

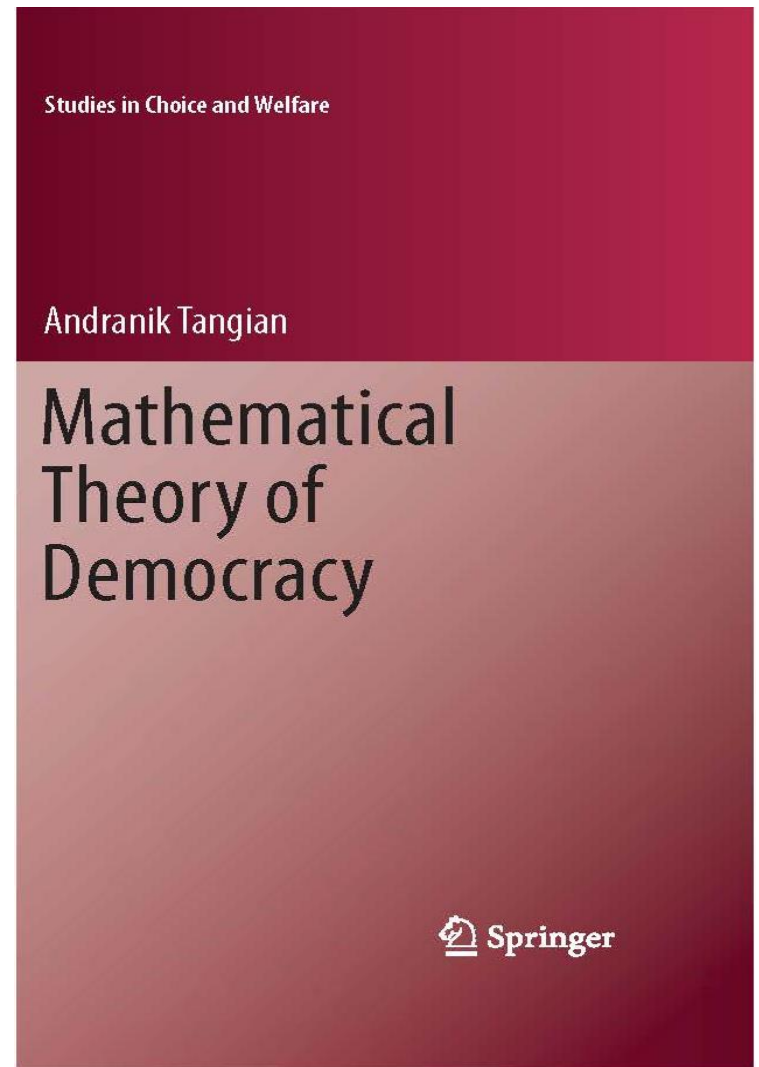
Mathematical Theory of Democracy

Book presentation

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I have come to the conclusion that politics is too serious a matter to be left to the politicians

Charles De Gaulle (1890–1970)

Introduction and Acknowledgements

Voting on proposals and propositions
Reidefinition of democracy in 19th century
Universal suffrage → universal voting rule
Perplexed scholars → Voting paradoxes
Ultrafilters (BA Efimov)
Math theory of democracy (NN Vorobjev)
Real politics (Th Ngakoutou)
Applications (J Gruber, S Fuchs-Seliger)

P1 Theory

1 Athenian democracy

Historical path to democracy

Establishing democracy by Cleisthenes

Culmination of democracy under Pericles

Philosophy of democracy according to
Pericles, Plato and Aristotle

Aristotle's mathematical model of two types
of governance

2 Echoes of Democracy in Ancient Rome

Typology of states and the theory of mixed government

Rotation of government types by Polybius

The Roman Republic

Pliny the Younger's logical analysis of a Senate hearing

3 Revival of Democracy in Italian Medieval City-Republics

The Venetian Republic

The Florentine Republic

Theory of mixed government revitalized in the medieval studies

First mathematical approaches to elections in the works of Llull and Cusanus

4 Enlightenment and the End of Traditional Democracy

Montesquieu and the separation of powers

Rousseau and the General Will

The Borda method of election

Laplace's justification of the Borda method

Problems posed by the Borda method

Condorcet's social choice theory

Condorcet method versus Borda method

5a Modernity and Schism in Understanding Democracy

American and French constitutions

Principles of political representation

Tocqueville's understanding of democracy.

Evolution of representative government in
the 19th and 20th centuries

Universal suffrage and democracy

5b Modernity and Schism in Understanding Democracy

General commitment to democracy in the modern Western world

Democratic perspectives: e-democracy and deliberative democracy

Voting theory in the age of representative government

From egalitarianism in voting to hierarchies in Arrow's model

P2 Theory

6a Direct Democracy

Politician (candidate for leader)	Questions q			Indicators, in %	
	1	2	3	P_c	U_c
	Remove powers from the Areopagus	Pay for political participation	Help Sparta to put down a rebellion	Popularity: average representativeness	Universality of representing a majority
Pericles' opinion	+	+	+		
Ephialtes' opinion	+	+	-		
Cimon's opinion	-	-	+		
Weight of protagonists in the society, in %	66.7	66.7	66.7		
Pericles' representativeness, in %	66.7	66.7	66.7	66.7	100.0
Ephialtes' representativeness, in %	66.7	66.7	33.3	55.6	66.7
Cimon's representativeness, in %	33.3	33.3	66.7	44.4	33.3
Average indicator values, in %				P=55.6	U=66.7

