Effort Complementarity and Firm Size

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Abstract

We build a model of team production where complementarity of workers' efforts depends on size of the firm. As the workers become more specialized, their incentives improve by virtue of a decrease in free-riding. Firms become larger up to the point where specialization effect is overwhelmed by market size effect. We show that usually size of the firm increases when switching from effort maximization to workers' utility maximization and from the last one to output maximization. Then we provide comparative statics on the effect of ouside equity, market size, importance of "learning by doing" on the firm size.

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1 Introduction

In 1914, Ford Motor started using conveyor to assemble its famous Model T. This dramatically decreased time to assemble a car. Only 11 cars per month were produced before the conveyor, while after its introduction a car was assembled within 83 minutes. This led to one of the greatest commercial successes in history: in a few years, half of the cars used in the world were Fords. In our view, we interpret this increase in productivity as a respond to increase in specialization of workers. Greater specialization meant harder substitution of efforts at differen specialities across workers, as a failure of one worker could stop production of the whole conveyor. Smaller elasticity of substitution provides better incentives in case of imperfectly monitorable individual efforts and hence amplified the increase in their productivity. This story motivates our "Model T" theory of firms size: the size of the firm grows up to a point where the gain from incentives fostered by the complementarity of efforts is outweighted by decreasing returns to scale or market size effects.

Most models find that provision of effort in teams is suboptimal due to production externalities and point out the mechanisms that mitigate such problems, such as optimal contracting and peer-pressure. Our approach is different: we find out how effort provision is related to the technology and the size of the partnership. We argue that as long as degree of speceialization, and hence, complementarity of efforts is related to the size of the team, the size can provide incentives. On the other hand, after some point, the workers start being allocated to the same tasks, which reduces complementarity of efforts and hence spoils the incentives. Hence, we derive conditions the optimal size of the team, depending on the degree of complementarity of workers' efforts.

2 Literature

The paper contributes to two strands of the literature. The first strand is moral hazard in teams introduced by Holmstrom (1982). He showed that provision of effort in teams will be generally suboptimal due to externalities in production and impossibility of monitoring individual efforts perfectly. Legros and Matthews (1993) showed that the problem might be effectively mitigated in some special cases. Kandel and Lazear (1992) suggest peer-pressure to mitigate what they call $1/N$ effect: increased number of workers suggests smaller shares of output per worker which results in smaller motivation. When the firm is getting larger, they argue, the output is divided between larger quantity of workers, while they bear the same individual costs. Hence, the effort of each worker should be decreasing as firms grow larger. Adams (2006) showed that the $1/N$ effect may not occur if the efforts of workers are complementary enough. However, as he uses CES production function, his model predicts trivial, either unit or infinite, firm sizes, depending on the value of elasticity of substitution. Winter (2004) shows that excessive complementarity and increasing returns to scale can result in asymmetic discriminatory equilibria which might have non-trivial effects on firm size.

Theories of firm boundaries are classified as technological, organizational and institutional (the classification is borrowed from Kumar, Rajan and Zingales (1999)). This separation is, of course, quite discrete, and most theories lie in between, combining elements of the approaches. This paper is to present a purely technological argument, however it is originally motivated by organizational one by Alchian and Demsetz (1972). As they noted:

> Team production is used if it yields an output enough larger than the sum of separable production to cover the costs of organizing and disciplining team members.

Moreover, our paper can be easily connected to other theories, which would result in richer insights.

The technological theories explain the firm size by the productive inputs, and ways the valuable output is produced. Basically, there are five technological factors that are taken into account describing the firm size: market size, gains from specialization, management control constraints, limited workers' skills, loss of coordination. For example, Adam Smith (1776) explained the firm size by benefits from specialization limited by the market size. By his logic, workers can specialize and invest in narrower scope of skills, hence economizing on the costs of skills. Becker and Murphy (1992) focus on the tradeoff between specialization and coordination costs. The larger is the firm, the larger are the costs of management to put them together to produce the valuable output.

