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Estimating the Needed Volume of Investment in a Public–Private Partnership to Develop a Regional Energy/Freight Transportation Infrastructure

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ABSTRACT
A decision-support tool for estimating the volume of investment in developing a regional energy/freight transportation infrastructure is proposed. The tool provides the estimates of the required investment volume and those of the expected amount of revenue that the infrastructure functioning may generate. These estimates are key ones in negotiations with private investors on forming a potential public–private partnership to finance the infrastructure development. The tool includes (a) a mathematical model underlying the formulations of three optimization problems on its basis depending on the information available to the decision-makers—two mixed programming problems and a minimax problem, which is proven to be reducible to a mixed programming one with all integer variables being Boolean, (b) a standard software package for solving mixed programming problems, and (c) a software package for processing data. The results of testing the proposed tool on sets of model data taken from open sources are discussed.

KEYWORDS
Access road; bilinear functions of vector arguments; investment in developing a regional energy/freight transportation infrastructure; minimax problems; mixed programming problems; public–private partnership; transportation hub

Introduction

One of the major functions of systems analysis consists of (a) finding similarities in systems of different natures, (b) studying classes of similar problems, and (c) presenting features attributed to all the problems from one and the same class. Transport and energy systems give examples of such different systems for which similarities in functioning are obvious and allow one to study this functioning on the basis of the same mathematical models. For instance, a problem of transmitting electricity from several power stations to several objects can mathematically be formulated as a (single-product) transportation problem. A battery swapping station providing services for several points (a) at each of which depleted batteries of different chemical foundations, capacities, and sizes are collected and delivered to this station, where these batteries are recharged, and (b) to each of which the recharged batteries are transported can be viewed as a transportation hub processing cargo. Here problems of the interaction of different types of the batteries with different types of the technological equipment deployed in recharging processes are similar to those of the interaction of different types of cargo transport at transportation hubs. Finally, various calculations for electric networks, especially for those with both traditional and renewable sources of electric energy, turn out to be similar to corresponding calculations for cargo transportation networks.

Though, certainly, energy and transportation have problems that are specific only either to one of these two kinds of systems or to the other, it seems reasonable to study problems that may appear in both of them. This is the case even if, currently, at least some of these problems are either the subject of separate intensive studies in each of these two systems or are studied in only one of them though they have already started drawing attention of researches studying the other one. Problems associated with creating and developing infrastructures in energy and transportation systems form a class of such problems.

Particularly, problems of (a) allocating transportation hubs in a region, (b) choosing the capacities of and the schemes of moving cargo via these hubs, (c) estimating the total expenses that are expected to be needed for all the construction and maintenance activities associated with these hubs, and (d) estimating the

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expected volume of the revenue that the functioning of these hubs may generate are typical for transportation systems. These problems are of interest to both public administrations and private investors. Thus, developing a tool that could allow one to make calculations associated with all the above-mentioned activities would help every public administration negotiate with private investors their potential involvement in large-scale projects important to society. Practically the same problems are to be considered with respect to battery swapping stations. That is, (a) allocating battery swapping stations within a particular region or municipality, (b) choosing the capacities of and the schemes of moving depleted and recharged batteries via these battery swapping stations, (c) estimating the total expenses that are expected to be needed for the construction and maintenance activities associated with these battery swapping stations, and (d) estimating the expected volume of the revenue that these battery swapping stations may generate look as problems almost identical to those to be considered for transportation hubs.

In the present article, all the problems under consideration are formulated in "transportation terms" only since this terminology seems to the authors to be more natural. However, one can easily reformulate these problems in "energy terms" in which the obtained results can be interpreted. The authors hope that publishing this article in the present collection of articles may draw the attention of (a) public administrations dealing with energy problems, particularly, with those associated with electric vehicles, and (b) applied mathematicians modeling energy problems.

In developing regional freight transportation infrastructures, building a set of new transportation hubs with access roads to them is one of the two key parts of an engineering project that regional administrations may offer to finance to their potential partners from the private sector. Another key part is associated with effectively managing thus developed transportation infrastructure. Only these two parts of a project on developing a regional freight transportation infrastructure—providing a set of construction and engineering works associated with building transportation hubs and access roads to them and effectively managing these elements of the freight transportation infrastructure—are the subject of consideration in the present article.

It is well known that applied mathematics and computer-aided systems help a great deal in analyzing and solving logistic, operational, and managerial problems in transportation systems. However, investment problems associated with the logistics underlying a regional freight transportation infrastructure have their own specifics. These specifics complicate the use of general financial engineering approaches in solving these strategic management problems, which every regional administration faces. So the question is: Can one provide a decision-support tool that would reflect these specifics in estimating the volume of investment needed by regional administrations, particularly, from the private sector, for developing and effectively managing regional transportation infrastructures?