6b Direct Democracy

Geometric interpretation of the indices

Decisive bodies selected from the society

Ambiguous effects of the enlargement of
decisive bodies

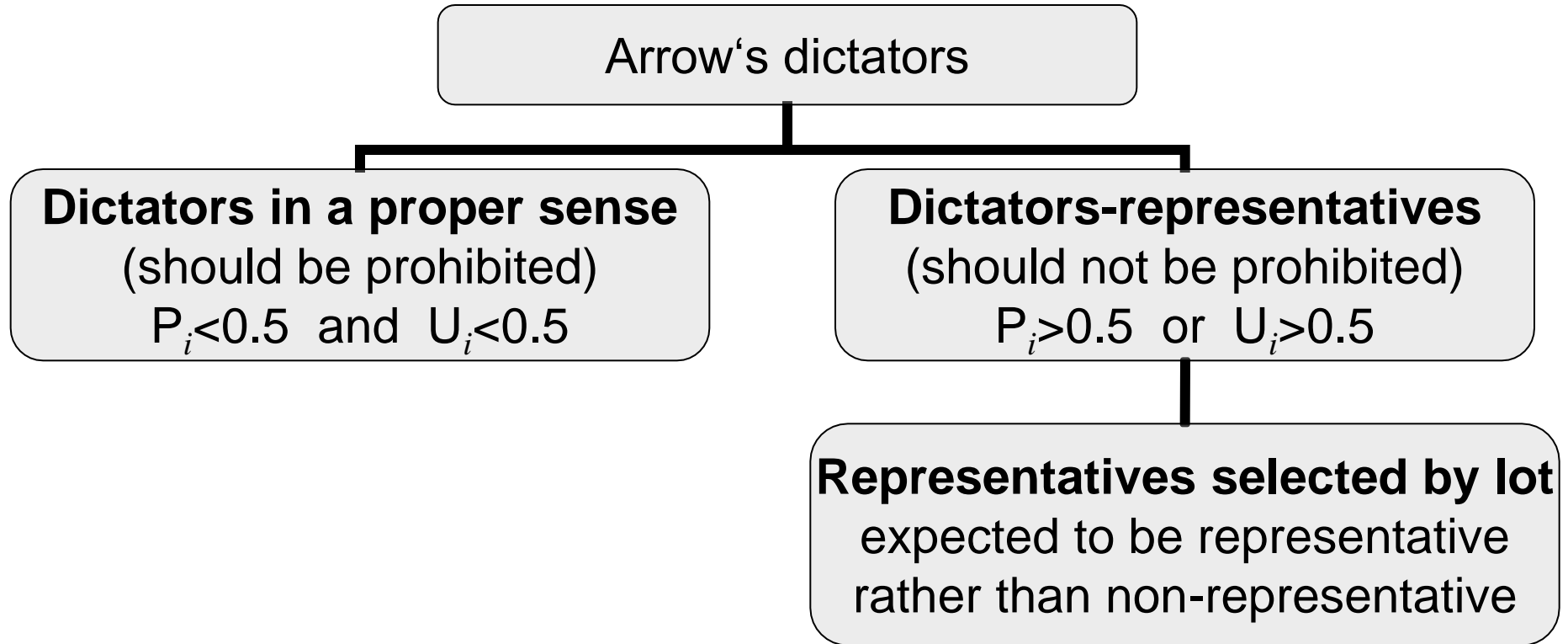
Magistrate versus parliament with regard to
their functions

Inefficiency of democracy in an unstable
society

7a Dictatorship and Democracy

Number of individuals	3
Number of alternatives	3
Number of preferences	13
Number of preference profiles	$13^3 = 2197$
Total number of questions (6 for a preference)	13182
Number of elements in the opinion matrix A	39546
Popularity of a dictator (mean % of individuals represented)	68.44%
Universality of a dictator (% of majority opinions represented by the dictator)	76.33%

7b Dictatorship and Democracy



The left-hand branch (Arrow's paradox) *can* be empty

The right-hand branch (no paradox) *is never* empty

7c Dictatorship and Democracy

When an event is taking place people express their opinions and wishes about it, and as the event results from the collective activity of many people, some one of the opinions or wishes expressed is sure to be fulfilled if but approximately. When one of the opinions expressed is fulfilled, that opinion gets connected with the event as a command preceding it.

Men are hauling a log. Each of them expresses his opinion as to how and where to haul it. They haul the log away, and it happens that this is done as one of them said. He ordered it. There we have command and power in their primary form.

L.Tolstoy. *War and Peace*

8 Representative Democracy

Representativeness of parties and Bundestag

Representativeness of party coalitions

Inconsistency of voters with own political profiles

Superficiality of voting by name (implicit paradox of voting)

9 Statistically Testing the Representative Capacity

Can a high representativeness be attained by chance?

Geometric method

Algebraic method

Probabilistic method

Interpolation methods

10 Bridging Representative and Direct Democracies

Leadership qualities	A	A	A	B	B	→	A
Competence & experience	A	A	B	A	B	→	A
Communication skills	A	A	B	B	A	→	A
	↓	↓	↓	↓	↓		↓
	A	A	B	B	B	→	B
							A

↓ → **Individual determination** (voting by name):

B wins with only 6 out of 15 partial votes

→ ↓ **Public determination** (voting by criteria):

A wins with 9 out of 15 partial votes

P3 Applications

11 Simple Applications

Why to consider non-political applications?

Planning a public opinion poll with no cyclic majorities

Planning a representative public opinion poll

Invitation to a round table.