Williamson (1971), Calvo and Wellisz (1978) and Rosen (1982) use loss of control for explaining firm size. Williamson points out that a size of hierarchical organization may be limited by loss of control, assuming the intentions of managers are not fully transmitted downwards from layer to layer. Calvo and Wellisz (1978) show that the effect of the problem is largely dependent on the structure of monitoring. If the workers do not know when the monitoring occurs, the loss of control doesn't hinder the firm size, while it may if the monitoring is scheduled. Rosen (1982) highlights the tradeoff between increasing returns to scale in management and the loss of control. As highly qualified managers foster the productivity of their workers, managers with higher abilities should have larger firms firm. However, the attention of managers is limited, hence having too much workers results in loss of control and decreases the productivity of their team substantially. The optimal firm size in this model is when the value produced by the new worker is less than the losses due to attention diverted from his teammates.

Kremer (1993) focuses on the tradeoff between specialization and probability of failure associated with low skill of workers. He assumes that the the value of output is directly proportional to the number of tasks needed to produce it. Larger number of workers and hence tasks tackled allows to produce more valuable output, but each additional worker is a source of risk of spoiling the whole product. Hence, the size of the firm is explained by the probability of failure by the workers which is proportional their skill.

3 The Model

Production is conducted in firms of size N. Workers are identical. Each worker contributes effort $e_i \in R^+$, $i \in \{1, ..., N\}$. Efforts of workers $e_1, ..., e_N$ are aggregated by the aggregator function $g(e_1, ..., e_N|N)$. The output is $f(g(e_1, ..., e_N|N))$, which is split equally among the workers.

Assumption 1 $g(e_1, ..., e_N|N)$ is differentiable, symmetric and homogeneous of degree 1 in efforts for each given N.

Assumption 2 $f(\cdot)$, $c(.)$ are twice differentiable, $f' > 0$, $c' > 0$, $c'' > 0$

Hence the worker's maximization problem is:

$$\max_{e_i} \frac{1}{N} f(g(e_1, ..., e_N|N)) - c(e_i)$$

4 Equilibrium

The equilibrium concept is Nash. It is a collection of efforts e_i, $i \in \{1, ..., N\}$. The workers simultaneously decide on their effort during production. The first order condition for workers' problem is:

$$\frac{1}{N} f'(g(e_1, ..., e_N|N)) \cdot g'_i(e_1, ..., e_N|N) = c'(e_i). \tag{4.1}$$

Assumption 3 $f(\cdot)$, $g(\cdot)$, $c(\cdot)$ are such that there is one solution of the FOC and there is a symmetric equilibrium, $e_i = e^* \ \forall i \in \{1, ..., N\}$.

Hence the equilibrium effort e^* is given by the condition:

$$\frac{1}{N} f'(g(e^*, ..., e^*|N)) \cdot g'_i(e^*, ..., e^*|N) = c'(e^*).$$

Denote $g(1, ..., 1|N) = h(N)$. This function characterizes the economy of scale in workers' effort. In the following analysis, we will assume that $h(N)$ can be completed for non-integer N as a continuous, differentiable, increasing function.

Fact 1. *The first order conditions can be expressed as*

$$\frac{1}{N^2} f'(e^* \cdot h(N)) \cdot h(N) = c'(e^*). \tag{4.2}$$

Proof. By homogeniety, $g(e^*, ..., e^*|N) = e^* \cdot g(1, ..., 1|N) = e^* \cdot h(N)$. Moreover, $h(N) = g(1, ..., 1|N) = (e^* \cdot g(1, ..., 1|N))' = (g(e^*, ..., e^*|N))' = N \cdot g'_i(e^*, ..., e^*|N)$.

□

(4.2) gives the equilibrium effort as a function of quantity of workers N. We use $e^*(N)$ to denote the solution of (4.2) even when the argument is not integer.

$e^*(N)$ is continuous and differentiable by virtue of implicit function theorem and assumptions 2-3. (4.2) is refered to thereafter as Equilibrium Effort Condition (EEC).

5 Comparative Statics

We consider three possible targets: setting the number of workers to maximize the utility of the group members, their efforts, total output or some combination of those.

Maximizing utility would be a kernel allocation if we considered the two stage game: the workers first form partnerships then produce and split output equally. It might be also regarded as a solution of a firm problem in case of price taking on labor market and its inability to monitor individual efforts.

Effort maximization is important in case of significant "learning by doing" effects. This may be the case in education or industries where the human capital is of greatest importance (such as consulting).

Output maximization might be the case if the payoff of the manager is a function of target output only. This might be the case for "red directors" in the Soviet Union, Economics department heads, empire building top managers and others. Of course, many real objective functions, will have some combination of the listed motives.