This article demonstrates that with respect to developing regional freight transportation infrastructures, the answer is "yes," provided corresponding analytical means are properly chosen and correctly applied. Particularly, a mathematical model underlying a decision-support tool for estimating the needed volume of investment in developing a regional freight transportation infrastructure is proposed. This tool allows a regional administration to start negotiations with potential investors from the private sector on financing the corresponding project. The proposed model is a nonlinear generalization of the known facility location problem. It reflects the legal, engineering, and financial capabilities of the regional administration to offer to the private sector its cooperation in the framework of, for instance, a potential public–private partnership. On the basis of this (generalized) model, estimating the expenses associated with implementing the project can be done with the use of standard optimization software packages. Moreover, solutions to the corresponding optimization problems can quickly be obtained when a part of or even all the data reflecting the geography of a corresponding region can be known only approximately. This reflects the uncertainty conditions under which the above-mentioned expenses are estimated.

In addition to the Introduction, the article contains eight more sections and three Appendices.

Section I contains (a) the problem statement, (b) the features of mathematical models to formalize this problem in two situations depending on the assumptions on what information available to the regional authorities can be used in formalizing this problem, and (c) certain observations to bear in mind in solving optimization problems formulated on the basis of these models.

Section II presents a review of two groups of mathematical problems, close in formulation to those under consideration in this article, and a classification of these problems for one of the groups.

Section III provides mathematical formulations of the problems considered in Section I, and two forms of these mathematical formulations are proposed. The first form is a mathematical programming problem with mixed variables. This kind of optimization problems is a formalized description of the problem on estimating the investment volume needed for developing a regional freight transportation infrastructure under two scenarios. The first scenario takes place when all the components of the vectors of the coefficients in the goal (objective) function of the optimization problem are considered as known numbers. The second scenario appears when at least some components of these vectors, reflecting the demand on cargo flows in the region, are considered as variables. The second form of the problem mathematical formulation is a robust (minimax) optimization problem with mixed variables and the system of constraints having a linear structure. This form of a formalized description of the above-mentioned estimation problem is used when all the vectors of the coefficients in the goal function of the optimization problem are considered as variables. Solving this minimax problem allows the regional administration to estimate the investment volume in the "worst-case scenario" of the uncertain input data value combinations and to choose its best economic strategy in developing a regional freight transportation infrastructure.

Section IV presents the formulation of the Basic Assertion, which allows one to reduce the minimax problem, formulated in Section III, to a mixed programming problem with the goal function and constraints having a linear structure.

Section V discusses the results of testing the proposed decision-support tool (for estimating the investment volume needed to develop a regional freight transportation infrastructure) on model data. Several sets of the data needed to form the input information for both the mixed programming problems and the minimax problem, considered in Section III (soling which is reducible to solving a mixed programming problem), were prepared with the use of open sources. In the course of the testing, corresponding mixed programming problems were solved by the MATLAB software package, and solutions to these problems were compared. The applicability of the testing results in making economic decisions by regional administrations and the role of the proposed decision-support tool in making such decisions are discussed.

Section VI offers the authors' viewpoint on why the present article makes a contribution in the research field to which the topic of this article belongs. This section provides some estimates of the numbers of Boolean variables that may appear in real optimization problems formalizing the problems of estimating the investment volumes needed for developing freight transportation infrastructures. It contains methodological recommendations for using the proposed decision-support tool by both regional and federal administrations in their negotiations with potential investors from the private sector on forming public–private partnerships for developing freight transportation infrastructures. This section discusses the requirements that the decision-support tool should meet to be helpful in solving problems associated with developing regional freight transportation infrastructures.

Section VII briefly summarizes the research results reflected in the article.

Section VIII contains concluding remarks.

Appendix 1 offers the proof of the Basic Assertion from Section IV, Appendix 2 presents tables with numerical test results from Section V, and Appendix 3 illustrates some of these test results graphically.

The problem statement, features of mathematical models to formalize this problem, and specifics of optimization problems formulated on their basis

Usually, the geography of the region and the already existing country's transportation infrastructure there determine potential places in which new transportation hubs could be built. If this is the case, the expected volumes of, for instance, yearly cargo flows via these new hubs help roughly estimate the desirable capacities of the new hubs. However, the capacities of both new transportation hubs and access roads to them affect the distribution of the expected total cargo flows in the new freight transportation infrastructure as a whole, which is planned to be developed. So, in considering the development of a new regional freight transportation infrastructure, a decision-support tool for analyzing

- how many new transportation hubs should be built in the region,
- where these new transportation hubs should be located,
- what capacities the new transportation hubs and access roads to them should have,
- what schemes for moving cargo via new and already functioning transportation hubs and access roads to them could be viewed as optimal for the region and for the country as a whole,
- what total expenses associated with building new transportation hubs and access roads to them and with maintaining all the elements of the planned regional freight transportation infrastructure one should expect, and
- what volume of the revenue the planned regional freight transportation infrastructure should generate in the form of taxes to allow the regional administration to offer this revenue as (at least a part of) its financial contribution to, for instance, a public–private partnership with potential investors

would be extremely helpful for both federal and regional administrations. Such a tool would allow regional administrations to analyze the effectiveness of the regional freight transportation infrastructures, both existing and those to be developed to meet the demand for transportation services in the region.

