НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ
ВЫСШАЯ ШКОЛА ЭКОНОМИКИ

Международный Институт Экономики и Финансов

МАГИСТЕРСКАЯ ДИССЕРТАЦИЯ

по образовательной программе высшего профессионального образования, направление 080100.68 Экономика

На тему "Predicting Abnormal Returns on Financial Markets Using Search Volume Data"

Студент 2 курса магистратуры
Яцишин Илья Витальевич

Научный руководитель
профессор PhD Фантаццини Деан

Москва 2013

Прогнозирование сверхдоходностей акций на финансовых рынках с использованием данных поисковых запросов

Развитие информационных технологий, в частности Интернета, привело к изменению поведенческих шаблонов. Интернет стал использоваться в качестве основного источника информации, и данные по количеству поисковых запросов могут отражать реальные процессы. Индекс поисковых запросов(SVI), публикуемый Google, успешно использовался в ряде работ для прогнозирования распространения болезней и продаж различных категорий товаров(Ginsberg et al., 2009, Varian and Choi, 2009).

На основании гипотезы Barber and Odean (2008) повышенный интерес к компании со стороны непрофессиональных инвесторов приводит к положительному давлению на цену её акций. Da et al. (2009) показали возможность использования SVI для измерения внимания инвесторов наряду с традиционными методами и для предсказания сверхдоходностей акций(Joseph et al., 2011). Одна из основных проблем SVI — его нормированность к максимальному значению интенсивности поисковых запросов. Данная методология не позволяет проводить сравнительный анализ известности различных компаний и оперирует только с величиной аномального изменения количества поисковых запросов. Доступность данных Яндекс по абсолютному количеству поисковых запросов за период позволяет провести в данном исследовании сравнительный анализ различных индексов внимания инвесторов: абсолютного количества запросов(ABSS), аномального изменения объема запросов(ABSS) и темпа роста количества запросов(RGS). В качестве активов используются данные по 182 акциям, торгуемым на Московской Бирже(ММВБ и РТС).

На основании методологии Joseph et al. (2011) проведено посторение портфелей имитирующих соответствующие факторы. Доходности полученных портфелей были использованы для модификации модели Fama and French (1993)—Carhart (1997) для предсказания доходностей акций. По результатам выбора лучшей модели для предсказания доходности превосходящими характеристиками обладает портфель, построенный на основании абсолютного количества поисковых запросов. При этом необходимо отметить, что индекс аномального изменения количества поисковых запросов, показавший значимые результаты в предыдущих исследованиях, также значимо улучшает базовую модель. Однако при одновременном использовании двух индексов ABNS перестает быть значимым. Модель с абсолютным количеством поисковых запросов превосходит аналогичную с ABSS. Данные результаты показывают значимость известности отдельной компании для предсказания доходности по сравнению с изменениями во внимании инвесторов. Данные результаты не могли быть получены ранее с использованием данных Google.

В дополнении к данному анализу было проведено исследование сверхдоходностей квартильных портфелей, построенных для различных факторов. Квартильные портфели не обладают значимыми премиями, таким образом, отсутствует различие в необъясненной доходности для различных уровней индекса. Однако при анализе портфелей с нулевыми первоначальными инвестициями были получены значимые сверхдоходности, соответствующие положительной премии для большего уровня индекса.

Полученные результаты демонстрируют значимую зависимость доходности акций от известности соответствующих компаний, оцененных с помощью абсолютного количества поисковых запросов. В работе Bank et al. (2011) показана возможность прогнозирования ликвидности с помощью поисковых индикаторов, что означает наличие зависимости между ликвидностью и вниманием инвесторов. Таким образом, полученная премия может соответствовать премии за ликвидность акций. В дальнейших исследованиях необходимо проверить значимость абсолютного количества запросов при предсказании доходности, контролируя показатели ликвидности акций.

Contents

Introduction

As the development of theory of asset pricing was driven by evolution of financial markets, it is naturally that almost all efforts in understanding mechanics of markets were concentrated on the small number of leading ones. Remaining markets, operating in developing economies, are still insufficiently studied. The key issue is that underdeveloped market mechanisms may cause inability to use classic asset pricing models and need deeper analysis to test applicability of classic hypotheses. After the merger of MICEX and RTS total capitalization of the Russian financial market became $698,7 billion[1], what is comparable with the capitalization of the single company Apple Inc.($541.07 billion[2]) that is traded on NASDAQ. However thorough research into Russian market could reveal mechanics of developing financial markets. Current research has as its object the formation of a milestone in complete analysis of Russian financial market.

In past decades development of human behavior has tended to increase the significance of the Internet as the source of information. Nowadays when person is interested in knowing something particular it is likely that he would search in the Internet for it. Congregation of statistics about such searches allows to register changes in attention and comparative level of interest. The first search engine that provided access to such statistics for everyone was Google in 2007. Currently the search market share of Google is about 65% in US and more than 90% in some other countries.[3] Large market share allows to extrapolate results at least to a sample or even to the population. However in Russia search market share of Google is only 25% what requires additional prerequisites for representativeness of Google search data in Russia.[4] Fortunately, the leading Russian search engine Yandex with a market share of about 60% also provides access to search volume data, what allows to use both sources of data virtually covering all users of web search in Russia.

Emergence of the new source of data lead to development of new field of research based on behavioral pattern described above. Search volume data was used to predict flu spread(Ginsberg et al., 2009), travel, automotive, home and retail sales(Varian and Choi, 2009), box-office revenues(Goel et al., 2010) and abnormal stock returns(Joseph et al., 2011). This source of data has enormous potential in all fields of science that are based on human choices and actions. Despite of traditional survey based monitoring, search engines provide cheap source of data with large sample that can be obtained for previous periods. Search volume data are also free from a disadvantage that is present in most social surveys. Users do not feel being monitored, thus, they have no incentive to misrepresent their object of interest. Moreover, the search engine aims at generation of better content based on choices of users. That means each person has to act genuinely to

[1] As on 05.06.2012. `http://arsagera.ru/`
[2] As on 09.06.2012. `http://finance.yahoo.com/`
[3] As on 12.06.2012. `http://comscore.com/`
[4] As on 21.06.2012. `http://liveinternet.ru/`

get better utility from further searches.

Current research is based on the reasoning of Barber and Odean (2008) that abnormal returns of stocks are likely to exist due to positive price pressure of retail investors attention. Da et al. (2009) showed that search volume index can be used to estimate investors attention. Probably, positive price pressure is formed by less sophisticated investors, that do not use thorough research to choose shocks to invest in(Joseph et al., 2011). The Russian financial market was found in 1992 and due to its short history and still moderate size it has a large share of unqualified investors. That allows to hypothesize that approaches of Da et al. (2009) and Joseph et al. (2011) can be useful to predict abnormal returns in Russian financial market.

An increase in investor attention leads to higher trading volume as well as positive price pressure. Therefore, analysis of liquidity of stocks can be carried out using search volume data. Bank et al. (2011) focused on liquidity of German stocks using several measures of illiquidity, when Da et al. (2009), Joseph et al. (2011) considered only abnormal trading volume. Another contribution made by Bank et al. (2011) is testing search volume indices for names and formal tickers separately. The authors show that both indices catch investor attention.