Sufficiency of limited information for decision making

12 Application to Collective Multicriteria Decisions

Specifying travel alternatives for an outing

Selecting one travel alternative

Selecting several travel alternatives

Measuring the individual satisfaction

13 Application to Stock Exchange Predictions

Dow Jones stocks as representatives of DAX stocks

Testing standalone predictors

Testing the best predictor out of many

Testing a predictor selected by lot

Groups of Dow Jones stocks as collective predictors

14 Application to Traffic Control

Traffic forecasts using the model of representatives

Testing a standalone forecaster





Testing the best forecaster out of many

Testing a forecaster selected by lot

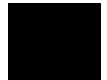




Groups of intersections as collective forecasters

Recent:

Election to the 2013 Bundestag

	Votes %
 CDU/CSU (conservators)	41.6
 SPD (social democrats)	25.8
 Left-Party (left social democrats & communists)	8.6
 Green (ecologists)	8.4
23 minor parties (<5% each)	15.7

Source data: 36 Y/N questions from Wahl-o-mat)

	Opinions of parties and unions					Question weights		Survey results,	
	 CDU 41.6	 SPD 25.8	 Linke 8.6	 Grüne 8.4	 DGB -	Google 7.10.13 log2	1nd expert 0-3	Prota- gonists %	Anta- gonists %
Minimul wage	No	Yes	Yes	Yes	Yes	2,020,000	3	86	12
Compensation for parents	Yes	No	No	No	No	1,010,000	2	20	77
Speed limit on Motorways	No	No	Yes	Yes	Yes	415,000	2	53	45
Retain EUR as German currency	Yes	Yes	Yes	Yes	Yes	676,000	2	69	27
State control over electricity tariff	No	Yes	Yes	?	Yes	95,600	2	90	10

