Fair Division: Microeconomics meets Computer Science

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formal modeling of fair division starts 70 years ago

- *mathematicians:* the cake-division model; Steinhaus 1948

- *game theorists:* axiomatic bargaining: Nash 1950; cooperative games: Shapley 1953

- *economists:* No Envy and fair competitive trade; Foley 1967, Varian 1974

key issue: equal division is *fair*, but typically *inefficient* (example: 3 toys for 2 children)

differences in individual preferences $\Longrightarrow$ opportunities for mutual benefits above and beyond equal split

how then to define fairness when individual shares must differ ?

other important issues

can the elicitation of preferences be incentive compatible ?

is our division rule easily computable ?

examples

family heirlooms: silverware, paintings

seats in overdemanded classes

family chores

work shifts, teaching loads

divorce, dissolution of a partnership: assets and liabilities more recent examples: *peer to peer* Fair Division on the Internet

to share computing resources

to share memory space

online barter for goods and services

the interface of microeconomics and internet science

- about 2000: Algorithmic Mechanism Design and Computational Social Choice

- the ACM *Electronic Commerce* conferences (EC) start in 1999, followed by the *Web and Internet Economics* conferences (WINE) in 2004, and by the *Computational Social Choice* conferences (COMSOC) in 2010

- in 2014 EC becomes the 15th *Economics and Computation* conference

- and the ACM Society launches a new journal: *Transactions on Economics and Computation*

Example 1: cake-cutting algorithms

as old as the hills: the Divide and Choose mechanism

- ensures *No Envy* (under any continuous preferences)
- requires only one cut and one query (no complicated report)
- efficiency is another matter

Problem: to generalize D&C to more than two agents, under additive utilities

- Selfridge algorithm does this for three agents, with at most 3 cuts and 5 queries

- Brams and Taylor (1995) find a general *crumbly* algorithm, with a potentially unbounded number of cuts

- Aziz and McKenzie (2016) find a bounded algorithm

but those algorithms produce impractical crumbs

while real life applications require topological (connected shares) and geometric (no gerrymandering) constraints

Example 2: dividing complementary inputs

cloud computing is managed by a "dispatcher" distributing CPUs, memory, bandwidth, etc., to simultaneous users

a typical user needs these resources in fixed proportion $\Longrightarrow$ the family of *Leontief preferences*

$$u_1(a_1, b_1, c_1) = \min\{\frac{a_1}{3}, \frac{b_1}{5}, \frac{c_1}{2}\}$$

Ivan needs 3 units of CPU (good a) for 1 of Memory (good b)

$$u_I(a_1, b_1) = \min\{\frac{a_1}{3}, b_1\}$$

Dimitri: $u_D(a_2, b_2) = \min\{\frac{a_2}{2}, b_2\}$

Yulia: $u_Y(a_3, b_3) = \min\{a_3, \frac{b_3}{2}\}$

to divide: 6 units of CPU and 4 units of Memory the *Egalitarian solution* equalizes the *relative utilities*

$$\frac{\text{utility of my share}}{\text{my utility of all the resources}}$$

easy to compute: find the critical *overdemanded* commodity (ies)

in the example it is CPU:

$$\frac{\frac{a_1}{3}}{\min\{\frac{6}{3}, 4\}} = \frac{\frac{a_2}{2}}{\min\{\frac{6}{2}, 4\}} = \frac{a_3}{\min\{6, \frac{4}{2}\}} = \frac{3}{7}$$

$$\Longrightarrow \text{ Ivan: } (\frac{18}{7}, \frac{6}{7}) \text{ ; Dimitri: } (\frac{18}{7}, \frac{9}{7}) \text{ ; Yulia: } (\frac{6}{7}, \frac{12}{7})$$

with some Memory to spare

this solution is miraculous (Ghodsi et al. 2011, Xue and Li 2013)

- everyone is guaranteed at least $\frac{1}{n}$-th of the whole cake: *Fair Share*
- the allocation is Envy-Free
- strategyproofness: nobody ever benefits from reporting incorrect ratios of needs
- ditto if a group of agents try a coordinated misreport
- the solution is easy to compute

Example 3: dividing substitutable goods

dividing assets in a divorce or dissolution of a partnership

contractors with substitutable skills divide a set of desirable jobs with different characteristics: teachers $\rightarrow$ classes, lawyers $\longrightarrow$ clients, etc..