The present article proposes a mathematical model underlying a variant of a decision-support tool capable of answering the above-listed six questions by determining

- optimal (from the regional administration's viewpoint) locations of new cargo transportation hubs in the region,
- total expenses associated with building both new transportation hubs in the chosen (optimal) locations and access roads to them, and
- the revenue expected to be generated by the functioning of thus developed regional freight transportation infrastructure in any planning period being of interest to the regional administration.

This determination is done by solving three optimization problems in each of which the following data are used as problem parameters for any particular planning period that interests the regional administration: (a) the total demand for cargo flows at each place of cargo origin, (b) the total demand for cargo at each cargo destination point, (c) the total demand for cargo flows in the region, (d) the cost values for building new transportation hubs, (e) the cost values for building new access roads to these new hubs, (f) the cost values for transporting cargoes between every place of the cargo origin and every point of the cargo destination via each transportation hub (both already existing and that to be built), and (g) the maintenance cost values for both the transportation hubs and access roads to them.

Depending on the values of which of the listed parameters are considered as known numbers by decision-makers, two situations should be considered.

Situation 1

Either the values of all the parameters listed in (a)–(g) or those of only a part of these parameters are considered by decision-makers to be known numbers for a particular planning period.

Situation 2

Only the areas to which the values of all the parameters listed in (a)–(g) belong are know to the decision-makers for a particular planning period.

If the values of the parameters listed in (a)–(g) are considered by decision-makers to be known numbers for a particular planning period (*Situation 1, Case A*), the decision-support tool makes the above-mentioned determination by solving a mathematical programming problem with mixed variables and with the constraints and the goal function having a linear structure, which can be solved by standard software packages for solving optimization problems with mixed variables. A different mathematical programming problem though of the same type–that is, a mathematical programming problem with mixed variables and with the linear constraints and the goal function having a linear structure—should be solved if only the areas to which the values of a part of the parameters, listed in (a)–(c), belong are known to the decision-makers (*Situation 1, Case B*).

If for all the parameters listed in (a)–(g), only the areas to which their values belong are known to the decision-makers (*Situation 2*), the decision-support tool makes the above-mentioned determination by solving a minimax problem. This minimax problem is the one with the system of constraints having a linear structure, mixed variables, and a bilinear goal function of two vector arguments one of which has only integer coordinates. Appendix 1 offers a proof that this minimax problem can be reduced to a mathematical programming problem with mixed variables and with the system of constraints and the goal function having a linear structure, which can also be solved by standard packages for solving mathematical programming problems with mixed variables.

For processing the available data to obtain the input information needed (a) for the calculations in both *Situation 1* and *Situation 2*, and (b) for graphically depicting the calculation results, other standard software tools should be used.

In both cases of *Situation 1*, the regional administration intends to spend as little as possible for (a) building both the new cargo transportation hubs and access roads to them, and (b) providing the maintenance for both the existing transportation hubs and access roads to them and the new ones. These are the expenses that the regional administration would like private investors to cover in the framework of a potential public–private partnership. At the same time, the regional administration offers its financial contribution to this potential partnership. This contribution (or a part of it) comes in the form of the expected cash flow volume to be generated by the taxes to be paid to the regional budget by cargo owners and cargo carriers. These regional taxes are to be paid by these customers of the (new) freight transportation infrastructure for using the transportation hubs and access roads to them as elements of this infrastructure.

Thus, the total expenses reduced by the expected amount of the revenue to be received in the form of the above-mentioned regional taxes are to be minimized in both cases of *Situation 1*. Corresponding optimization problems are mathematically formulated as mathematical programming ones with mixed variables and the system of constraints having a linear structure. In these two optimization problems, both the expenses and the revenue are mathematically described by linear functions of mixed variables. Solutions to these problems determine (a) an optimal location of new transportation hubs to be built, along with their optimal capacities, (b) an optimal set of access roads to these new transportation hubs to be built, along with their types and capacities, and (c) an optimal distribution of the cargo flows via both existing transportation hubs and the new ones corresponding to the goal function minimum in each case, i.e., in *Situation 1, Case A* and in *Situation 1, Case B*.

In *Situation 2*, the goal function in the third optimization problem is still the difference between the expenses and the expected amount of the revenue to be received in the form of the above-mentioned regional taxes. However, in this situation, the goal function of this optimization problem is mathematically described by the maximum function of the algebraic sum of five bilinear functions of vector arguments in finite-dimensional spaces.