It worth noting two facts that will be used extensively throughout the rest of the paper:

Fact 2. *Let $u(x),\ v(x) > 0$ for $x \neq 0$ and differentiable. Denote the elasticity of q at the point x as $\epsilon_q(x) = \frac{xq'(x)}{q(x)}$. Than $\epsilon_{v(x)\cdot u(x)}(x) = \epsilon_v(x) + \epsilon_u(x)$ and $\epsilon_{v(u(x))}(x) = \epsilon_v(u(x)) \cdot \epsilon_u(x)$.*

Fact 3. *For each $v(x)$ such that $v(x) > 0$ and $x > 0$ implies that the sign elasticity of $v(x)$ equals the sign of its derivative $sgn(\epsilon_v(x)) = sgn(v'(x))$.*

Fact 3 implies that we can use elasticities instead of derivatives for solving maximization problems. Denote $\epsilon_f(x) = \frac{xf'(x)}{f(x)}$ the elasticity of function f taken at the point x and $r_f(x) = -\frac{xf''(x)}{f'(x)}$ is the relative-risk aversion of f taken at the point x and $r_c(x) = \frac{xc''(x)}{c'(x)}$.

To ensure single crossing we have to assume:

Assumption 4. $r_f(\cdot)$ is increasing, $\epsilon_h(\cdot)$ is decreasing, and at least one of them is strictly decresing or increasing. Moreover, $r_c(e^*) > -r_f(e^*(N) \cdot h(N))$.

5.1 Effort maximization

The formal statement of the effort maximization problem is:

$$\max_N e^*(N)$$

We denote the solution of this problem for continuous argument N_1. Here and thereafter we consider optimal sizes up to a continuity correction, i. e. if we found "optimal size" N_1, the real optimal size N_1^* is either $[N_1]$ or $[N_1]+1$, where $[\cdot]$ denotes the integer part.

The elasticity of equilibrium effort with respect to N is found by taking the full elasticity of the equilibrium effort condition and rearranging (see Appendix for details):

$$\epsilon_{e^*}(N) = \frac{[1 - r_f(e^*(N) \cdot h(N))] \cdot \epsilon_h(N) - 2}{r_c(e^*(N)) + r_f(e^*(N) \cdot h(N))}, \tag{5.1}$$

As the sign of elasticity is equal to the sign of derivative, the necessary condition for effort maximization is:

$$[1 - r_f(e^*(N_1) \cdot h(N_1))] \cdot \epsilon_h(N_1) - 2 = 0.$$

5.2 Output maximization

$$\max_N f(e^*(N) \cdot h(N))$$

Denote the output maximizing size as N_2 and the amount of efficient effort as $E = e^*(N) \cdot h(N)$. It's elasticity is

$$\epsilon_E(N) \quad = \epsilon_{e^*}(N) + \epsilon_h(N) = \tag{5.2}$$

$$\frac{[1 - r_f(e^*(N) \cdot h(N))] \cdot \epsilon_h(N) - 2}{r_c(e^*(N)) + r_f(e^*(N) \cdot h(N))} + \epsilon_h(N) =$$

$$\frac{[1 - r_f(e^*(N) \cdot h(N))] \cdot \epsilon_h(N) - 2 + \epsilon_h(N) \cdot [r_c(e^*) + r_f(e^*(N) \cdot h(N))]}{r_c(e^*(N)) + r_f(e^*(N) \cdot h(N))} =$$

$$\frac{\epsilon_h(N) \cdot [r_c(e^*(N) \cdot h(N)) + 1] - 2}{r_c(e^*(N)) + r_f(e^*(N) \cdot h(N))}.$$

Output $f(e^* h(N))$ is just a monotonic transformation of efficient effort $e^*(N) \cdot h(N)$, hence the necessary condition for output maximization is:

$$\epsilon_h(N_2) \cdot [r_c(e^*(N_2)) + 1] - 2 = 0$$

To show that N_2 is unique, we note that $\epsilon_E(N) = \epsilon_{e^*}(N) + \epsilon_h(N)$ is decreasing in N as both $\epsilon_{e^*}(N)$ and $\epsilon_h(N)$ are decreasing. Moreover, $\epsilon_E(1) = \epsilon_{e^*}(1) + \epsilon_h(1) > 0$ and $\epsilon_E(\infty) = \epsilon_{e^*}(\infty) + \epsilon_h(\infty) < 0$, hence $\epsilon_E(N)$ changes sigh only once, hence N_2 exists and unique.