In previous studies the Google index was used, but the normalization of each index series to its maximum value does not allow to compare interest to different companies. Probably characteristics of data were the reason why hypotheses about abnormal search volume were tested. In the current research the analysis of both abnormal search volume and relative interest in different companies is conducted.

Previous studies

Investor attention

In the study Barber and Odean (2008) showed several behavioral principles concerning emotionally involved common investor that contradict principles of classic models of investor[5]. For actual investor it is easier to sell stocks that have grown to gain profit than to sell stock that lost its value and admit loss. When unsophisticated investor wants to invest money into stock he faces thousands of assets. As an agent with bounded rationality investor simplifies a problem of picking assets from thousands to a much easier problem of picking out of dozens of alternatives that he heard about. Thus investor chooses to buy stocks of companies that recently caught his attention and sells those, which he already bought before. The authors omit possibility of having a short position for common investors as the data available in that research showed that retail investors have only 0.29% of short positions. It is shown that stocks have additional driver of positive price pressure during the increase in attention to them. Considering the financial market it is necessary to analyze the other side of transaction — a buyer in case of high attention to some stock. The buyer in such cases is likely to be a professional investor who having fundamentally evaluated the stock react on abnormally high price and sells it. To examine the hypothesis of attention-driven abnormal stock returns the authors sort stocks on the following characteristics:

- abnormal trading volume, as the higher attention can be the possible reason of it;
- extreme one day returns, as investors are likely to see such stocks in analytic reports concerning the best and the worst performing stocks. Investors can notice extreme returns itself. The high volatility is likely to be driven by some events or news, that can also grab attention;
- mentioning in the news.

Results of the analysis show that hypothesized behavior pertains investors in large discount brokerage firms. They buy both extremely positive and extremely negative stocks and assets with abnormally high volume or that were mentioned in the news. Investors trading at large retail brokerage firms and at small discount brokerage firms have less attention-based behavior and they don't buy previously extremely profitable stocks. The chosen strategy of individual investors does not outperform the market. Stocks they buy also do not outperform those they sell. The study of Barber and Odean (2008) shows that such active trading strategy do not allow agents to be profitable comparing with the market return and it is more likely that promising stocks that would fetch high profit do not catch attention.

[5]For example, models by Grossman and Stiglitz (1980), Kyle (1985).

Search volume data

Google search volume index(SVI) was presented to scientific community by Varian and Choi (2009) as an instrument that can be used not only by search engine optimizers(SEO), but also in econometric studies. The main concept presented in this study was the potential of SVI in predicting the present, as the search volume data are updated weekly while majority of statistics are published monthly. SVI becomes a useful tool for predicting the current level of some index that would be announced in the end of the month using corresponding weekly SVI. Authors show examples of possible studies building standard seasonal autoregressive and fixed effect models of travel destination arrivals, automotive, retail and home sales. Models with Google data are compared with simple models by mean absolute error. In some cases, the predictive power of advanced models is not improved, but there are ones, that have significant decrease in prediction error. Thus, H. Choi and H. Varian showed that search volume index is a promising instrument in econometric studies and should be examined more thoroughly.

Proposal of further research was followed by series of more scientific studies in different areas. SVI had remarkable results in predicting flu outbreaks in Ginsberg et al. (2009). The authors found 45 related searches, that allowed to identify influenza outbreaks up to two weeks earlier than Center of Decease Control and Prevention(CDC) could. The first study concerning financial markets that used search volume data was written by Da et al. (2009). Research is based on hypothesis of Barber and Odean (2008) that investor attention specifies behavior of noise traders. The authors are considering SVI as a direct measure of investor attention, that has less disadvantages. Existing measures of investor attention are based on a prerequisite that certain events guarantee that investors paid attention to them. Mentioning of a firm in the news does not necessary attract attention as currently "a wealth of information creates a poverty of attention." Abnormal return or turnover can be driven by factors, that are not related to attention. The authors made the hypothesis that less sophisticated investors are those who search information about companies using the Internet. Professional investors use subscriptions to specific resources and less likely use ordinary search engines to get information engaged in decision making. That allows to consider SVI as a measure of attention of noise traders. Using the list of Russell 3000 stocks the authors test three hypotheses:

- SVI is an appropriate measure of investor attention and is correlated with existing indirect measures.
- SVI captures attention of retail investors.
- Individual investors are net buyers of attention-grabbing stocks. Increase in attention leads to positive price pressure. (Barber and Odean, 2008).

Correlations between SVI and other measures of investor attention[6] are low (beyond 9%).

[6]The measures are: (1) *Absolute abnormal return* — absolute value of concurrent week abnormal return; (2) *Abnormal turnover* — standardized abnormal turnover as in Chordia et al. (2007); (3) *News* — number of news stories in Dow Jones news archive in concurrent week; (4) *Chunky news* — number of stories published multiple times in Dow Jones news archive in concurrent week; (5,6) *Frac_Neg_H4* and

However construction of vector autoregression(VAR) with variables that are available weekly provide reassuring results. The authors use block bootstrap procedure to estimate p-values. SVI turns out to lead other proxies of investor attention, what shows that SVI indicates increase of attention prior to other measures. Lagged abnormal return is significantly related to SVI, what can indicate that extreme dynamics during concurrent week can grab investors attention. Verification of the second hypothesis is carried out by making regression analysis of trade dynamics for market centers, that differ in participation of retail investors in them. The authors consider as dependent variables changes in numbers of orders of individual investors and changes in turnover. As authors use Dash-5 reports that provide trading statistics, disaggregated into categories by a number of shares involved in a transaction. As the Dash-5 report does not include statistics concerning transactions with more than 10000 stocks, activity of professional investors are not included in the analysis. As the measure of extreme changes in attention *abnormal search volume index*(ASVI) is used:

$$ASVI_t = log(SVI_t) - log(Med\{SVI_{t-1}, SVI_{t-2}; ...; SVI_{t-8}\})$$

Median level of search volume index, computed using eight prior weeks, identify "natural" level of interest; thus, clearing data from fluctuations. It also allows to remove time trends. Regressions on $ASVI_{t-1}$ controlling other measures of attention, market capitalization and returns provide significant results concerning differences between analyzed market centers. Orders of less sophisticated investors often went to now defunct Madoff Securities LLC, what corresponds with higher sensitivities of number of orders and its turnover on ASVI. On the contrary, the same sensitivities at New York Stock Exchange (NYSE) for NYSE stocks and Archipelago for NASDAQ stocks are much less that fit the behavior of institutional traders. Testing of Barber and Odean (2008) model showed that SVI can predict increase in stock prices in following two weeks and price reversal within a year. More promising result was the possibility of large first-day return prediction during IPO and overall IPO returns when controlling on first-day IPO returns. SVI is available prior the IPO and it can be used to measure investor attention, when other indicators are unavailable. Stocks with similar IPO returns, having high level of attention, will experience higher price reversal comparing with firms, that had no attention-driven price pressure during the IPO.