Components of the vector arguments of these bilinear functions are

a. the expenses associated with (the cost values for) building new transportation hubs at each of the potential locations,
b. the expenses associated with (the cost values for) building access roads to new transportation hubs at these potential locations,
c. the maintenance expenses associated with (the maintenance cost values for) both the new transportation hubs and access roads to them,
d. the volumes of cargo transportation flows expected to be moved between all the transportation hubs of the developed transportation network (both existing and those to be built) and all the places of cargo origin/destination points,
e. Boolean variables determining whether a new transportation hub of a particular capacity should be built at a particular location, and
f. Boolean variables determining whether a new access road of a particular type to a new transportation hub should be built (or the existing access roads are sufficient to allow this transportation hub to function in full capacity) and the capacity of each new access road to be chosen to be built.

Continuous variables from the vectors mentioned in (a)–(d) of this (a)–(f) list belong to polyhedra described by compatible systems of linear equations and inequalities. It is natural to assume that each of the polyhedra to which each vector variable belongs is a subset of a parallelepiped in a corresponding Euclidean space. Boolean variables mentioned in (e) and those mentioned in (f) of the above (a)–(f) list form two vectors, each belonging to a unit cube (different for each of these two vectors) in a corresponding Euclidean space.

A review of scientific publications studying problems close to those under consideration in the present article

Scientific publications that are close to the subject of the present article form two groups. The first group includes publications traditionally considered in studies associated with the hub location problem in various formulations. The second group includes publications dealing with public–private partnership investments in developing transportation infrastructures. Both groups are briefly reviewed in this section of the article. For the first group of publications, the text to follow mostly only cites the papers in which brief or detailed reviews of the hub location problem studies are offered.

A review of publications on hub location problems

A variety of formulations of the hub location problems can be structured, for instance, based upon several characteristics of the hubs and the places to be connected with them. One can view these characteristics as parameters of the corresponding mathematical models.

(1) *The type of the mathematical formulation of the problem*. In the framework of a "discrete" formulation, places for hub locations in a region are to be chosen within a set of a finite number of particular places in the region. In the framework of a "continuous" formulation of the problem, the hubs can be placed anywhere in the region.
(2) *The goal function type in the optimization problem*. Two major types of the goal functions are usually considered: The maximum cost of services for all the origin-destination pairs that is to be minimized (the minimax criterion), and the sum of all the costs that is to be minimized (the mini-sum criterion). In addition to the costs, the goal function may include profits from providing services. Also, in some cases,

non-financial objectives, reflecting the service level, are among the criteria.

(3) *The available data on the number of hubs*. The number of hubs in a particular problem can be either an exogenous parameter or the one to be determined in the course of solving the problem.

(4) *The cost of placing the hubs*. Three types of the cost are considered in the hub location problems: The zero cost, the fixed cost, and the variable cost.

(5) *A connection type between the hubs and the places connected with the hub*. There are two types of the connection between the hubs and such places: a single connection and a multiple connection. Under the single connection, each place (a sender or a recipient) may be associated with (or assigned to) the only service hub. Under the multiple connection, each place can be connected with (assigned to) several service hubs.

(6) *The cost of connecting the hubs to the customers (places)*. As in the case of the cost of placing the hubs, three types of the connection cost are considered: The zero cost, the fixed cost, and the variable cost.

(7) *The existence of special conditions on the connections among the hubs (the types of subgraphs formed by sets of the hubs)*. Among major assumptions on such conditions, the four assumptions on a subgraph of the hubs—(a) a complete graph, (b) a star, (c) a tree, and (d) a line—dominate.

(8) *The existence of restrictions on the capacities of either the hubs or their connections with the places (or both).*

(9) *The existence of flows between particular origin-destination pairs in the sets of the hubs and in the set of the places connected to them.*

(10) *The existence of service level constraints.*

(11) *The existence of uncertainty in parameters of the network such as, for instance, costs and demands.*

(ReVelle & Swain, 1970) published one of the first papers in which the problem of optimally locating service centers in a region was studied. The problem formulated there has become known in applied mathematics as the "p-median problem," and it received this name due to its similarity with that of finding the median in a graph. (In (Hakimi, 1964), in the median graph problem, the median is understood as the graph vertex that minimizes the weighted sum of the distances between this vertex and all the other vertices of the graph.) (Daskin & Maas, 2015) consider the p-median problem in which a location of the service centers minimizing the average distance between the locations to be serviced and the nearest of the service centers to be placed is searched. (ReVelle & Swain, 1970) and (Cornuejols et al., 1990) analyzed this problem in the case of no capacity limitations put on the service centers to be placed though capacitated versions of the problem are also known. (Garey & Johnson, 1979) proved that, generally, all these problems are *NP*-hard.

Hub location problems have intensively been studied in the last several decades. Almost every recent publication, particularly, in the network analysis cites surveys on this subject in (Krarup & Pruzan, 1983), (Campbell, 1994a), (O'Kelly & Miller, 1994), (Labbé, Louveaux, Dell'Amico, Maffioli, & Martello, 1997), (Klincewicz, 1998), (Campbell et al., 2002), (Alumur & Kara, 2008), (Campbell & O'Kelly, 2012), (Farahani et al., 2013), (Contreras, 2015), and (Zabihi & Gharakhani, 2018).