Bundestag Factions

CDU/CSU

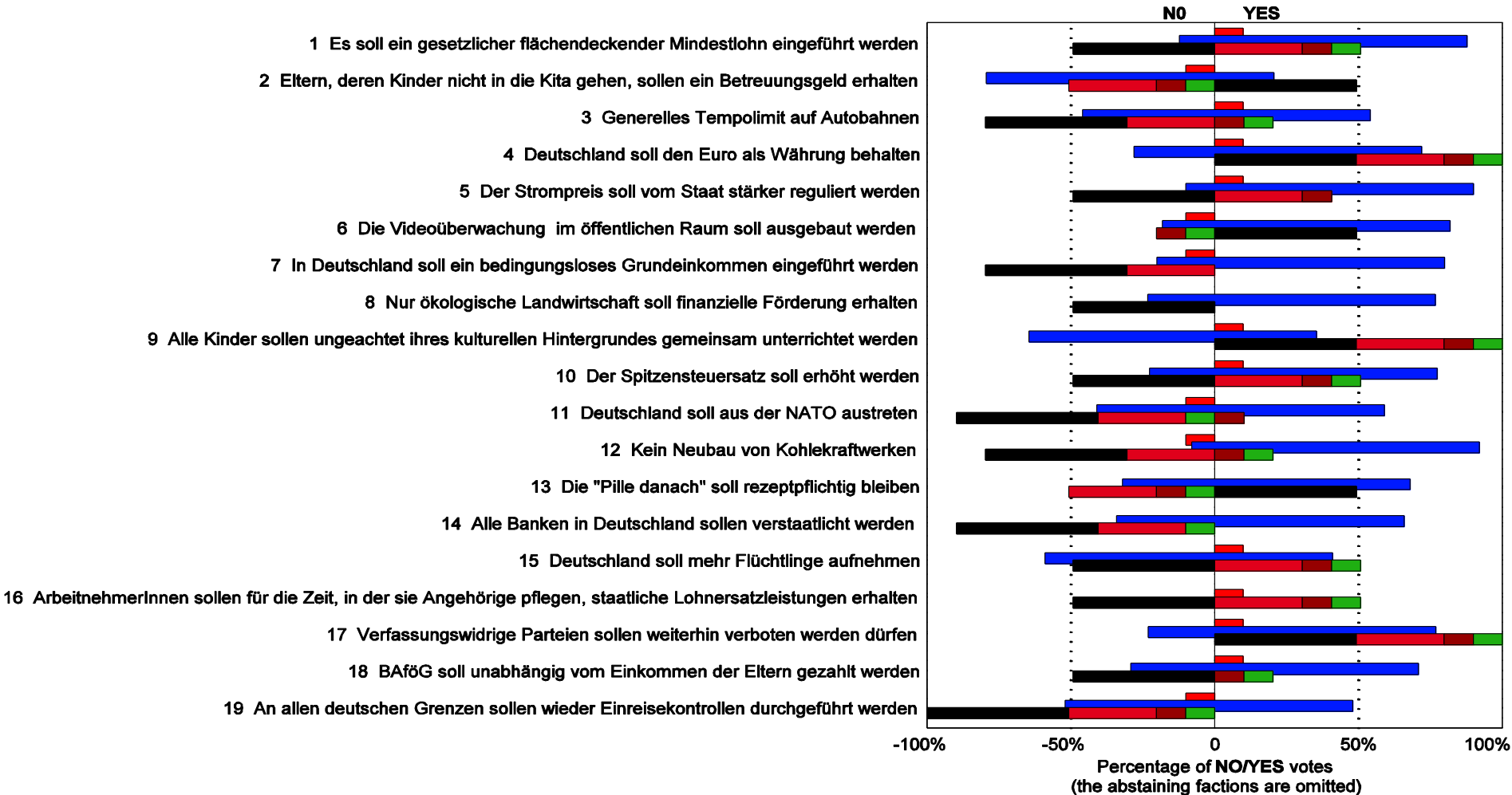
SPD

DIE LINKE

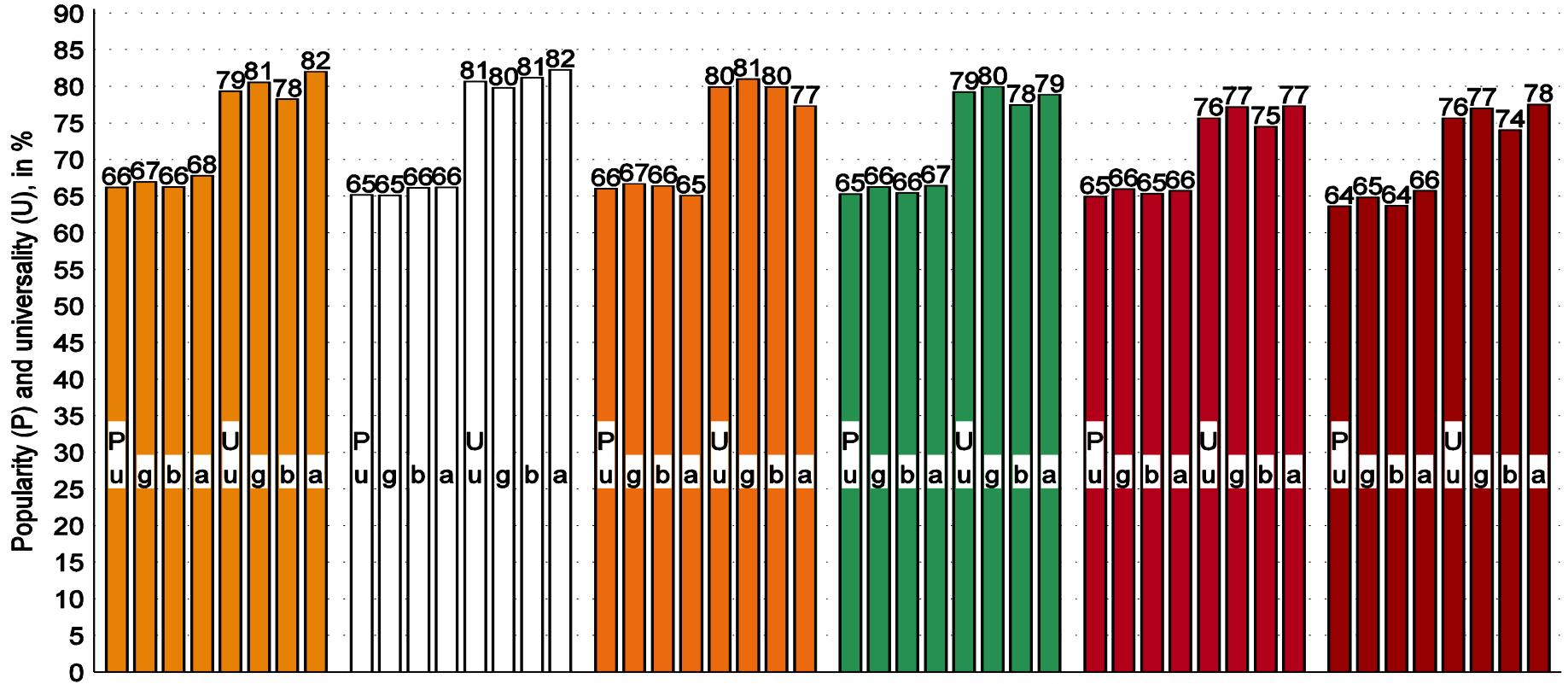
GRÜNE

DGB

Public opinion



Indices 1-6



Bündnis 21/RRP
 Mean index: 73.51
 Mean P: 66.89
 Mean U: 80.12
 Votes: 0.02%

Volksabstimmung
 Mean index: 73.40
 Mean P: 65.74
 Mean U: 81.07
 Votes: 0.07%

Nichtwähler
 Mean index: 72.88
 Mean P: 66.14
 Mean U: 79.63
 Votes: 0.03%

DIE FRAUEN
 Mean index: 72.46
 Mean P: 65.96
 Mean U: 78.96
 Votes: 0.03%

Die PARTEI
 Mean index: 70.92
 Mean P: 65.59
 Mean U: 76.26
 Votes: 0.18%

DIE LINKE
