21}) +$$
$$+ \quad (\alpha_{12}P'_{12} + \alpha_{22}P'_{22})u'(2 - P_{12} - P_{22}) - e_{22}c'(x_{22}) = 0$$

Let the candidates and the voters be symmetric with respect to the two issues. Let $e_{11} = e_{22} = e_2$, $e_{21} = e_{22} = e_2$, $\alpha_{11} = \alpha_{22} = \alpha$, $\alpha_{12} = \alpha_{21} = 1 - \alpha$. Assuming that $x_{11} = x_{22} = x_1$ and $x_{12} = x_{21} = x_2$, we denote

$$P_{11} = P((2\alpha - 1)(x_1 - x_2) + \delta) = P_1$$
$$P_{12} = P((2\alpha - 1)(x_1 - x_2) - \delta) = P_2$$
$$P_{21} = P(-(2\alpha - 1)(x_1 - x_2) + \delta) = 1 - P_2$$
$$P_{22} = P(-(2\alpha - 1)(x_1 - x_2) - \delta) = 1 - P_1$$

$$(10)$$

and

$$u_1 = U(P_{11} + P_{21}) = U(2 - P_{12} - P_{22}) = u(1 + P_1 - P_2)$$
$$u_2 = U(P_{12} + P_{22}) = U(2 - P_{11} - P_{22}) = u(1 - P_1 + P_2).$$

18

By symmetry of P, we have $P'_{11} = P'_{22} = P'_1$, $P'_{21} = P'_{12} = P'_2$, $P''_{11} = -P''_{22} = P''_1$, and $P''_{21} = -P''_{12} = P''_2$. The first-order conditions can be rewritten as:

$$
\begin{aligned}
H_1 &= (\alpha P'_1 + (1-\alpha)P'_2)u'_1 + \\
&+ ((1-\alpha)P'_1 + \alpha P'_2)u'_2 - e_1 c'(x_1) = 0 \quad (11) \\
H_2 &= ((1-\alpha)P'_1 + \alpha P'_2)u'_1 + \\
&+ (\alpha P'_1 + (1-\alpha)P'_2)u'_2 - e_2 c'(x_2) = 0. \quad (12)
\end{aligned}
$$

Differentiating (12) with respect to x_1 gives us

$$
\begin{aligned}
\frac{\partial H_1}{\partial x_1} &= (2\alpha - 1)(\alpha P''_1 + (1-\alpha)P''_2)u'_1 + \\
&+ (2\alpha - 1)(P'_1 + P'_2)(\alpha P'_1 + (1-\alpha)P'_2)u''_1 + \\
&+ (2\alpha - 1)((1-\alpha)P''_1 + \alpha P''_2)u'_2 + \\
&- (2\alpha - 1)(P'_1 + P'_2)((1-\alpha)P'_1 + \alpha P'_2)u''_2 - e_1 c''(x_1). (13)
\end{aligned}
$$

We have $P''_1 < 0$ if $x_1 - x_2 \geq 0$. If $x_1 - x_2 \geq \frac{\delta}{2\alpha-1}$, then $P''_2 \leq 0$. As $P'_2 > P'_1$, we have $\alpha P'_1 + (1-\alpha)P'_2 < (1-\alpha)P'_1 + \alpha P'_2$. $u''_2 \geq u''_1 \geq 0$ guarantees us that $\frac{\partial H_1}{\partial x_1} < 0$ as long as $x_1 - x_2 \geq \frac{\delta}{2\alpha-1}$. As H_1 is differentiable in both x_1 and x_2, we can define a differentiable implicit function $x_1(x_2)$ that gives us (12) for any $\tilde{x} \geq \frac{\delta}{2\alpha-1}$.

Now substitute $x_1(\tilde{x})$ into (12). As $H_1 = 0$, we must have

$$
H_2 = H_1 + H_2 = (P'_1 + P'_2)(u'_1 + u'_2) - e_2 c'(x_2) - e_1 c'(x_1(x_2)). \quad (14)
$$

Take $x_2 = \frac{\delta}{2\alpha-1}$. If it is sufficiently small, we must have $H_2 > 0$, as $H_1 = 0$ and $c'(0) = 0$. If x_2 is sufficiently large, then $H_2 < 0$, as $(P'_1 + P'_2)(u'_1 + u'_2)$ is bounded from above. Thus, there exists x_2^* such that (14) is equal to zero. It follows that x_2^*, $x_1(x_2^*)$ satisfy (12), (12). This proves the first statement of the theorem.