Numerous publications consider the uncapacitated multiple allocation p-hub median problem (UMAp HMP), first presented in (Campbell, 1992). Its modifications are presented in (Campbell, 1994b), (Skorin-Kapov et al., 1996), including the uncapacitated multiple allocation hub location problem with fixed costs (UMAHLP), considered in (Campbell, 1994b). Exact and heuristics algorithms to solve these problems are proposed, for instance, in (Campbell, 1996), (Klincewicz, 1996), (Ernst & Krishnamoorthy, 1998a), (Ernst & Krishnamoorthy, 1998b), (Ebery et al., 2000), (Mayer & Wagner, 2002), (Boland et al., 2004), (Hamacher et al., 2004), (Marin, 2005), and (Canovas et al., 2007), and these algorithms are applicable to solving both the UMAHLP and the UMApHMP problems. A review of a number of heuristic algorithms for solving the p-median problem is presented in (Mladenovic et al., 2007).

Other hub location problems are formulated (a) for networks of particular structures such as a line structure (Martins de Sa et al., 2015), a tree structure (Contreras et al., 2010), a star structure (Labbe & Yaman, 2008), (Yaman, 2008), and (Yaman & Elloumi, 2012), structures with a particular number of connections (r-allocation) (Yaman, 2011), and structures with an incomplete hub network (Nickel et al., 2001), (Yoon & Current, 2008), (Calik et al., 2009), and (Alumur et al., 2009), (b) under a number of assumptions on the transportation cost and cargo flows such as the economies of scale (O'Kelly & Bryan, 1998), (Horner & O'Kelly, 2001), and (Camargo et al., 2009), different discounting policies (Podnar et al., 2002), (Campbell et al., 2005a), and (Campbell et al., 2005b), and under the presence of arcs with fixed setting costs (O'Kelly et al., 2015), (c) assuming a possibility to select the capacity of a hub (Correia et al., 2010), (d) for multimodal hub location problems with different transportation modes (Kelly & Lao, 1991), (Racunica & Wynter, 2005), (Limbourg & Jourquin, 2009), (Ishfaq & Sox, 2011), (Meng & Wang, 2011), and (Alumur et al., 2012a), (e) under price sensitive demands (O'Kelly et al., 2015), (f) assuming a sequential addition of competing hubs (Mahmutogullari & Kara, 2016), (g) for dynamic multi-period hub location problems (Gelareh et al., 2015), and (h) for hub-and-spoke models dealing with disruptions at the stage of designing transportation networks with backup hubs and alternative routes (An et al., 2015).

Most of the papers on the hub location problem consider the case in which all the data is assumed to be known exactly. In papers addressing the uncertainty in the data, the existence of particular probability distribution over the uncertain parameters is assumed (Marianov & Serra, 2003), (Sim et al., 2009), (Yang, 2009), (Contreras et al., 2011), (Alumur et al., 2012b), (Adibi and Razmi 2015), and (Yang, Yang, & Gao, 2016).

A recognized direction of dealing with the uncertainty in parameters of, particularly, networks, including transportation ones, in optimizing both network design and work consists of formulating corresponding problems as robust optimization ones. In these problems, the best solutions in the worst-case combination of parameters assuming values from particular sets is searched. Numerous authors, for instance, (Belenky, 1981), (Ben-Tal & Nemirovski, 1998), (Ben-Tal & Nemirovski, 1999), (Yaman et al., 2001), (Bertsimas & Sim, 2003), (Bertsimas & Sim, 2004), (Ben Tal et al., 2004), (Atamturk, 2006), (Ordonez & Zhao, 2007), (Yaman et al., 2007), (Ben-Tal & Nemirovski, 2008), (Mudchanatongsuk et al., 2008), (Shahabi & Unnikrishnan, 2014), (Merakli & Yaman, 2016, 2017), (Yang & Yang, 2017), (Zetina et al., 2017), and (Talbi & Todosijevic, 2017) exercise this approach for optimization problems in which sets of uncertain parameters are those described by systems of linear equations and inequalities.