 Mean index: 70.36
 Mean P: 64.56
 Mean U: 76.15
 Votes: 8.59%



PARTEI DER NICHTWÄHLER
 DIE ALTERNATIVE ZUR STIMMENTHALTUNG



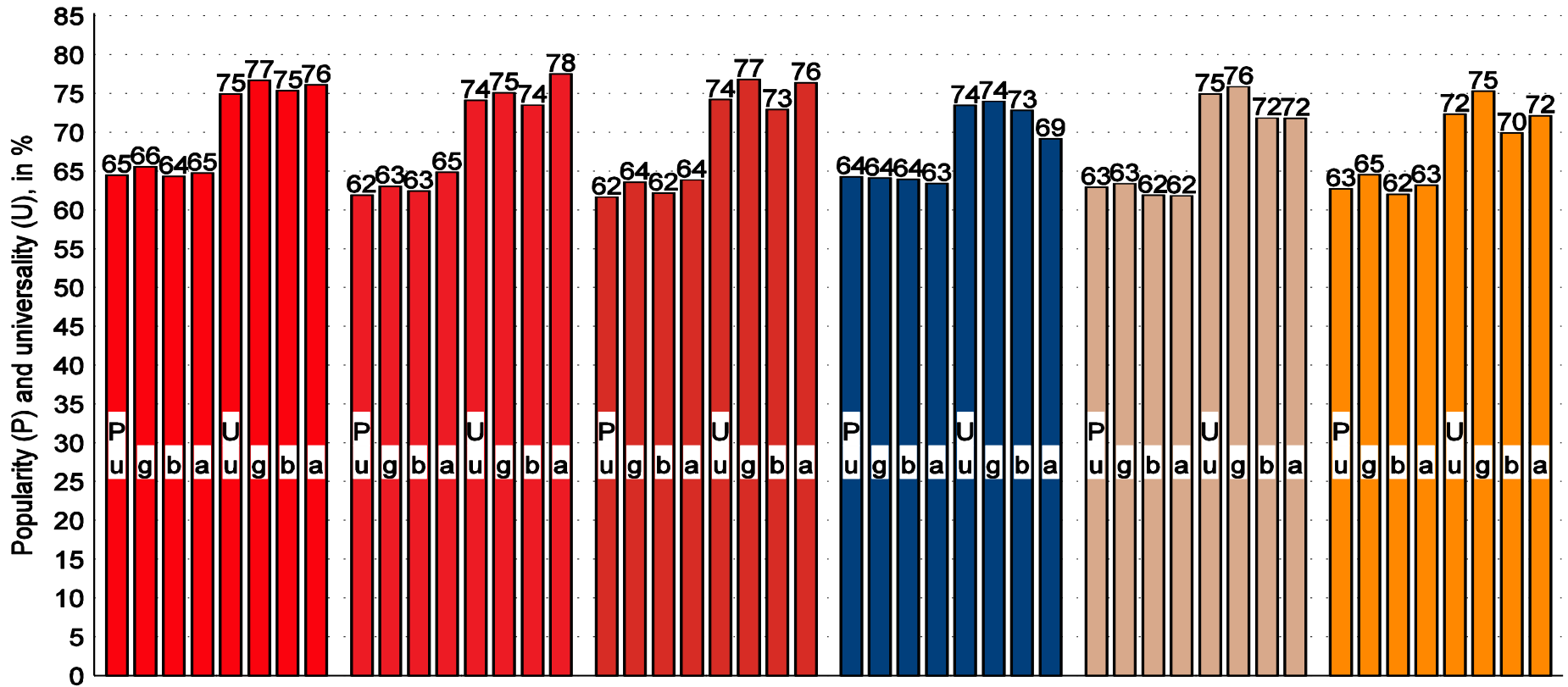
Feministische Partei
DIE FRAUEN



Die PARTEI

DIE LINKE.

Indices 7-12



NPD
 Mean index: 70.35
 Mean P: 64.85
 Mean U: 75.85
 Votes: 1.28%

MLPD
 Mean index: 69.14
 Mean P: 63.14
 Mean U: 75.13
 Votes: 0.06%

PSG
 Mean index: 69.03
 Mean P: 62.89
 Mean U: 75.17
 Votes: 0.01%

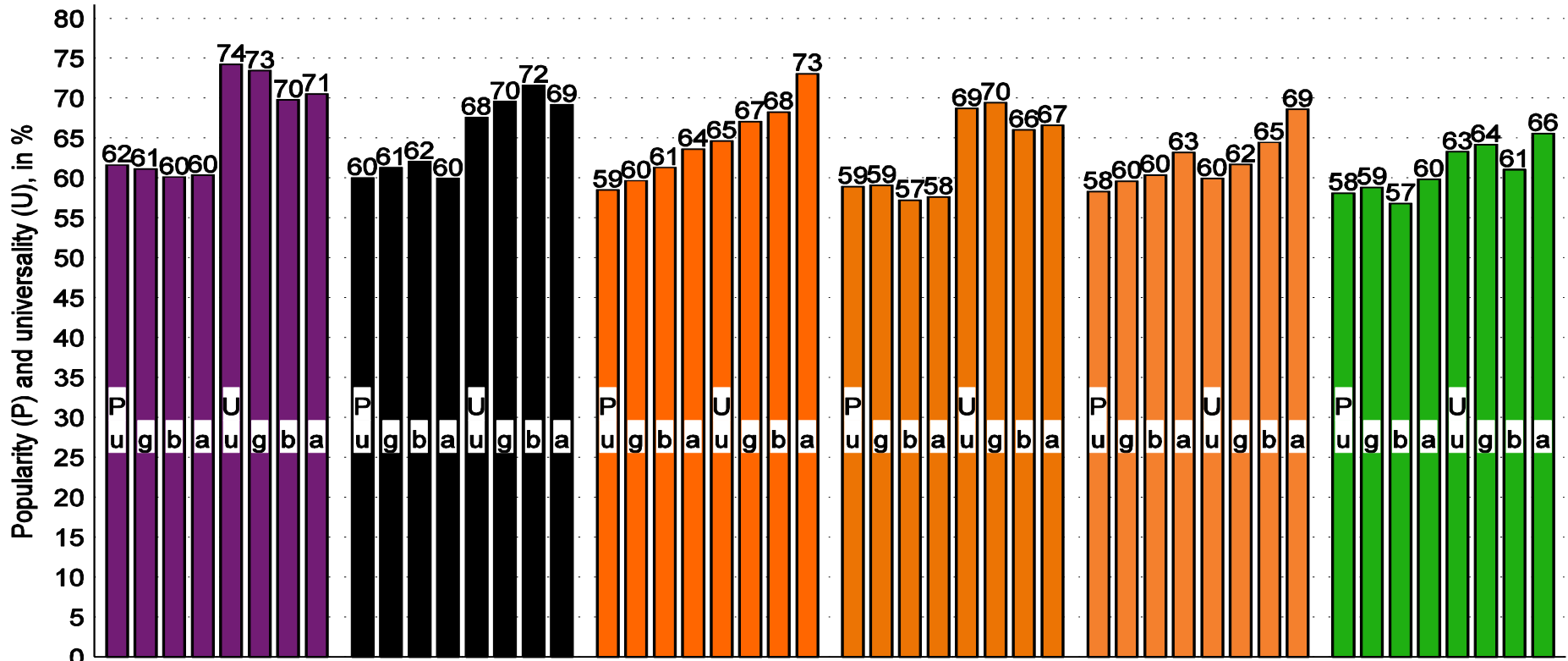
Tierschutzpartei
 Mean index: 68.22
 Mean P: 64.01
 Mean U: 72.43
 Votes: 0.32%