5.3 Utility maximization

Maximizing the utilitise subject to the equilibrium effort condition results in a problem:

$$\max_N \frac{1}{N} f'(e^*(N) \cdot h(N)) - c(e^*(N))$$

Taking the first order conditions:

$$\frac{1}{N} f'(e^* \cdot h(N)) \cdot [(e^*(N))' \cdot h(n) + e^*(N) \cdot h'(N)] - c'(e^*) \cdot (e^*(N))' - \frac{1}{N^2} f(e^* \cdot h(N)) = 0$$

Plugging $c'(e^*)$ from the EEC:

$$\frac{1}{N} f'(e^* \cdot h(N)) \cdot [(e^*(N))' \cdot h(n) + e^*(N) \cdot h'(n)] -$$

$$\frac{1}{N^2} f'(e^* \cdot h(N)) \cdot h(N) \cdot (e^*(N))' = \frac{1}{N^2} f(e^* \cdot h(N))$$

Rewriting in terms of elasticities and canceling $f(e^*(N) \cdot h(N))$ out:

$$\epsilon_f(e^*(N) \cdot h(N)) \cdot [\epsilon_h(N) + \epsilon_{e^*}(N)] - \frac{1}{N}\epsilon_f(e^*(N) \cdot h(N)) \cdot \epsilon_{e^*}(N) = 1$$

Finally, we obtain the condition for utility maximizing size N_3:

$$\epsilon_f(e^*(N_3) \cdot h(N_3))[\epsilon_h(N_3) + \frac{N_3 - 1}{N_3}\epsilon_{e^*}(N_3)] = 1 \tag{5.3}$$

To ensure that there is a unique N_3 satisfying (5.3), we assume that $\epsilon_f(e^*(N) \cdot h(N))$ is small enough, i. e.

$$\epsilon_f(e^*(N) \cdot h(N))[\epsilon'_h(N) + \frac{N-1}{N}\epsilon'_e(N) + \frac{1}{N^2}\epsilon_{e^*}(N)] + \tag{5.4}$$

$$+\epsilon'_f(e^*(N) \cdot h(N))[\epsilon_h(N) + \frac{N-1}{N}\epsilon_{e^*}(N)] < 0$$

5.4 Synthesis

The comparative statics results can be summarized by following propositions.

Proposition 4. *The output maximizing team is larger than effort maximizing, $N_1 < N_2$.*

Proof. $\epsilon_E(N_2) = \epsilon_{e^*}(N_2) + \epsilon_h(N_2) = 0$, while $\epsilon_h(N_2) > 0$, hence $\epsilon_{e^*}(N_2) < 0$. $\epsilon_{e^*}(N)$ is monotonic in N by Assumption 4 and (5.1), hence $N_2 > N_1$.

□

Proposition 5. *Suppose that $f(x) = x^\alpha$, the production function has constant elasticity and $N_2 > 2$. Than $N_1 < N_3 < N_2$.*

Proof. See Appendix.

□

5.5 Changes in productivity

Suppose that payoffs for the workers change relatively to the costs:

$$\max_{e_i} \frac{L}{N} f(g(e_1, ..., e_N|N)) - c(e_i)$$

Proposition 6. *An increase in relative revenue L leads to a decrease of optimal N for utility maximizing, effort maximizing and revenue maximizing firm.*

Updating the EEC , one gets:

$$\frac{L}{N^2} f'(e^* \cdot h(N)) \cdot h(N) = c'(e^*). \tag{5.5}$$

Equilibrium effort is increasing in relative implortance of revenue for every size N. By implicit function theorem one might see that:

$$\epsilon_e(L) = \frac{1}{r_c(e^*) + r_f(e^* h(N))} > 0.$$

The maximizing sizes $N_1,\ N_2,\ N_3$ do not depend on the revenue importance L other than though equilbrium effort.