Results that are close to those presented in this article are discussed in (Merakli & Yaman, 2016), (Serper & Alumur, 2016), and (Alibeyg et al., 2016). (Merakli & Yaman, 2016, 2017) propose a hub location model with a demand uncertainty described by systems of linear constraints. Similar to (Belenky, 1981), they formulate a minimax optimization problem on two polyhedra and apply the dual transformation to linearize it and find the best solution in the worst-case of the demand combinations. To solve the linearized problem on CAB, AP, and the Turkish network data, the CPLEX software package, along with particular variants of the Benders decomposition algorithms, is used. An approach presented in (Zetina, Contreras, & Cordeau, 2017) and (Talbi & Todosijevic, 2017) differs from those proposed in (Merakli & Yaman, 2016, 2017). That is, in (Zetina, Contreras, & Cordeau, 2017) and in (Talbi & Todosijevic, 2017), the change of some problem's parameters (demand, transportation cost) is allowed, and the objective is to find the best solution under the worst-case set of these parameters. The cardinality of this set can be interpreted as a budget of uncertainty, but this approach is limited and cannot be used for modeling complex relationships among uncertain parameters. (Serper & Alumur, 2016) consider the capacitated hub location model with different vehicle types and variable hub capacities. The model lets choose (a) transportation modes (air, ground) and the vehicle type (airplane, trailer, truck) for both hub-to-hub and hub-to-node transportation, and (b) the capacity level at a hub for each transportation mode.

An approach to modeling variable hub capacities is used in (Alumur et al., 2018), where the authors propose a framework for modeling congestions at hubs in hub location problems with a service time limit. (Alibeyg, Contreras, & Fernandez, 2016) introduce a class of hub network design problems with profit-oriented goal functions, which reflect the tradeoff between the profits obtained from moving the commodities and the costs of building transportation networks. In (Alibeyg et al., 2018), the authors propose an exact algorithmic framework for solving profit-oriented hub location problems. In this framework, a Lagrangian relaxation is used to obtain efficient bounds at the nodes in a branch-and-bound method taking into account the structure of the goal function. The resulting exact algorithms appear to solve more instances of the problems in a limited period of time than CPLEX can solve.

Also, there are publications that do not address the hub location problem itself while studying models related to those used in the hub location problem, which may eventually, be helpful in studying this problem. For instance, (Wang, 2016) presents a theoretical study of the optimal hubs network topology, and (Redondi et al., 2011), (Czerny et al., 2014), (Bracaglia et al., 2014), and (Teraji & Morimoto, 2014) consider a competition among the hubs. (Small & Ng, 2014) study optimization problems of choosing a capacity and the type of access roads to transportation hubs, whereas (Nagurney et al., 2015) and (Li & Nagurney, 2015) apply a game theory approach to finding equilibrium prices in supply chain networks under competition conditions. That is, (Nagurney, Saberi, Shukla, &

Floden, 2015) consider supply chain networks with competing manufacturers and freight service providers, whereas (Li & Nagurney, 2015) consider supply chain networks with competing suppliers of product components to be assembled by the purchasing firms, which may eventually manufacture some of these components on their own.

A review of publications on public–private partnership in transportation

(Rouhani et al., 2016) propose a particular framework for analyzing public–private partnership investment projects in transportation from the public welfare viewpoint. In these projects, a share of the revenue that is generated by transportation systems developed as a result of implementing any particular project is returned to the citizens who own the public infrastructure involved in the public–private partnership project. (Geddes & Nentchev, 2013) assert that such a strategy may increase a public support for a system-wide pricing of the existing roads. The proposed public welfare framework estimates the benefits and the costs of using the investment approach for an urban transportation network with respect to all the major project stakeholders (residents, users, government, and the private sector). (Carpintero & Siemiatycki, 2016) study how various political factors affect the formation and the effectiveness of public–private partnership projects with respect to Spain light rail transit projects, and conclude that they affect them significantly.

(Aerts et al., 2014) propose to use a multiactor analysis to identify factors being critical for success in implementing public–private partnerships in developing the infrastructure of a port. Based on the results presented in several surveys, the authors assert that (a) the concreteness and preciseness of the concession agreement, (b) the ability to appropriately allocate and share risk, (c) the technical feasibility of the project, (d) the commitment made by the partners, (e) the attractiveness of the financial package, (f) a clear definition of responsibilities, (g) the presence of a strong private consortium, and (h) a realistic cost/benefit assessment are such factors. (Panayides et al., 2015) also consider ports in a study of the influence of institutional factors on the effectiveness of the public–private partnership. An empirical analysis provided by the authors in their paper allows them to suggest that (a) “the regulatory quality, (b) the market openness, (c) the ease of starting a business, and (d) the enforcement contracts” are important institutional determinants of the effectiveness of port public–private partnership projects.

(Wang & Zhang, 2016) study a road pricing problem in networks belonging to public–private partnerships in the form of a game. Two types of the players are considered by the authors in their game model: (a) a set of individual travelers each of whom tries to find her/his own path with the minimal travel cost, and (b) a set of transportation firms that cooperate among themselves in an attempt to minimize the total operational cost for every firm. The model allows the authors to find road charging schemes for the players that yield the optimum flows for players of both kind. Also, several other publications dedicated to studying the road pricing problem are listed in that paper. Particularly, among the listed ones, there are (a) (Yang & Zhang, 2002), where the authors study the tolling design conducted to secure a certain level of the social equity, (b) (Sumalee & Xu, 2011), where the authors consider optimal pricing schemes under an uncertain demand for services on a transportation network, (c) (Zhang & Yang, 2004), where the authors research a cordon-based congestion pricing (determining the payment for the right to travel inside a particular city zone), (d) (Liu et al., 2014), where the authors analyze a model from (Zhang & Yang, 2004) and modify it to take into consideration both the travelling time and the parking time inside the zone, (e) (Zhang et al., 2008), where the authors suggest to determine particular prices as components of equilibria in a game model similar to (Wang & Zhang, 2016)—where a stochastic nature of the player payoff functions is taken into consideration—and (Meng et al., 2012)—where cordon-based congestion pricing problems are considered, and stochastic equilibria for heterogeneous users are analyzed.