Joseph et al. (2011) base their research on Barber and Odean (2008) and Da et al. (2009) studies. Considering former results, the authors posit search volume index as a valid proxy of investors attention. Empirical analysis on the first step consists in division of all stocks in the S&P 500 into five quintiles using search intensity in a previous week. Such quintiles are built in a first day of a week and derived portfolios are analyzed for differences in weekly returns. Using Fama and French (1993) three factor model with

Frac_Neg_LM — number of words with "negative sentiment", defined using Harward IV-4 dictionary and Loughran and McDonald dictionary respectively, in the total number of words in news articles recorded in Dow Jones Newswire database.

Carhart (1997) extension Joseph et al. (2011) compute daily abnormal return for each quintile controlling the excess return on the market $R_m - R_f$ the return difference between portfolios of "small" and "big" stocks (SMB), the return difference between portfolios of "high" and "low" book-to-market stocks (HML), the return difference between a portfolio of stocks with high returns in the past year and a portfolio of stocks with low returns in the past year (UMD). A portfolio that is long on high search intensity quintile and has short position on those stocks that are in low search intensity quintile shows 0.14% 5-day implied return, what corresponds to 7.2% annually. Considering the same upper and lower quintiles abnormal trading volume[7] is 158% higher for firms with higher search intensity comparing with those with low search intensity. The authors also hypothesize that sensitivity of returns to search volume depends on potential to arbitrage. Highest abnormal returns correspond to stocks that are difficult to arbitrage in order to reverse positive price pressure caused by an increase of attention. Abnormal return from search intensity and volatility dual-sorted portfolios has no relationship with search intensity for low volatility firms. In contrast, there is a relationship between return and search volume for medium and high volatility stocks; and a relationship between return and volatility for firms that have high and medium search intensity, when there is no such relationship for low search intensity firms. Considering longer investment horizons of search intensity based portfolios the authors found significant further price reversal after four weeks.

Following the studies of Barber and Odean (2008), Da et al. (2009) and Joseph et al. (2011), Bank et al. (2011) tried to test previous results on a new dataset of German stocks. The authors use the naive search volume index of firm's name instead of formal ticker. Although such searches can be reflecting other events and trends, that are not related to investor attention and sentiment, the authors show that they are still a valid proxy for investor attention. The research is focused on the relationship between search traffic and stock liquidity as according to previous studies increase in investor attention lead to abnormal trading volume. Unlike other authors Bank et al. (2011) consider several indicators of illiquidity as the measure of trading activity. The first measure of illiquidity is *trading volume*:

$$TV_{i,y,d} = ln(VO_{i,y,d} \cdot P_{i,y,d}) \tag{1}$$

Where $TV_{i,y,d}$ — trading volume of stock i in year y and day d, $VO_{i,y,d}$ — number of shares traded, $P_{i,y,d}$ — price of respecting stock.

$$TO_{i,y,d} = \frac{VO_{i,y,d}}{NOSH_{i,y,d}} \tag{2}$$

Turnover rate is a reciprocal of average holding period and equal to fraction of shares traded($VO_{i,y,d}$) to number of outstanding shares($NOSH_{i,y,d}$).

$$ILLIQ_{i,y,d} = \frac{|R_{i,y,d}|}{TV_{i,y,d}} \tag{3}$$

[7] *Abnormal volume* $AV_{it} = (V_{it} - V_{i,avg})/V_{i,avg}$, where V_{it} is the trading volume for firm i on day t, and $V_{i,avg}$ is the average daily volume over the entire sample period.

$ILLIQ$ is a measure that reflects the price impact of one Euro of trading volume that was introduced by Amihud (2002). For illiquid stocks that have low trading volume $ILLIQ$ will be higher considering equal returns. Due to illiquidity stocks should fetch higher expected returns as it is harder to find counterpart when needed and secure the gain or loss. That means higher returns are likely to be corresponded to illiquid stocks. The monthly and weekly $ILLIQ$ indices are computed as averages.

$$ILLIQ_{i,y,w} = \frac{1}{D_{i,y,w}} \sum_{d=1}^{D_{i,y,w}} ILLIQ_{i,y,w,d} \tag{4}$$

$$ILLIQ_{i,y,m} = \frac{1}{D_{i,y,m}} \sum_{d=1}^{D_{i,y,m}} ILLIQ_{i,y,m,d} \tag{5}$$

Where $D_{i,y,w}(D_{i,y,m})$ is the number of days when the stock i was trading in week w (month m) in year y. d is the number of a day in corresponding week(month). Monthly and weekly indices of trading volume and turnover rate are computed as simple averages of those obtained from equations (1) and (2).

While performing the robustness check Bank et al. (2011) computes several alternative measures of illiquidity:

$$TPI_{i,y,d} = \frac{|R_{i,y,d}|}{TO_{i,y,d}} \tag{6}$$

Turnover price impact($TPI_{i,y,d}$) is quite similar to $ILLIQ$ index, introduced by (Florackis et al., 2011). As turnover rate is used in computation instead of trading volume, the index does not depend on price changes[8] and inflation.

$$R\_IMP_{i,y,d} = \frac{ROLL_{i,y,d}}{TV_{i,y,d}} \tag{7}$$

$R\_IMP_{i,y,d}$ introduced by Goyenko et al. (2009) shows the average spread relative to trading volume. $ROLL_{i,y,d}$ measure of spread was introduced by Roll (1984). $ROLL_{i,y,d} = 2\sqrt{-cov(\Delta P_t; \Delta P_{t-1})}$ if correlation is negative and $ROLL_{i,y,d} = 0$ otherwise. Roll (1984) assumes that price of transaction depends on fundamental value of asset which is random walk with zero mean and σ standard deviation and transaction costs. Costs are equal to half of bid-ask spread and add to or subtract from fundamental value if asset is bought and sold, respectively. P_t is a daily price of stock.

$$S\_REL_{i,y,d} = \frac{PA_{i,y,d} - PB_{i,y,d}}{(\frac{PA_{i,y,d} + PB_{i,y,d})}{2})} \tag{8}$$

S_REL was introduced by Amihud and Mendelson (1986) and shows the relative bid-ask spread to estimate transaction costs. $PB_{i,y,d}$ and $PB_{i,y,d}$ are bid and ask prices in the end of trading day, respectively.