Using the maximizing firm size condition from above:

$$\epsilon_h(N_1) \cdot [1 - r_f(e^*(N_1, L)h(N_1))] - 2 = 0$$

As L increases, by IFT one obtains that firm size decreases:

$$\frac{dN_1}{dL} = \frac{\epsilon_h(N_1) r_f'(e^*(N_1, L)h(N_1))h(N_1)}{F} \frac{de}{dL} < 0$$

where

$$F = \epsilon_h'(N_1) \cdot [1 - r_f(e^*(N_1, L)h(N_1))] +$$

$$\epsilon_h(N_1) \cdot r_f'(e^*(N_1, L)h(N_1))[e^*(N_1, L)_1' h(N_1) + e^*(N_1, L)h'(N_1)]] < 0$$

Similar analysis results in $\frac{dN_2}{dL} < 0$ and $\frac{dN_3}{dL} < 0$

6 Discussion

6.1 Assumptions

6.1.1 Production function

There are two possible set of factors affecting the production function: technology and market power. Technological factors are usual arguments on returns to scale.

The market argument looks as follows. Let $f(x) = x \cdot P(x)$ be the revenue from selling x units, where is $P(x)$ is the inverse demand for firms output. In this case the elasticity of f is $\epsilon_f(x) = 1 + \epsilon_p(x)$ and $r_f(x) = \epsilon_p(x)\frac{r_p(x)-2}{1+\epsilon_p(x)} > 0$ for $r_p(x) < 2$.

6.1.2 The aggregator

Setting the structure of aggregator function lets us to incorporate most of the models in the literature. For example, if $g(e|N) = \sum e_i$ we have perfectly substitutable efforts, which leads to rapidly decreasing incetives as firm size grows. On the other hand, $g(e|N) = v(N) \cdot \min_{e_i}\{e\}$ results in perfect complementarity of efforts, hence inducing optimal effort supply. Moreover, having $g(e|N) = v(N) \cdot (\prod_{i=1}^{N} e_i)^{1/N}$ will result in Kremer-like Cobb-Douglas production function. Equally, one can have CES production function $g(e|N) = (\sum e_i^{\rho})^{1/\rho}$ which would be in line with Adams (2006).

6.2 Implications

Our framework and comparative statics results might be useful to analyse how the incentives of the social planner will determine firm size. For example, one could obtain that larger market size increases the firm size for large enough elasticities of marginal revenue, while it might decrease for small ones. The inverse holds for outside equity. Importance of "learning by doing" will decrease firm size when the elasticities of marginal revenue are low, but increase otherwise. This, for example, implies that younger or longer contracted workers will work in smaller and larger teams for low and high elasticities respectively.

7 Conclusion

We build a specialization-based model of team production and find out optimal sizes of teams given different objectives of the social planner. We show that effort maximizing social planner will choose smaller firm sizes than output maximizing one. Moreover, we show that while usually utility maximizing size lies between those, this relationship might mix up for too large or too small returns to scale. This framework enables us to explain team and, hence, firm size depending by either planner's target function or technology used in production. However, we admit it lacks detailization: the impact of particular factors such as worker skills, market size and contracting is a topic for future research.

References

[1] Christopher P Adams. Optimal team incentives with CES production. *Economics Letters*, 92(1):143–148, 2006.

[2] Armen A Alchian and Harold Demsetz. Production, information costs, and economic organization. *The American economic review*, 62(5):777–795, 1972.

[3] Gary S Becker. *Human capital: A theoretical and empirical analysis, with special reference to education*. University of Chicago Press, 2009.

[4] Gary S Becker and Kevin M Murphy. The division of labor, coordination costs, and knowledge. *The Quarterly Journal of Economics*, 107(4):1137–1160, 1992.

[5] Kenneth Burdett and Randall Wright. Optimal firm size, taxes, and unemployment. *Journal of Public Economics*, 39(3):275–287, 1989.

[6] Guillermo A Calvo and Stanislaw Wellisz. Supervision, loss of control, and the optimum size of the firm. *The Journal of Political Economy*, 86(5):943–952, 1978.

[7] Joseph Farrell and Suzanne Scotchmer. Partnerships. *The Quarterly Journal of Economics*, 103(2):279–297, 1988.

[8] Bengt Holmstrom. Moral hazard in teams. *The Bell Journal of Economics*, 13(2):324–340, 1982.

[9] Eugene Kandel and Edward P Lazear. Peer pressure and partnerships. *Journal of Political Economy*, 100(4):801–817, 1992.