(Zhang & Durango-Cohen, 2012) present a game-theoretic model of a concession agreement for examining how a government’s tax policy affects the interest of private investors to invest in a transportation infrastructure.

Organizational problems associated with forming public–private partnerships for Indian dry (inland) ports are reported in (Haralambides & Gujar, 2011) based on the interviews with various stakeholders that the authors have conducted. According to the authors, the excess capacity of the ports, limit pricing policies, and a weak legal framework for setting and running a public–private partnership are among the major obstacles in this field. (Cabrera et al., 2015) consider similar problems for ports in Spain, where the authors list what they believe are primary concerns for public–private partnership schemes in this area of freight transportation services. An improper risk allocation in tendering processes, the failure to meet expectations of the demand for services, and concerns associated with turning the transportation enterprise into a monopoly are listed and discussed there.

(Dementiev & Loboyko, 2014) propose a game-theoretic approach to analyzing the chances of establishing public–private partnerships in Russia's suburban railway sector of passenger transport. (Dementiev, 2016) further develops this approach by considering the idea to delegate some regulatory functions in public transportation to a public–private partnership. The implementation of this idea is expected to help balance social and commercial interests in line with a predetermined objective. Thus, this idea presents a certain theoretical interest from the viewpoint of welfare comparisons for alternative organizational structures in the public transport sector. Certain optimal corporate structures for a public–private partnership are determined there depending on local costs for public funds and society preferences.

(Carmona, 2010) considers general problems of developing a transportation infrastructure in a country in the context of economic regulations in public–private partnership settings. The author proposes to take into account three particular measures of the efficiency. That is, (a) the dynamic allocation efficiency (determining whether the whole life-cycle social benefits exceed the costs of the infrastructure provisions), (b) the utilization efficiency of the transportation infrastructure (determining whether charging the price that promotes the best possible use of available infrastructure capacity positively affects the infrastructure functioning), and (c) the productive efficiency (determining how the services provided by the road infrastructure minimize the transportation cost).

(Galilea & Medda, 2010) analyze to what extent and how economic and political statuses of a particular country (mainly the presence of corruption and democracy) may contribute to success of a public–private partnership in developing transportation infrastructures.

One should notice that most of the publications related to public–private partnership problems, in particular, in the field of transportation, are those of general considerations. These publications do not address quantitative approaches to studying issues underlying these problems. Certainly, there are publications in which mathematical models associated with forming public–private partnerships and analyzing their effectiveness are proposed for general interactions of the public and the private sector. Also, there are those related to such an interaction in areas other than transportation. However, neither take into consideration any specifics of transportation services considered, particularly, in the present article, and for this reason, they are not considered in the presented brief review.

For instance, (Belenky, 2014) considers such models in the form of three-person games in which a state, and investor, and a developer of a project (or a set of projects) interact—in an attempt to find a mutually acceptable conditions for the partnership. In those models the players proceed from (a) the minimum volume of investment required for each project from the viewpoint of the state, (b) the volume of investment that the state can afford to contribute, (c) the preferences and requirements of the developer for the compensation of its services, and (d) the volume of investment that the investor can afford to contribute. Some other financial factors are also taken into consideration. For this type of the games, under linear constraints describing a set of strategy for each player, necessary and sufficient conditions for the equilibria are established. These conditions allow one to find equilibria in the problem under consideration there by solving linear programming problems forming a dual pair. However, the results presented in that publication are not directly applicable to the problems under consideration in this article. This is the case due to the presence of Boolean variables in the model underlying mathematical formulations of the problems to be considered in Section III of the present article.

Thus, the presented review shows that there are classes of problems with the formulations being close to the problems mentioned in Section I, which are under consideration in this article. These close problems have not been modelled and studied in a manner allowing one to use results from the reviewed publications in working out decisions by the parties negotiating a potential private–public partnership on developing a regional freight transportation infrastructure.