Each week the authors divide stocks into three groups according to changes in search volume in previous week. Derived portfolios have significant differences in changes of standard measures of illiquidity(TV, TO, $ILLIQ$), standard deviation of daily returns within a week(month) and weekly(monthly) returns. Portfolio with high attention change

[8] Market return still reflects changes in prices, but there in no memory of previous price changes.

has higher turnover rate, trading volume changes and lower $ILLIQ$, what corresponds with the hypothesis of reduction of illiquidity due to attention rise. Moreover, standard deviation of daily returns is higher in high interest portfolio, what shows increasing trading activity. Weekly return also differs between portfolios, what conforms with studies of Barber and Odean (2008), Da et al. (2009), Joseph et al. (2011). The authors consider two panels of dual-sorted portfolios, by market value and changes in search intensity and by turnover and changes in search intensity. Both panels show that turnover rate and trading volume changes differ significantly between small and high search intensity change groups, regardless of the market value or turnover rate group. However changes are higher for firms with lower market value and for firms with higher turnover rate. Estimation of panel autoregression models of $ILLIQ$ controlling for different illiquidity measures and market value showed significant relationship between $ILLIQ$ and search volume index in all examined models. Robustness check carried out with alternative measures of illiquidity showed similar results; thus, search volume index is a good indicator of future changes of illiquidity. Moreover it was shown that the firm name can be used for purposes of attention measurement as well as ticker.

Russian Financial Market

In Goriaev (2004) different risk-factors analysis was made. CAPM model was strongly rejected for Russian stocks and high-beta stocks had negative 10% p.a. returns premium, and other factor models showed significant results. Country risk factor provided 59% p.a return, corporate governance factor – 25% p.a. This factors decreased its values in 2002-03. SMB and dollar factors provided 33% and 39% p.a. returns respectively.

Data

Search volume

Google Inc. introduced their service Google Trends[9] in 2007. It provides Search Volume Index(SVI) for a given keyword that in normalized to the total number of searches in chosen area. Such normalization eliminates changes of index due to increase in overall search activity and variation of Google search market share. The data are further scaled to maximum search intensity of a given keyword, what makes comparison between different search keywords impossible. As the result search volume index can be only used to analyze changes in search intensity. Normalized index is then scaled to fit 0—100 interval. Google SVI has several advantages. Repeating searches from the same user during the short time interval are treated as one, what allows to consider no bias due to them. Google defines the geographic area of search; thus, in the study of Russian financial market it is possible to use searches done in Russia. Google also uses additional proprietary normalization algorithm that utilizes number of users in different regions, but there is no definite explanation of its mechanics. The data is provided from 2004.

Alternatively to Google, Yandex Wordstat[10] provides number of searches, that is not scaled or normalized. That makes comparative analysis of different keywords possible, but it is necessary to take account of time trend that can be present in the data. Yandex provides two years of monthly data and year of weekly data, what makes impossible to get long time series immediately. However due to the format of provided data different time frames of series can be appended easily. Yandex distinguishes geographical location of search also.

Further Yandex absolute search volume index is denoted as ABSS. Using ABSS two more indexes are computed:

- ABNS — Abnormal Search Volume:

$$ABNS_t = ln(ABSS_t) - ln(mean(ABSS_{t-1}, \ldots, ABSS_{t-4})) \tag{9}$$

- RGS — Growth Rate of Search Volume:

$$RGS_t = ABSS_t / ABSS_{t-1} \tag{10}$$

For current study ABSS series were gathered for the 10.10.2011-05.04.2011 (71 weeks) time period. As the keyword both ticker and company name are used. As the search volume for some companies is small, all searches for a specific company are summed to create one indicator. This data is collected for all companies listed on the Moscow Exchange.

[9] http://trends.google.com/
[10] http://wordstat.yandex.ru/

Financial time series

As the current Russian financial market is a result of a merger of two market there are several boards that are used to trade stocks. Main market and Standard are those, that are used mostly by unsophisticated investors(through broker firms), examined by Barber and Odean (2008). Main market is the board that has been a part of MICEX recently. Transactions are made in rubles, when Standard board(formerly RTS Standard) allows to use foreign currency. In current study daily financial time series of post-crisis period 05.10.2009-05.04.2011 are used[11]. Most of stocks in Russia are very illiquid. For purposes of current research following liquidity requirements are imposed for stocks. There should be deals during at least half of trading days to add it into sample. Only 182 assets satisfy this criterion. Further restrictions on sample are not imposed as in previous research samples of S&P500 and Russel 2000 stocks are used and there are less stocks traded on the whole Russian market.

HML represents book-to-market ratio. For construction of HML indicator(Fama and French, 1993) annual financial statements are used to get book value of the company.[12] SMB represents the size of the company.

CAPM factors

Analysed multifactor models are based on CAPM model (Lintner, 1965a,b, Mossin, 1966, Sharpe, 1964).

$$R_{pt} - R_{ft} = \alpha + \beta_{mp}(R_{mt} - R_{ft}) + \varepsilon_{pt} \tag{11}$$

For estimation of this model risk-free rate R_f and excess market return $R_m - R_f$ should be defined. As a proxy for R_f REPO rate is used[13]. Market return R_m is defined as MICEX index return. For further convenience excess return is defined as $XSRET = R_m - R_f$ and risk-free rate variable is called $CASH$.

[11] Source of trade data, RosBusinessConsulting `http://export.rbc.ru/`
[12] Source of financial statements, `http://stocks.investfunds.ru/`
[13] Source: The Central Bank of the Russian Federation `http://cbr.ru`

Factor mimicking portfolios

To compute factor returns the methodology of computing zero-investment factor mimicking portfolios is used. This approach requires availability of short selling and it do not take into account costs of rebalancing of the portfolio and cost of money to maintain short position. However it allows to compute theoretical premium of one of the factors.

To build a factor mimicking portfolio assets are sorted in descending order using corresponding indicator on the date of recombination. For example for ABSS index each weekend assets are sorted according to number of searches during the previous week. Then assets are split into four quartile portfolios. This portfolios are fixed until next recombination. In given example assets remain in same quantile until the next weekend. Then the portfolio that is long in highest quartile assets and short in lowest quartile is concerned. Assets are included with equal weights, thus this portfolios do not require initial investments. Factor mimicking portfolio return for further periods is treated as a factor premium of corresponding index that was used to construct it. It is crucial that portfolio is not fixed once as it can include different assets after each recombination. For Fama and French (1993) factors long position in first 'letter' and short in last is used. For example, to buy SMB factor mimicking portfolio small company stocks should be bought and large company stocks should be sold. For the purposes of the current study daily factor returns are used. As portfolios are sorted according to some index in a single period thus control for changes in overall search activity is unnecessary. Additionally sorting procedure by itself uses relative measures and thus there is no need in control for changes in search intensity like during holidays e.t.c.

For most of the factors recombination is accomplished as frequent as the data allows. For search volume indices factor portfolios are recombined weekly. For SMB and HML factors portfolios are recombined daily, but such frequency is determined only by changes in market value as book value is updated annually for HML. UMD factor should reflect momentum effect. Past performance of an asset can be defined over different windows, thus for purposes of current research two variants of Carhart (1997) momentum with weekly(UMDW) and daily(UMD) recombination windows are used.

According to the methodology described above factor returns are computed and descriptive statistics of this series are represented in the Table 1. Here and in further sections the name of the indicator would represent factor returns that were built using corresponding variable. It can be seen that daily HML factor has the highest in magnitude average premium(as well as highest t-statistic) with negative sign. Second largest average premium has HML with positive return.