Now let x_1 and x_2 satisfy (12) and (12). Since we have

$$
H_1 - H_2 = (1-2\alpha)(P'_2 - P'_1)(u'_1 - u'_2) - e_1 c'(x_1) + e_2 c'(x_2) = 0 \quad (15)
$$

and, by assumption, $u_1' < u_2'$, from $e_1 < e_2$ and $c'' > 0$ it must follow that $x_1 > x_2$.

Proof of Theorem 2.

Suppose that x_1, x_2 are such that (12) and (12) are satisfied; moreover, let $u_1'' = u_2'' = 0$ and $\delta = 0$. It follows that $P_1'' = P_2'' = P''$ and

$$H_{11} = \frac{\partial H_1}{\partial x_1} = (2\alpha - 1)P''(u_1' + u_2') - e_1 c''(x_1) \qquad (16)$$

$$H_{12} = \frac{\partial H_1}{\partial x_2} = -(2\alpha - 1)P''(u_1' + u_2') \qquad (17)$$

$$H_{21} = \frac{\partial H_2}{\partial x_1} = (2\alpha - 1)P''(u_1' + u_2') \qquad (18)$$

$$H_{22} = \frac{\partial H_2}{\partial x_2} = -(2\alpha - 1)P''(u_1' + u_2') - e_2 c''(x_2). \qquad (19)$$

$$(20)$$

We also have

$$
\begin{aligned}
H_{1\alpha} &= (P_1' - P_2')(u_1' - u_2') + 2(x_1 - x_2)P''(u_1' + u_2'), & (21) \\
H_{2\alpha} &= (P_2' - P_1')(u_1' - u_2') + 2(x_1 - x_2)P''(u_1' + u_2'), & (22) \\
H_{1\delta} &= (2\alpha - 1)P''(u_1' - u_2'), & (23) \\
H_{2\delta} &= (2\alpha - 1)P''(u_2' - u_1'), & (24) \\
H_{1e_1} &= -c'(x_1), \ H_{1e_2} = 0, \ H_{2e_1} = 0, \ H_{2e_2} = -c'(x_2). & (25)
\end{aligned}
$$

The implicit function theorem gives us the following:

$$
\begin{pmatrix}
x_{1\alpha} & x_{2\alpha} \\
x_{1\delta} & x_{2\delta} \\
x_{1e_1} & x_{2e_1} \\
x_{1e_2} & x_{2e_2}
\end{pmatrix}'
= -\frac{1}{D} \cdot
\begin{pmatrix}
H_{22} & -H_{12} \\
-H_{21} & H_{11}
\end{pmatrix} \times
$$

$$
\times
\begin{pmatrix}
H_{1\alpha} & H_{1\delta} & H_{1e_1} & H_{1e_2} \\
H_{2\alpha} & H_{2\delta} & H_{2e_1} & H_{2e_2}
\end{pmatrix},
\qquad (26)
$$

where

$$D = H_{11}H_{22} - H_{12}H_{21}, \tag{27}$$

or

$$D = e_1 e_2 c''(x_1)c''(x_2) + (2\alpha - 1)P''(u'_1 + u'_2)(e_1 c''(x_1) - e_2 c''(x_2)). \tag{28}$$

As $P'' < 0$, $\alpha > \frac{1}{2}$, $e_1 < e_2$, and $x_1 > x_2$, it follows that $D > 0$ as long as $c'''(\cdot) < 0$.

We have

$$\frac{\partial x_1}{\partial \alpha} = \frac{1}{D} 2e_1 c''(x_1)(x_1 - x_2)P''(u'_1 + u'_2) < 0 \tag{29}$$

$$\frac{\partial x_2}{\partial \alpha} = \frac{1}{D} 2e_2 c''(x_2)(x_1 - x_2)P''(u'_1 + u'_2) < 0 \tag{30}$$

$$\frac{\partial x_1}{\partial \delta} = -\frac{1}{D} H_{1\delta}(H_{22} - H_{21}) \tag{31}$$

$$\frac{\partial x_2}{\partial \delta} = -\frac{1}{D} H_{1\delta}(-H_{11} - H_{21}) \tag{32}$$

$$\frac{\partial x_1}{\partial e_1} = -\frac{1}{D} H_{22} H_{1e_1} \tag{33}$$

$$\frac{\partial x_1}{\partial e_2} = -\frac{1}{D}(-H_{12})H_{2e_2} < 0 \tag{34}$$

$$\frac{\partial x_2}{\partial e_1} = -\frac{1}{D}(-H_{21})H_{1e_1} > 0 \tag{35}$$

$$\frac{\partial x_2}{\partial e_2} = -\frac{1}{D} H_{11} H_{2e_2} < 0. \tag{36}$$

The sign of $\frac{\partial x_2}{\partial \delta}$ is equal to the sign of $u'_1 - u'_2$. If $|P''|$ is sufficiently small, then the sign of $\frac{\partial x_1}{\partial \delta}$ is equal to the sign of $u'_2 - u'_1$, and $\frac{\partial x_1}{\partial e_1} < 0$.