FAMILIE
 Mean index: 68.13
 Mean P: 62.57
 Mean U: 73.69
 Votes: 0.02%

PIRATEN
 Mean index: 67.84
 Mean P: 63.19
 Mean U: 72.50
 Votes: 2.19%



Indices 13-18



DIE VIOLETTEN
 Mean index: 66.46
 Mean P: 60.87
 Mean U: 72.05
 Votes: 0.02%

pro Deutschland
 Mean index: 65.21
 Mean P: 60.88
 Mean U: 69.54
 Votes: 0.17%

RENTNER
 Mean index: 64.58
 Mean P: 60.83
 Mean U: 68.33
 Votes: 0.06%

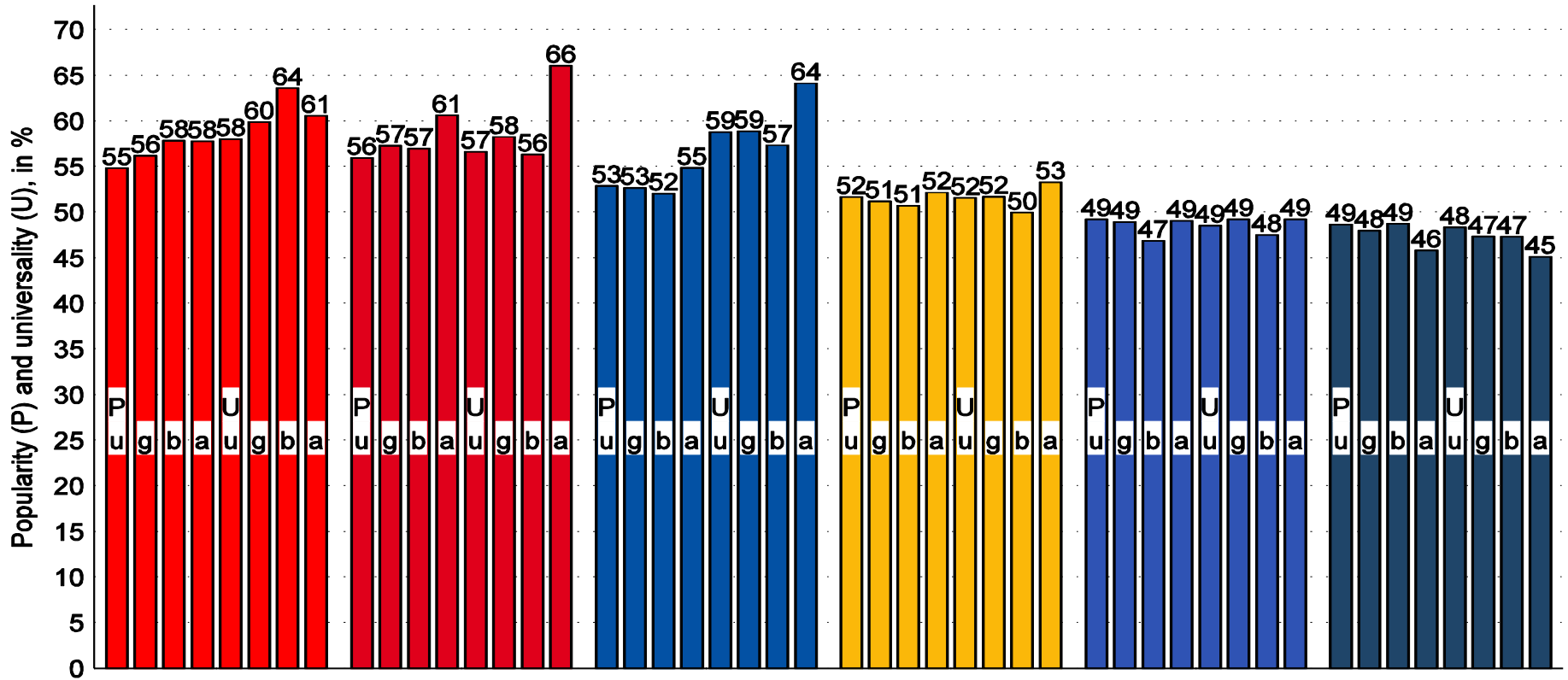
ÖDP
 Mean index: 63.02
 Mean P: 58.27
 Mean U: 67.76
 Votes: 0.29%