In Table 2 the correlation matrix for factor returns is presented. Pairwize correlations are computed using longest sample available for its estimation. Among factors correlations the largest magnitude have RGS-ABNS and SMB-HML pairs. First one captures effect of the same shocks in search volume, thus it is consistent with factors definition. The second pair is more likely to be correlated due to small number of stocks used in portfolios

Table 1: Descriptive statistics of factor returns

Statistic	N	Mean	St. Dev.	Min	Max	t-stat
CASH	858	0.000704	0.039323	-0.073243	0.079207	0.017901
XSRET	858	-0.000463	0.040912	-0.136658	0.095645	-0.011314
HML	876	0.009914	0.098390	-0.416576	0.688679	0.100763
SMB	876	0.009533	0.154346	-0.676923	1.335510	0.061764
UMD	875	-0.065864	0.132000	-0.801498	0.837842	-0.498972
ABSS	376	0.003750	0.132543	-0.523994	0.422235	0.028292
ABNS	358	0.001517	0.039573	-0.199882	0.255573	0.038345
RGS	372	0.000004	0.000532	-0.003868	0.003020	0.007723
UMDW	872	-0.000131	0.001743	-0.011119	0.009273	-0.074982

specification combined with the specifics of Russian market. Probably there are many companies that are included in both portfolios, what means that smaller companies tend to have higher book-to market value. Another factors have much lower correlations and thus it can be considered they either catch different effects or have no defined return pattern.

Table 2: Correlation matrix for factor returns

Factor	CASH	XSRET	HML	SMB	UMD	ABSS	ABNS	RGS
XSRET	-0.951							
HML	-0.004	-0.075						
SMB	0.007	-0.183	0.444					
UMD	0.055	-0.095	-0.050	0.029				
ABSS	-0.031	0.032	-0.006	0.009	0.054			
ABNS	0.008	-0.010	0.022	-0.034	-0.009	-0.063		
RGS	0.000	0.028	-0.062	-0.034	0.035	-0.044	0.568	
UMDW	0.027	-0.077	0.043	0.030	0.170	-0.060	0.063	-0.009

Carhart four factor model

Russian financial market is still at the development stage and has not yet well studied. Thus, it is impossible to use results of Bank et al. (2011), Da et al. (2009), Joseph et al. (2011) without the validity check of the previous hypotheses or identity of Russian financial market and key financial markets. To verify the hypothesis of Barber and Odean (2008) the benchmark model should be tested. Then the check of possibility to use search volume data as proxy of investor attention and for return prediction could be done.

In the begging of current research CAPM model for Russian stocks is estimated. For estimation seemingly unrelated regression(SUR) is used. Methodology of SUR regression estimation can be found in Appendix A. It can be seen on Figure 1 that Russian stocks vary in CAPM coefficients. Some of them have much larger alpha than others do, and there are also negative alphas present. Estimated betas are within $(0.8, 1.15)$ interval.

Further the Carhart four-factor model using the same methodology is estimated.

$$R_{pt} - R_{ft} = \alpha + \beta_{mp}(R_{mt} - R_{ft}) + \beta_{SMBp}SMB_t + \beta_{HMLp}HML_t + \beta_{UMDp}UMD_t + \varepsilon_t \quad (12)$$

The coefficients are represented on Figure 2. Here it can be seen that higher difference in magnitude of market beta: it lies within $(0.5, 1.25)$ interval. To test the significance of added factors likelihood-ratio test for nested model is performed to test hypothesis that all betas at tested factor are equal to zero. Such methodology was chosen due to several issues. There are 182 coefficients for each factor and as significance of certain factor is tested, the coefficients should not be tested for assets separately. This research is meant to test the significance of investors sentiment in predicting abnormal returns and the applicability of certain variations of Fama-French and Carhart models should be tested additionally. The results of significance test presented on Table 3. The likelihood ratio statistic is 2858, what corresponds to 0.001 significance level. For 546 degrees of

Figure 1: Alphas and betas for 182 Russian stocks in CAPM model

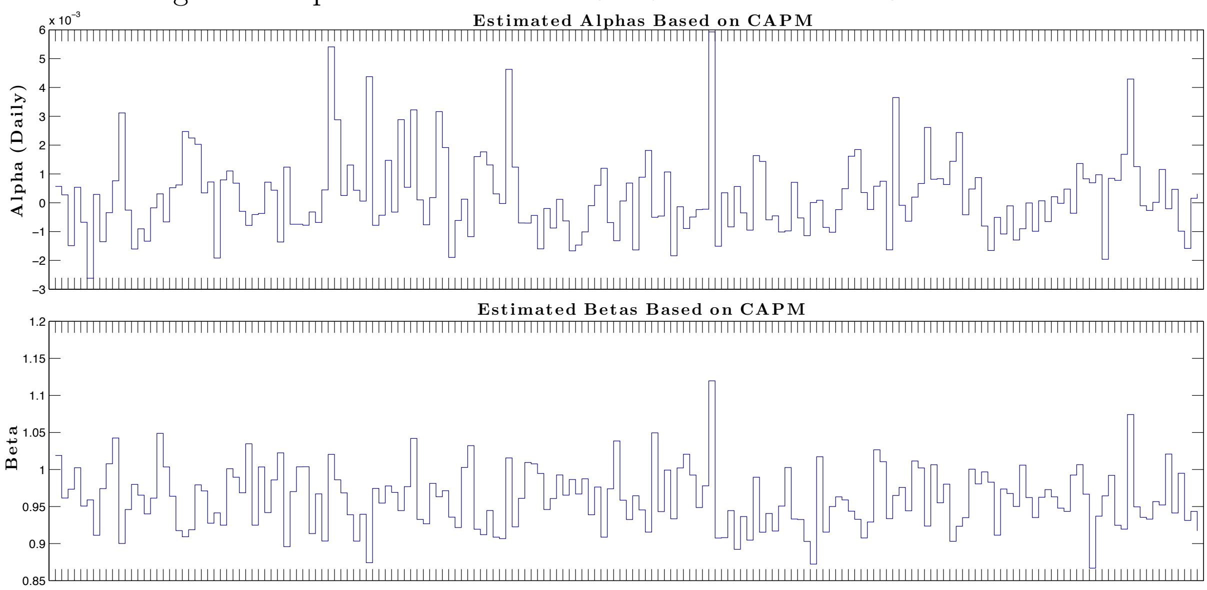
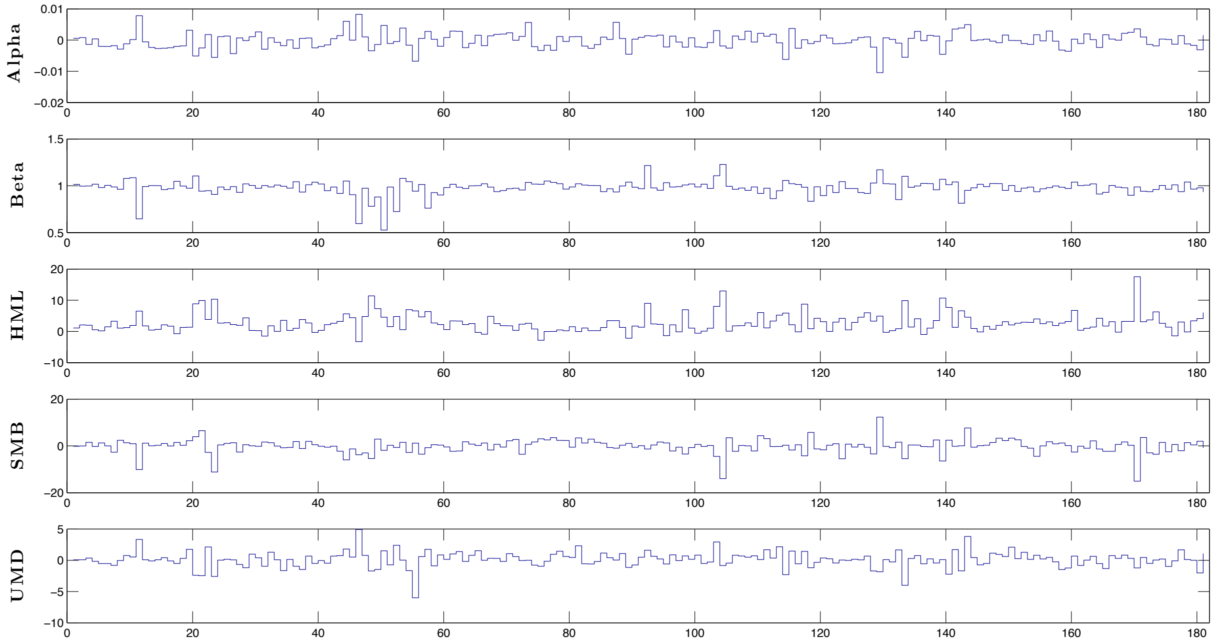