Proof of Theorem 3. Differentiating (12) and (12) by u'_1 and u'_2, we get

$$H_{1u'_1} = H_{2u'_2} = \alpha P'_1 + (1 - \alpha)P'_2 \tag{37}$$

$$H_{2u'_1} = H_{2u'_1} = \alpha P'_2 + (1 - \alpha)P'_1 \tag{38}$$

By the implicit function theorem we have

$$\frac{\partial x_1}{\partial u_1'} = -\frac{1}{D}((2\alpha - 1)^2 P''(u_1' + u_2')(P_2' - P_1') -$$
$$- H_{1u_1'} e_2 c''(x_2)) > 0 \tag{39}$$

$$\frac{\partial x_2}{\partial u_1'} = -\frac{1}{D}((2\alpha - 1)^2 P''(u_1' + u_2')(P_2' - P_1') -$$
$$- H_{1u_2'} e_1 c''(x_1)) > 0 \tag{40}$$

$$\frac{\partial x_1}{\partial u_2'} = -\frac{1}{D}((2\alpha - 1)^2 P''(u_1' + u_2')(P_1' - P_2') -$$
$$- H_{1u_2'} e_2 c''(x_2)) \tag{41}$$

$$\frac{\partial x_2}{\partial u_2'} = -\frac{1}{D}((2\alpha - 1)^2 P''(u_1' + u_2')(P_1' - P_2') -$$
$$- H_{1u_1'} e_1 c''(x_1)) \tag{42}$$

$$\frac{\partial x_1}{\partial u_1'} - \frac{\partial x_1}{\partial u_2'} = -\frac{1}{D}(2\alpha - 1)(P_1' - P_2')(H_{22} + H_{12}) \tag{43}$$

$$\frac{\partial x_2}{\partial u_1'} - \frac{\partial x_2}{\partial u_2'} = \frac{1}{D}(2\alpha - 1)(P_1' - P_2')(H_{11} + H_{21}). \tag{44}$$

If $|P''|$ is sufficiently small, then $\frac{\partial x_1}{\partial u_2'} > 0$, $\frac{\partial x_2}{\partial u_2'} > 0$, $\frac{\partial x_1}{\partial u_1'} - \frac{\partial x_1}{\partial u_2'} > 0$ and $\frac{\partial x_2}{\partial u_1'} - \frac{\partial x_2}{\partial u_2'} < 0$.

Proof of Theorem 4.

Assuming that $P_1' = P_2' = P'$ and differentiating (12) and (12) with respect to P', one has

$$H_{1P'} = H_{2P'} = u_1' + u_2'. \tag{45}$$

By the implicit function theorem we have

$$\frac{\partial x_1}{\partial P'} = -\frac{1}{D}(u_1' + u_2')(H_{22} - H_{12}) > 0 \qquad (46)$$

$$\frac{\partial x_2}{\partial P'} = -\frac{1}{D}(u_1' + u_2')(-H_{21} + H_{11}). \qquad (47)$$

If $|P''|$ is sufficiently small, then $\frac{\partial x_2}{\partial P'} > 0$.

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В данной работе приводится теоретико-игровой анализ влияния электоральной неопределенности на политические программы кандидатов. Предполагается, что каждый из двух кандидатов, участвующих в выборах, решает, сколько средств потратить на то, чтобы убедить избирателя в том, что он компетентен в решении каждой из двух политических задач. Предполагается, что у одного из кандидатов есть преимущество по первой задаче, у другого – по второй. В работе рассматриваются условия, при которых один из кандидатов будет «залезать на чужую территорию», т.е. позиционировать себя как способного решить политическую задачу, в отношении которой у него нет преимущества. Показано, что это может происходить при увеличении неопределенности относительно популярности кандидатов, но только в том случае, когда у кандидатов есть дополнительные стимулы побеждать на выборах с большим отрывом. Показано также, что усилия кандидатов будут выше в том случае, когда электорат более однороден в оценках важности двух политических проблем, когда у кандидатов есть больше информации относительно предпочтений избирателей, и когда ниже издержки, связанные с избирательными кампаниями.

Захаров Алексей Владимирович

Электоральная конкуренция с издержками при выборе тем

(*на английском языке*)