BIG
 Mean index: 62.08
 Mean P: 60.44
 Mean U: 63.73
 Votes: 0.04%

GRÜNE
 Mean index: 61.02
 Mean P: 58.46
 Mean U: 63.57
 Votes: 8.44%



Indices 19-24



DGB
 Mean index: 58.63
 Mean P: 56.71
 Mean U: 60.56
 Votes: 0.00%

SPD
 Mean index: 58.55
 Mean P: 57.75
 Mean U: 59.36
 Votes: 25.74%

FREIE WÄHLER
 Mean index: 56.49
 Mean P: 53.16
 Mean U: 59.83
 Votes: 0.97%

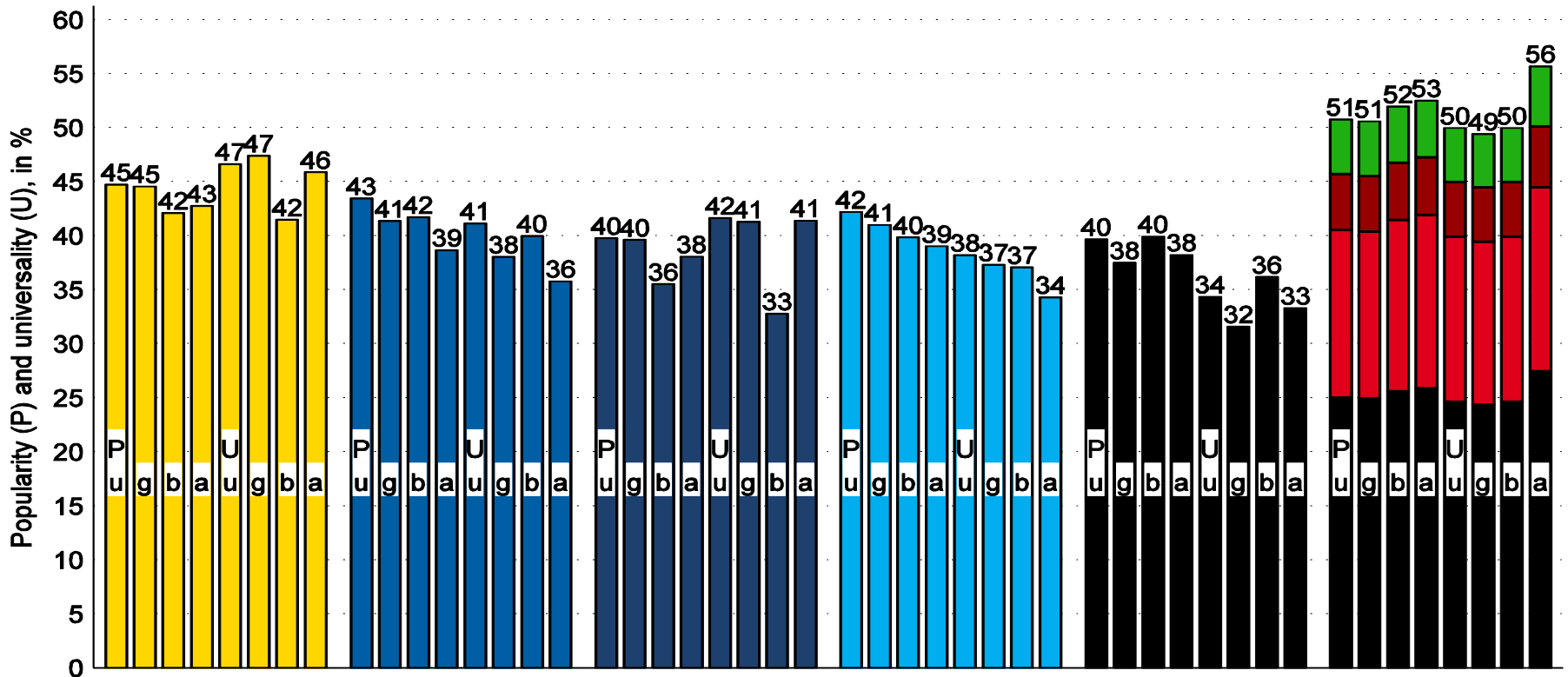
PBC
 Mean index: 51.57
 Mean P: 51.47
 Mean U: 51.67
 Votes: 0.04%

BP
 Mean index: 48.61
 Mean P: 48.56
 Mean U: 48.67
 Votes: 0.13%

BüSo
 Mean index: 47.46
 Mean P: 47.84
 Mean U: 47.07
 Votes: 0.03%



Indices 25-30



FDP
 Mean index: 44.47
 Mean P: 43.57
 Mean U: 45.38
 Votes: 4.76%

REP
 Mean index: 40.05
 Mean P: 41.33
 Mean U: 38.77
 Votes: 0.21%

Partei der Vernunft
 Mean index: 38.79
 Mean P: 38.27
 Mean U: 39.31
 Votes: 0.06%

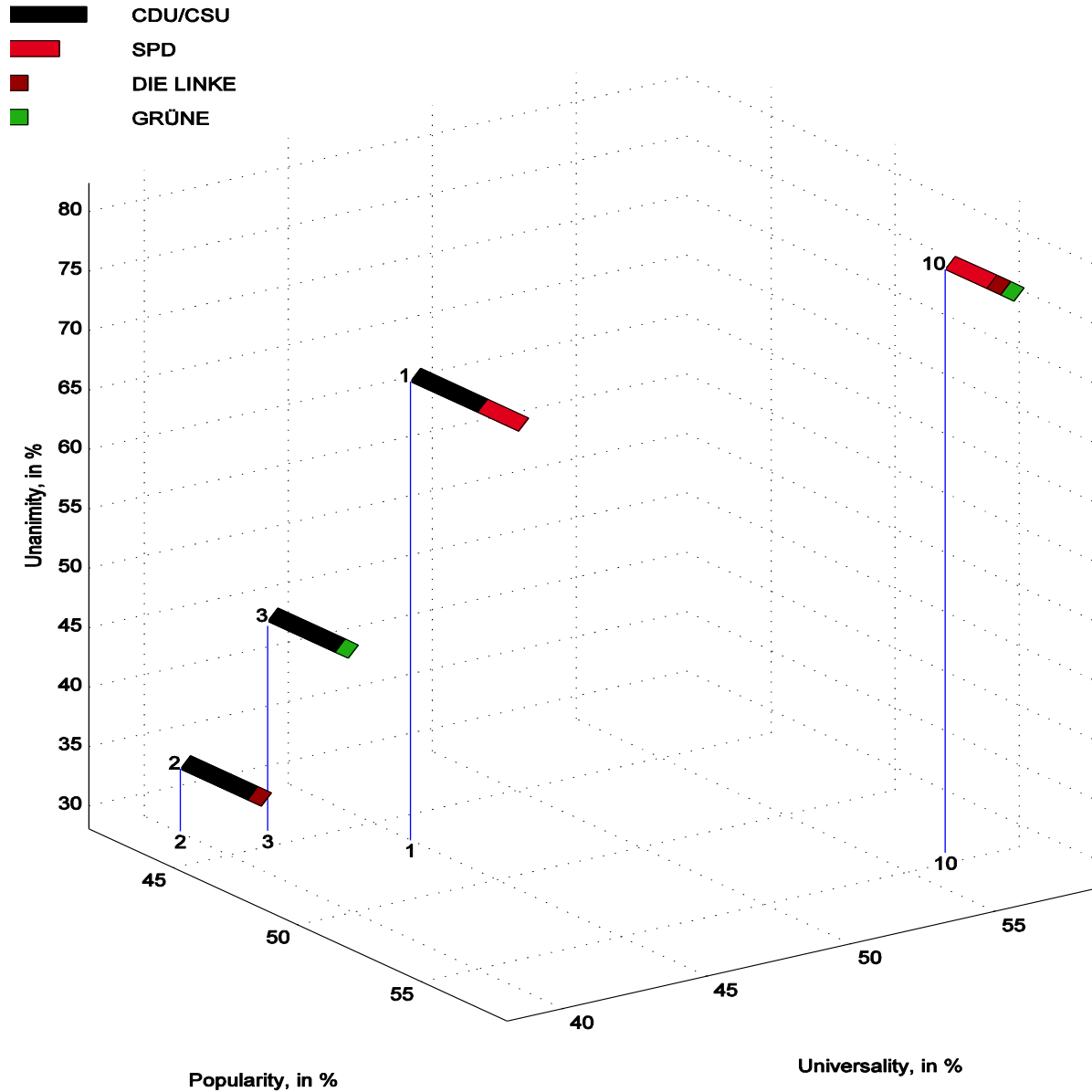
AfD
 Mean index: 38.65
 Mean P: 40.55
 Mean U: 36.75
 Votes: 4.70%