Figure 2: Alphas and betas for 182 Russian stocks in Carhart four-factor model



freedom 5% critical value is 598. Thus the null hypothesis that all factors do not improve quality of the model is rejected toward the hypothesis that there is at least one significant factor. That means in the further analysis Carhart four-factor model would be used as a benchmark.

Concerning another measures of model fitness it would be inappropriate to use measures as R^2 etc. as the multiple regression models are estimated simultaneously. Overall measure for 182 models would not show interpretable result as variation between different models is not taken into account. Separate analysis of different models would not show the performance of the model as CAPM and derivative models should explain variation of returns between all assets.

Additionally the test for zero abnormal return could be performed using the same methodology. The likelihood ratio statistic is 758, that is significant at 0.001 level. The corresponding critical value for 5% significance level with 182 degrees of freedom is 213. Thus there is still unexplained variation of returns between assets and additional risk factors could be searched for.

Table 3: LR test for H_0: factor coefficients = 0

H_0 \ Model:	Fama-French Carhart(FF)	**FF ABSS**	FF ABNS	FF RGS	FF ABSS&RGS	FF ABSS&ABNS	FF ABNS&RGS	FF All factors
		11545			741	750		752
ABSS		213			213	213		213
		0.0000			0.0000	0.0000		0.0000
			683			-10110	343	-7582
ABNS			213			213	213	213
			0.000			1	0.000	**1**
				8620	-2183		-7592	345
RGS				213	213		213	213
				0.000	**1**		1	0.000
	758	672	1087	353	653	792	1069	777
α	213	213	213	213	213	213	213	213
	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HML	2858							
SMB	598							
UMD	0.000							

	LR-stat
Note:	Critical value
	p-value

Model selection

In current research three proxies of investors attention are proposed. ABSS analyzes overall fame of the company. Thus the hypothesis is that more famous companies have higher returns than those that almost unknown. ABNS reflects sudden changes in search activity. For this factor Barber and Odean (2008) hypothesis is tested: companies that catch attention have positive price pressure and thus higher returns. RGS shows the growth rate of search volume for the last period. The underlying hypothesis is similar to previous one, but it reflects more frequent moves of search intensity. All three measures analyzed are relative in the same period thus no other corrections should be made.

To choose significant factors the following procedure is used. For each combination of factors in the model there should be no insignificant ones. Significance of factors is tested using likelihood-ratio test for nested models. Factor significance is tested for each of them separately thus the critical value for all tests would be 213 for 182 degrees of freedom and 5% significance level.

To start the analysis the model with all three factors included is considered. Results are presented in Table 3. In all-factors model ABNS is insignificant, thus this model is not the final one and further analysis should be performed. It should be mentioned that there are several negative values of LR-test statistic that are theoretically impossible, but there is no computational or other errors and the models with additional factor perform worse. That could be connected with bad convergence due to length of the dataset and estimates of covariance matrix. Reasons of such results should be analyzed further, but the model with additional variable is definitely not better then nested one.

On the next step model that is not worse than previous one is analyzed: with ABSS and RGS factors. There RGS factor is insignificant thus this model is not better than model with ABSS only. When ABSS model is considered it remains significant, thus it is a candidate for a best model.

To prove that the model outperforms other ones use the following logic of transitive relation is used. If one of the factors is insignificant in the model with two factors, while another is significant, that directly means that two factor model is not better than the model with significant factor only and better than a model with insignificant factor. Thus it is possible compare such models. To determine best model from the ones left the remaining two-factor models are considered. In a ABSS-ABNS pair model the abnormal search volume factor is insignificant. That provides two conclusions: the two-factor model is not significantly better than ABSS only one, and ABSS model is strictly better than ABNS model. The ABNS-RGS model has insignificant RGS factor that shows that RGS only model is worse than ABNS model and ABNS-RGS model is not better than ABNS only one.

This comparisons provide one best model: ABSS — absolute search volume. And controlling for this factor all other ones become insignificant. It could mean there is the evidence of the fact that fame itself is more significant than its changes for Russian companies. There is a premium for being well-known and its characteristics should be analyzed.

One more crucial result is that abnormal search volume is significant when used alone, what coincides with previous results, but when absolute number of searches is used it becomes insignificant. Such results could not be obtained before using the Google data. Availability of Yandex data allowed to test significance of search interest against changes in it, thus making this research unique. Among other results the test of significance of alpha shows that abnormal return could become less significant.

Abnormal returns for quartile portfolios

In previous section absolute search volume factor significantly improved model for returns. In the current section the analysis of characteristics of factor returns is performed. The key measure that has practical use is abnormal factor return or alpha. It shows the premium of a corresponding factor in portfolio returns that is not explained by other factors.

On a first step the analysis of quartile portfolio returns is performed. The occurrence of significant difference in alpha(constant term) between quartile portfolios would show the nature of premium. The result of factor returns regression on Fama-French and Carhart factors for different quartile portfolios is presented in Table 4. For ABSS quartile portfolios more than 93% in return variation is explained by other factors. Neither of quartile portfolios have significant alphas thus there is no abnormal return in quartile portfolios present. Practically this results show that neither long nor short position in one of quartile portfolios would not yield positive abnormal returns, but return of factor-mimicking portfolios should be tested also to show absence of factor premium.