additive utilities (preferences): fixed rates of substitution

$$u_1(a_1, b_1, c_1) = 3a_1 + b_1 + \frac{c_1}{2}$$

in practice: each participant must split 1000 points over the different goods the *Egalitarian* solution still equalizes $\frac{\text{utility of my share}}{\text{my utility for all the resources}}$

it is successfully challenged by

the *Competitive* solution

give the same budget to each person and find a price (necessarily unique) at which the competitive demands clear the resources

Ivan views 3 units of good a as equivalent to 1 of good b :

$$u_I(a_1, b_1) = a_1 + 3b_1$$

Dimitri: $u_D(a_2, b_2) = a_2 + 2b_2$

Yulia: $u_Y(a_3, b_3) = a_3 + b_3$

to divide: 40 unpopular goods (type a) and 80 popular ones (type b)

note: differences in tastes/preferences are subjective, agents held responsible for own tastes

Competitive division

utilities

	a (40)	b(80)
Ivan	1	3
Dima	1	2
Yulia	1	1

allocation

price	1	1
Ivan	0	40
Dima	0	40
Yulia	40	0

budget 40

Egalitarian division

utilities

	a (40)	b(80)
Ivan	1	3
Dima	1	2
Yulia	1	1

(rounded) allocation

	a	b
Ivan	0	36
Dima	0	38
Yulia	40	6

$\longrightarrow$ Ivan envies Dimitri

$\longrightarrow$ easy misreport of one's preferences: increase the relative worth of the good you do not get (nobody can misreport at the C solution, *for this particular example*)

$\longrightarrow$ No Envy at the C division is a weak form of incentive compatibility

the amazing Competitive solution

- maximizes the *Nash product of utilities* $\Longrightarrow$ essentially unique and easy to compute for any problem size

- picks an *Envy-free* allocation

- everyone benefits when the pile of goods increases (not true for EG)

- if a good becomes more attractive to me, I receive (weakly) more of this good (not true for EG)

- it is not strategyproof but no reasonable efficient rule can be

Example 4: dividing substitutable bads

family chores: cleaning, baby sitting, shopping

partners with substitutable skills divide a set of undesirable tasks with different characteristics: teachers $\rightarrow$ classes, lawyers $\longrightarrow$ clients, etc..

additive **dis***utilities*:

$$u_1(a_1, b_1, c_1) = 3a_1 + b_1 + \frac{c_1}{2}$$

To divide 40 popular bads (type a) and 80 unpopular ones (type b)

Ivan views 3 units of bad a as equivalent to 1 of bad b :

$$u_I(a_1, b_1) = a_1 + 3b_1$$

Dimitri: $u_D(a_2, b_2) = a_2 + 2b_2$

Yulia: $u_Y(a_3, b_3) = a_3 + b_3$

Egalitarian division

utilities

	a (40)	b(80)
Ivan	1	3
Dima	1	2
Yulia	1	1

(rounded) allocation

	a	b
Ivan	40	14
Dima	0	30
Yulia	0	36

where again there is Envy andagents have easy misreporting strategies *there are now two Competitive divisions !!*

disutilities

	a (40)	b(80)
Ivan	1	3
Dima	1	2
Yulia	1	1

allocation 1 — budget 40

price	1	1
Ivan	40	0
Dima	40	0
Yulia	0	40

(rounded) allocation 2 — budget 53

price	1	2
Ivan	53	0
Dima	27	13
Yulia	0	27

the Competitive solution is very appealing to divide goods

but when dividing chores (bads) the multiplicity issue is not an anomaly, and can be very severe

$\Longrightarrow$ *we do not know a normatively compelling single-valued competitive division of chores*

in fact *every single-valued efficient and envy-free division rule will be discontinuous in the utility parameters*

(Bogomolnaia, Moulin, Sandomirskiy and Yanovskaya (2017)) current research in Fair Division

- *the case of indivisible goods or bads*: dividing the family heirlooms: table, bicycle, Ipad, stuffed parrot,$\cdots$: how to approximate the desirable properties when the manna is divisible (Fair Share, No Envy, Competitive)

- *the assignment problem*: where each person must get a fixed total quantity of the items, goods or bads

Conclusion

- fair division methods eschews the need for property rights and direct bargaining or markets $\Longrightarrow$ they are centralized allocation rules with zero transaction costs

- implementation: free websites offering *provably fair* solutions: SPLIDDIT, Adjusted Winner

- currently limited to a handful of "iconic" division problems: sharing the rent between flatmates; sharing a taxi ride; distributing credit in a joint project;

abstract answers have the power of their normative properties

but only the adoption of these rules by real participants for real problems can vindicate them

Thank You