[10] Matthias Kräkel and Gunter Steiner. Equal sharing in partnerships? *Economics Letters*, 73(1):105–109, 2001.

[11] Michael Kremer. The O-ring theory of economic development. *The Quarterly Journal of Economics*, 108(3):551–575, 1993.

[12] Krishna B Kumar, Raghuram G Rajan, and Luigi Zingales. What determines firm size? Technical report, National Bureau of Economic Research, 1999.

[13] Patrick Legros and Steven A Matthews. Efficient and nearly-efficient partnerships. *The Review of Economic Studies*, 60(3):599–611, 1993.

[14] Canice Prendergast. The provision of incentives in firms. *Journal of economic literature*, 37(1):7–63, 1999.

[15] Raghuram G Rajan and Luigi Zingales. Power in a theory of the firm. *The Quarterly Journal of Economics*, 113(2):387–432, 1998.

[16] Sherwin Rosen. Authority, control, and the distribution of earnings. *The Bell Journal of Economics*, 13(2):311–323, 1982.

[17] Oliver E Williamson. The vertical integration of production: market failure considerations. *The American Economic Review*, 61(2):112–123, 1971.

[18] Eyal Winter. Incentives and discrimination. *The American Economic Review*, 94(3):764–773, 2004.

8 Appendix

8.1 Elasticity of effort derivation

Take the full elasticity w. r. t. N of the EEC:

$$\frac{1}{N^2} f'(e^*(N) \cdot h(N)) \cdot h(N) = c'(e^*) \Rightarrow \epsilon_{f'}(e^* \cdot h(N))[\epsilon_h(N) + \epsilon_e(N)] + \epsilon_h(N) - 2 = \epsilon_{c'}(e^*)\epsilon_e(N) \Rightarrow$$

$$-r_f(e^* \cdot h(N))[\epsilon_h(N) + \epsilon_e(N)] + \epsilon_h(N) - 2 = r_c(e^*)\epsilon_e(N) \Rightarrow$$

$$\epsilon_e = \frac{[1 - r_f(e^*(N) \cdot h(N))] \cdot \epsilon_h(N) - 2}{r_c(e^*) + r_f(e^*(N) \cdot h(N))}$$

8.2 Proof of Proposition 5

$f(x) = x^\alpha \Rightarrow r_f(e^*(N) \cdot h(N)) = 1 - \alpha,\ r_c(e^*(N)) > -r_f(e^*(N) \cdot h(N)) = \alpha - 1$, $\epsilon_f(e^*(N) \cdot h(N))) = \alpha$. The FOCs for the optimal size can be rewritten as

$$\alpha \cdot \epsilon_h(N_1) - 2 = 0$$

$$\epsilon_h(N_2) \cdot [r_c(e^*(N_2)) + 1] - 2 = 0$$

$$\alpha[\epsilon_h(N_3) + \frac{N_3 - 1}{N_3}\epsilon_e(N_3)] - 1 = 0$$

The first equation implies $\epsilon_h(N_1) = 2/\alpha$. The third expression is a derivative of individual worker's utility up to a positive factor. If we plug N_1:

$$sgn(u'(N_1)) = sgn(\alpha[\epsilon_h(N_1) + \frac{N_1 - 1}{N_1}\epsilon_e(N_1)] - 1) = sgn(2 - 1) = 1$$

. Hence utility is increasing in N at N_1, hence $N_1 < N_3$.

$$\epsilon_h(N_2) = \frac{2}{[r_c(e^*(N_2)) + 1]} < \frac{2}{\alpha}$$

Similarly,

$$sgn(u'(N_2)) = sgn(\alpha[\epsilon_h(N_2) + \frac{N_2 - 1}{N_2}\epsilon_e(N_2)] - 1) =$$

$$sgn(\alpha[\frac{1}{N_2}\epsilon_h(N_2) + \frac{N_2 - 1}{N_2}\epsilon_{\mathrm{E}}(N_2)] - 1) =$$

$$sgn(\alpha\frac{1}{N_2}\epsilon_h(N_2) - 1) \leq sgn(\frac{2}{N_2} - 1) = -1.$$

Hence, utility is decreasing at N_2. Combining with Proposition 1 we obtain $N_1 < N_3 < N_2$.