To test it zero-investment factor-mimicking portfolios are created the following way. In the portfolio assets that belong to higher quartile are taken in long position with equal weights of investments. Lowest quartile assets are taken in short position with equal weights thus there are no initial investments needed to create such portfolio. In Table 5 result of regression analysis of high-minus-low portfolios is presented. Unlike previous results such portfolios create positive abnormal returns that can be interpreted as positive premium of being well-known. The size of such premium is 0.12% per day, what corresponds to 0.61% weekly and 35.1% annual return. Thus there exists a positive abnormal return that can be explained using search volume data. However predicted alpha could not be transferred to actual investment return as this figures do not incorporate transaction costs during portfolio rebalancing. Moreover other factor portfolios should also be rebalanced on daily basis that would also add transaction costs. Some assets are not allowed to be sold short due to their illiquidity thus it makes impossible to replicate this theoretical yield. On the other hand it could be an explanation why there is still positive abnormal return for this factor. In Bank et al. (2011) search volume index allowed to predict changes in illiquidity and existing abnormal return could be the risk premium of illiquidity and further research of this effects using absolute search volume should be carried out.

The same analysis was performed for other two factors and there are similar results. Abnormal search volume and growth rate quartiles provide abnormal return, but its value is similar across them. Factor premium should differ between groups of different factor values. Formal test of zero-investments portfolio return provides existence of positive 0.13% premium for ABNS and 0.11% for RGS that is similar to ABSS return.

Table 4: Regression Results

	Dependent variable:											
	ABSS				ABNS				RGS			
	Q1	*Q2*	*Q3*	*Q4*	*Q1*	*Q2*	*Q3*	*Q4*	*Q1*	*Q2*	*Q3*	*Q4*
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
α	0.0010	0.0009	0.0009	0.0009	0.0011*	0.0011**	0.0011*	0.0011*	0.0011*	0.0011*	0.0011*	0.0010*
	(0.0006)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)
XSRET	−0.0022	−0.0009	−0.0010	−0.0015	0.0017	0.0009	0.0020	0.0017	0.0008	0.0009	0.0007	0.0012
	(0.0059)	(0.0057)	(0.0056)	(0.0057)	(0.0057)	(0.0057)	(0.0057)	(0.0057)	(0.0057)	(0.0057)	(0.0057)	(0.0057)
SMB	0.0451***	0.0435***	0.0422***	0.0422***	0.0451***	0.0420***	0.0415***	0.0423***	0.0434***	0.0427***	0.0424***	0.0432***
	(0.0038)	(0.0037)	(0.0037)	(0.0037)	(0.0037)	(0.0037)	(0.0037)	(0.0037)	(0.0037)	(0.0037)	(0.0037)	(0.0037)
HML	0.0103***	0.0097**	0.0098**	0.0097**	0.0095***	0.0091**	0.0090**	0.0092**	0.01021***	0.0098**	0.0099***	0.0101***
	(0.0039)	(0.0038)	(0.0038)	(0.0038)	(0.0038)	(0.0038)	(0.0038)	(0.0038)	(0.0038)	(0.0038)	(0.0038)	(0.0038)
UMD	0.9474***	0.9497***	0.9509***	0.9493***	0.9515***	0.9532***	0.9518***	0.9513***	0.9528***	0.9517***	0.9521***	0.9529***
	(0.0133)	(0.0130)	(0.0128)	(0.0129)	(0.0126)	(0.0126)	(0.0126)	(0.0126)	(0.0128)	(0.0126)	(0.0128)	(0.0128)
Observations	362	362	362	362	343	343	343	343	357	357	357	357
R^2	0.934	0.937	0.939	0.938	0.944	0.944	0.944	0.944	0.940	0.940	0.940	0.940
Adjusted R^2	0.933	0.937	0.938	0.937	0.943	0.943	0.943	0.943	0.939	0.939	0.939	0.939
Log likelihood	1156	1165	1170	1166	1117	1117	1187	1169	1155	1155	1154	1155
AIC	−2302	−2321	−2330	−2323	−2224	−2248	−2227	−2223	−2300	−2300	−2298	−2300
RMSE	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
F statistic	1260***	1330***	1370***	1340***	1260***	1420***	1430***	1410***	1380***	1380***	1370***	1380***

Note: *p<0.1; **p<0.05; ***p<0.01

Table 5: Quartile high-minus-low Index Regression Results

	Dependent variable:		
	Q4-Q1 portfolio returns		
	ABSS	*ABNS*	*RGS*
	(1)	(2)	(3)
implied weekly α	0.0061	0.0066	0.0059
α	0.0012*	0.0013**	0.0011*
	(0.0006)	(0.0006)	(0.0006)
XSRET	−0.0008	0.0017	0.0012
	(0.0062)	(0.0062)	(0.0062)
SMB	0.0447***	0.0441***	0.0445***
	(0.0040)	(0.0040)	(0.0040)
HML	0.0114**	0.0101**	0.0107***
	(0.0041)	(0.0041)	(0.0041)
UMD	0.9469***	0.9471***	0.9464***
	(0.0139)	(0.0136)	(0.0138)
Observations	362	343	357
R^2	0.929	0.935	0.93
Adjusted R^2	0.928	0.934	0.929
Log likelihood	1142	1091	1148
AIC	−2273	−2172	−2246
RMSE	0.010	0.010	0.010
F statistic	1160***	1210***	1170***

Note: *p<0.1; **p<0.05; ***p<0.01
implied weekly α is computed as $(1+\alpha)^5 - 1$

Conclusion

Recent studies provided evidence of predictability of asset returns due to investors sentiment. Barber and Odean (2008) proposed the hypothesis that increase in attention of noise traders could result in positive price pressure. According to Da et al. (2009) Google SVI could be appropriate proxy for investors attention. Google data that was used in previous research allowed to use only abnormal search volume index due to data normalized nature. This methodology could not measure comparative interest to different companies and tested effects of abnormal search index changes only. In current research this issue is analyzed. The results of Joseph et al. (2011) show that positive price pressure hypothesis is more likely to confirm on market with significant share of unsophisticated investors. Hence developing Moscow Exchange is a good candidate to test predictability of returns due to investors sentiment. As a data source Yandex Wordstat that provides absolute search volume is used. It allows to compare three search volume indicators: absolute search volume(ABSS), abnormal search volume(ABNS) and search volume growth rate(RGS).

Following the methodology of Joseph et al. (2011) factor portfolios are created and factor returns are used to explain assets returns. The crucial result is that for the analyzed data inclusion of ABNS as the factor allowed to significantly improve the basic Carhart (1997) model, what corresponds to previous research in this area. However the further analysis discover that among different factor models the one with absolute search volume index is the best one. In the model with two factors ABSS-ABNS, abnormal search volume index is insignificant and ABSS model is strictly better than ABNS one. Following results state that in previous research that used normalized Google data such conclusions could not be made. Overall search volume intensity and thus fame explain more return variation than abnormal changes in attention.