CDU/CSU
 Mean index: 36.37
 Mean P: 38.86
 Mean U: 33.88
 Votes: 41.55%

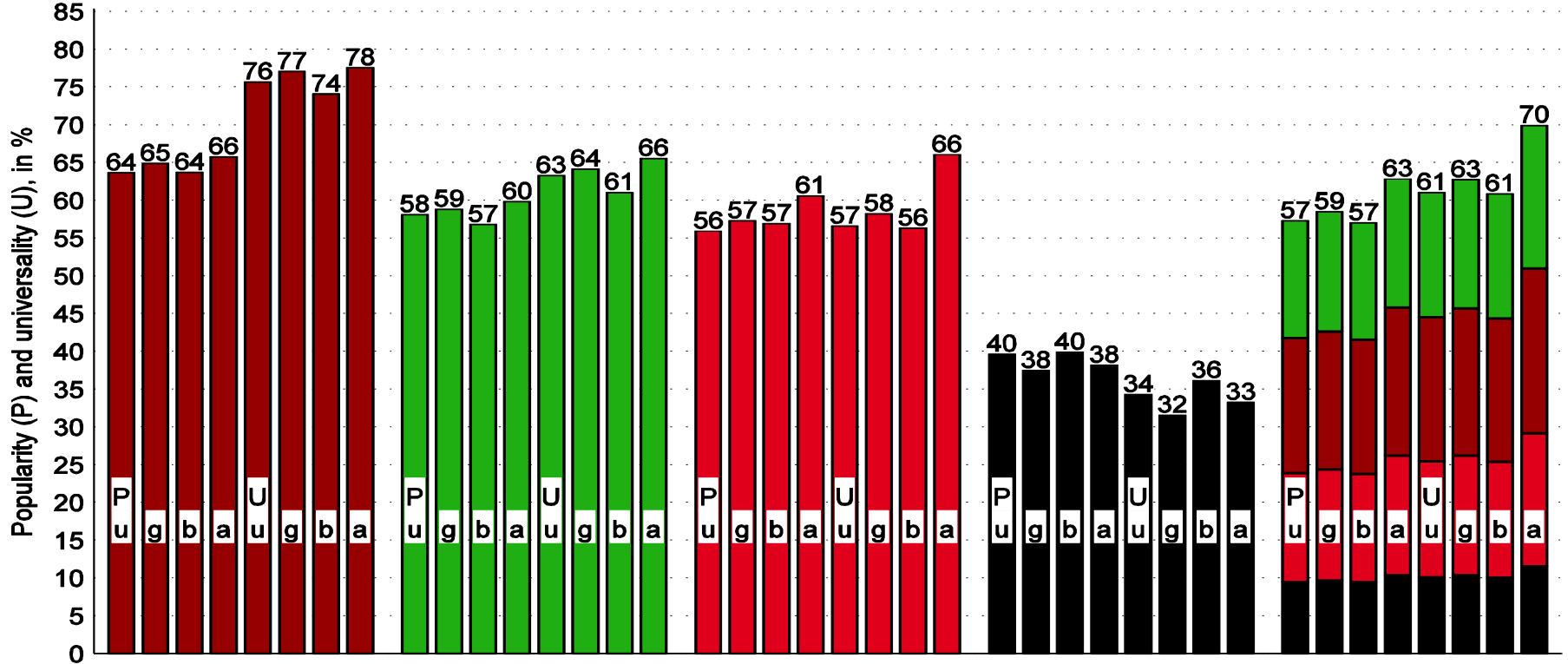
BUNDESTAG
 Mean index: 51.39
 Mean P: 51.48
 Mean U: 51.29
 Votes: 84.32%



Eligible coalitions



Alternative count: New Bundestag



DIE LINKE
 Mean index: 70.36
 Mean P: 64.56
 Mean U: 76.15
 New seats: 31.15%

GRÜNE
 Mean index: 61.02
 Mean P: 58.46
 Mean U: 63.57
 New seats: 27.14%

SPD
 Mean index: 58.55
 Mean P: 57.75
 Mean U: 59.36
 New seats: 25.16%

CDU/CSU
 Mean index: 36.37
 Mean P: 38.86
 Mean U: 33.88
 New seats: 16.55%

BUNDESTAG
 Mean index: 61.34
 Mean P: 58.97
 Mean U: 63.71
 New seats: 100.00%



Alternative count: New coalitions

