**Sphere packings, lattices, codes, and number theory**

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The problem of finding maximally dense sphere packings in the n dimensional Euclidian space $R^{n} $has been attracting the efforts of many mathematicians since the ancient times. The question is surprisingly difficult. For instance, a complete answer in the case of dimension 3 was obtained only in 1998 by Thomas Hales who succeeded in proving the conjecture on the optimality of the cannon ball packing, formulated by Johannes Kepler as early as in 1611. The corresponding problem in dimension 4 still remains unsolved.

In 2016 Maryna Viazovska made a spectacular breakthrough by proving the optimality of the $E\_{8}$ lattice packing in$ R^{8}$, as well as of the Leech lattice packing $Λ\_{24}$ in $R^{24}$, the last result being a joint work with H. Cohn, A. Kumar, S. D. Miller, and D. Radchenko. The proofs, in which methods from the theory of modular forms are used, are remarkably short and accessible, especially compared to the 300 pages proof of the Kepler’s conjecture.

The so called error correcting codes can be regarded as a finite analogue of sphere packings. Apart from their applications to the information transmission problems, the mathematical methods used to study them turn out to be remarkably rich and beautiful, ranging from combinatorics and analysis to number theory and algebraic geometry. Moreover, methods and results of the theory of error correcting codes turn out to have many applications to the sphere packing problem.

The main goal of the course is to provide an introduction to the theory of error correcting codes, lattices, and sphere packings by showing the wide variety of mathematical techniques used therein, with a particular accent on the modern number theoretical tools that seem to give the strongest results.

**Course Program**

1. The statement of the sphere packing problem.
2. Lattices and quadratic forms.
3. Some notions and results from the theory of error-correcting codes.
4. Construction of optimal packing low dimensions.
5. Linear programming approach to bounding parameters of codes and packings.
6. The kissing number problem.
7. Basic notions from the theory of modular forms.
8. Optimal sphere packings in dimension 8 and 24.
9. Asymptotic results on codes and packings.

**Prerequisites**

Familiarity with basic notions of analysis in $R^{n}$, one-dimensional complex analysis, and linear algebra.

**Final note**

An exam will be organized in the end of the course.