Additionally the analysis of quartile portfolios showed that there is no significant abnormal returns variation among them. However zero-investment factor-mimicking portfolios with long position in high search index stocks and short in low search index stocks provide significant positive abnormal return for all analyzed indexes. Obtained returns are theoretical ones as do not include transaction costs of portfolio rebalancing and unavailability of short sell for list of stocks. Attention premium could reflect liquidity issues. Bank et al. (2011) showed predictability of liquidity measures by SVI. In current study liquidity of stocks was not analyzed as a factor. Concerning Russian financial market liquidity could have significant risk premium that could be partially interpreted as fame that was captured by analyzed indexes. Thus further studies should cover analysis of predictability of liquidity measures and test of significance of attention based returns controlling for liquidity factor premium.

References

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets*, 5(1):31–56.

Amihud, Y. and Mendelson, H. (1986). Asset pricing and the bid-ask spread. *Journal of Financial Economics*, 17(2):223–249.

Bank, M., Larch, M., and Peter, G. (2011). Google search volume and its influence on liquidity and returns of German stocks. *Financial Markets and Portfolio Management*, pages 1–26.

Barber, B. M. and Odean, T. (2008). All That Glitters: The Effect of Attention and News on the Buying Behavior of Individual and Institutional Investors. *Review of Financial Studies*, 21(2):785–818.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance*, 52(1):57–82.

Chordia, T., Huh, S.-W., and Subrahmanyam, A. (2007). The Cross-Section of Expected Trading Activity. *The Review of Financial Studies*, 20(3):pp. 709–740.

Da, Z., Engelberg, J., and Gao, P. (2009). In Search of Attention. *AFA 2010 Atlanta Meetings Paper*.

Da, Z., Engelberg, J., and Gao, P. (2011). In Search of Fundamentals. *AFA 2012 Chicago Meetings Paper*.

Fama, E. and French, K. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1):3–56.

Florackis, C., Gregoriou, A., and Kostakis, A. (2011). Trading frequency and asset pricing on the London Stock Exchange: Evidence from a new price impact ratio. *Journal of Banking and Finance*.

Ginsberg, J., Mohebbi, M., Patel, R., Brammer, L., Smolinski, M., and Brilliant, L. (2009). Detecting influenza epidemics using search engine query data. *Nature*, 457(7232):1012–1014.

Goel, S., Hofman, J., Lahaie, S., Pennock, D., and Watts, D. (2010). Predicting consumer behavior with Web search. *Proceedings of the National Academy of Sciences*, 107(41):17486–17490.

Goriaev, A. (2004). Risk factors in the russian stock market. *Emerging Market Review*, 2(1):67–89.

Goyenko, R., Holden, C., and Trzcinka, C. (2009). Do liquidity measures measure liquidity? *Journal of Financial Economics*, 92(2):153–181.

Grossman, S. and Stiglitz, J. (1980). On the Impossibility of Informationally Efficient Markets. *The American Economic Review*, 70(3):393–408.

Joseph, K., Babajide Wintoki, M., and Zhang, Z. (2011). Forecasting abnormal stock returns and trading volume using investor sentiment: Evidence from online search. *International Journal of Forecasting*.

Kholodilin, K., M.Podstawski, and Siliverstovs, B. (2010). Do google searches help in nowcasting private consumption¿. Technical Report 256.

Kyle, A. (1985). Continuous Auctions and Insider Trading. *Econometrica*, 53(6):1315–1335.

Lintner, J. (1965a). Security prices, risk, and maximal gains from diversification*. *The Journal of Finance*, 20(4):587–615.

Lintner, J. (1965b). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The review of economics and statistics*, 47(1):13–37.

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica: Journal of the Econometric Society*, pages 768–783.

Roll, R. (1984). A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market. *Journal of Finance*, 39(4):1127–39.

Schmidt, T. and Vosen, S. (2009). Forecasting Private Consumption: Survey-based Indicators vs. Google Trends. Ruhr Economic Papers 0155.

Schmidt, T. and Vosen, S. (2010). A monthly consumption indicator for Germany based on internet search query data. Ruhr Economic Papers 0208.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk*. *The journal of finance*, 19(3):425–442.

Varian, H. R. and Choi, H. (2009). Predicting the Present with Google Trends.

Zellner, A. (1962). An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias. *Journal of the American statistical Association*, 57(298):348–368.

Data sources

1. Investfunds. `http://stocks.investfunds.ru/`
2. Google Trends. `http://trends.google.com/`
3. Google Insights for Search. `http://www.google.com/insights/search/`
4. RosBusinessConsulting. `http://export.rbc.ru/`
5. The Central Bank of the Russian Federation. `http://cbr.ru/`
6. Yandex Wordstat. `http://wordstat.yandex.ru/`

Appendix A. Seemingly unrelated regression

Seemingly unrelated regression(SUR) model is a generalization of linear regression model that was introduced by Zellner (1962). SUR model is a set of linear regression models that can have different dependent and independent variables and each of them could be estimated separately using standard OLS approach. Zellner (1962) proposed the procedure to estimate system of models simultaneously assuming that error terms could be correlated. Regression results, obtained using SUR methodology, are asymptotically more efficient than estimated using OLS for separate models. In case of uncorrelated error terms results are the same as using OLS. Each regression equation could be represented as:

$$y_m = X_m\beta_m + \varepsilon_m, \tag{13}$$

where m is the number of equation in the system of M equations, y_m is the $T\times 1$ vector of observations of independent variable, X_m is the $T \times k_m$ matrix of independent variables, k_m number of regressors in m'th equation, ε_m is $T \times 1$ vector of random error terms and β_m is $k_m \times 1$ number of coefficients.

The whole system can be represented as:

$$y = X\beta + \varepsilon \tag{14}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & X_M \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_M \end{bmatrix} \tag{15}$$

Variance-covariance matrix of u is

$$\Sigma = Var(u) = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \cdots & \sigma_{1,M} \\ \sigma_{2,1} & \sigma_{2,2} & \cdots & \sigma_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{M,1} & \sigma_{M,2} & \cdots & \sigma_{M,M} \end{bmatrix} \otimes I_T = \Sigma_c \otimes I_T \tag{16}$$

Σ_c is assumed constant i.e. for each pair of models $\sigma_{i,j}$ is constant for $\forall t$.

Estimation procedure is a two-step feasible generalized least squares method. On the first step OLS estimation for each equation is performed. Then the obtained residuals are used to estimate Σ_c:

$$\hat{\sigma}_{i,j} = \hat{\varepsilon}_i'\hat{\varepsilon}_j \tag{17}$$

On the second step GLS regression is estimated using $\hat{\Sigma}_c$. Coefficients estimates are equal to following expression:

$$\hat{\beta} = \left(X'(\hat{\Sigma}_c^{-1} \otimes I_T)X\right)^{-1} X'(\hat{\Sigma}_c^{-1} \otimes I_T)y \tag{18}$$

They are distributed as:

$$\sqrt{T}(\hat{\beta} - \beta) \xrightarrow{d} \mathcal{N}\left(0, \left(\frac{1}{T}X'(\Sigma_c^{-1} \otimes I_T)X\right)^{-1}\right) \tag{19}$$