Mathematical formulations of the problems under consideration

As mentioned in the Introduction, only financing the works associated with (a) building new transportation hubs and access roads to them, and (b) managing these elements of the new regional freight transportation infrastructure are the subject of the present article. This, particularly, means that neither the regional points of cargo origin/destination that do not use existing transportation hubs and will not use the new ones to be built nor access roads to these points, which are currently in use, are considered. Certainly, these points and these roads are elements of the existing freight transportation infrastructure and will remain such in the new one to be developed. However, it is assumed that they do not affect the demand for transportation services at the nodes of the regional transportation network to be connected with the transportation hubs from the freight transportation infrastructure to be developed. Thus, further in this article, developing the regional transportation infrastructure is understood as that of developing only a part of this infrastructure. This part consists of only the origin/

destination points connected with existing and with new transportation hubs to be built via existing and new access roads to these hubs. Here, some of the existing roads connecting the points that are not part of the regional freight transportation infrastructure with existing transportation hubs can, nevertheless, be currently used as access roads to these existing hubs (which are part of this infrastructure) to move cargoes to and from them. In the mathematical models considered in Section III, the capacities of these access roads (considered as parts of the regional freight transportation infrastructure to be developed) are those decreased by the flows of cargoes that are moved along these roads to and from the points on the network that are not existing transportation hubs.

Another feature of the mathematical models to be presented further in this section is associated with considering only "one type" of the cargoes moving via all the elements of the regional transportation infrastructure to be developed. Though, certainly, different types of cargoes are expected to be moved via these elements, these models operate with such parameters/variables as capacities of transportation hubs, capacities of access roads to them, and demands for cargo services at the cargo origin/destination points in the region for cargoes to be moved via these hubs. The models are not intended to be used to calculate any schemes of moving particular cargoes via elements of the freight transportation infrastructure to be developed. Usually, at the time of making strategic decisions on financing the development of new transportation hubs and access roads to them, neither the list of such cargoes nor the volumes of each cargo from such a list are known exactly. At the same time, (a) the expected total volumes of cargo to be moved via every transportation hub to be developed as part of the freight transportation infrastructure, and (b) the expected total volumes of cargo to be moved via each access road to each of these transportation hub can be estimated. Here, both exact expert estimates of these total volumes or at least the areas to which these total volumes are likely to belong can be obtained. The mathematical models proposed in this section of the article underlie optimization problems allowing the regional administration to receive the estimate of the investment needed to develop a new regional freight transportation infrastructure in both above cases of the available information about the total cargo volumes. One should, however, notice that the proposed mathematical models can easily incorporate any information about particular cargoes that the regional administration may eventually wish to take into account in calculating the investment estimates to be used in the course of negotiations with potential investors from the private sector.

Let

M be the number of locations (nodes) on the regional transportation network under consideration each of which is either a place of cargo origin or a cargo destination point (or both), for each of which both all coming in and all coming out cargoes are moved via transportation hubs,

N^{new} be the number of points (nodes) on the network suitable for locating new transportation hubs,

N^{exist} be the number of points (nodes) on the network with already functioning transportation hubs,

s_j be the expected yearly demand for (the volume of) cargo transportation services at node $j, j \in \overline{1,M}$ of the transportation network (from node j to transportation hubs in the new transportation network and from the hubs to that node),

s_j^{min} be the expected yearly minimal demand for cargo transportation services at node $j, j \in \overline{1,M}$ of the transportation network (from node j to transportation hubs in the new transportation network and from the hubs to that node),

s_j^{max} be the expected yearly maximal demand for cargo transportation services at node $j, j \in \overline{1,M}$ of the transportation network (from node j to transportation hubs in the new transportation network and from the hubs to that node),

S^{min} be the expected yearly minimal total demand for cargo transportation services at all the M locations in the planning period,

S^{max} be the expected yearly maximal total demand for cargo transportation services at all the M locations in the planning period,

ϵ_i be the number of variants of the yearly capacity that a new transportation hub to be built at node i may have, $i \in \overline{1, N^{new}}$,

μ be the number of the chosen variant of the transportation hub yearly capacity at node $i, i \in \overline{1, N^{new}}, \mu \in \overline{1, \epsilon_i}$,

$d_{i\mu}^{new}$ be the yearly capacity of a new transportation hub at node i under variant μ of the hub yearly capacity, $i \in \overline{1, N^{new}}, \mu \in \overline{1, \epsilon_i}$,

$d_{i\mu}^{new\ max}$ be the maximal yearly capacity of a new transportation hub at node i under variant μ of the hub yearly capacity, $i \in \overline{1, N^{new}}, \mu \in \overline{1, \epsilon_i}$,

$d_{i\mu}^{new\ min}$ be the minimal yearly capacity of a new transportation hub at node i under variant μ of the hub yearly capacity, $i \in \overline{1, N^{new}}, \mu \in \overline{1, \epsilon_i}$,

$d_{i'}^{exist}$ be the yearly capacity of the existing transportation hub at node $i', i' \in \overline{1, N^{exist}}$,

L_i be the number of types of all the new access roads to a new transportation hub at node $i, i \in \overline{1, N^{new}}$ that are planned to function on the transportation network as a result of its development during the planning period,

$l_{i'}$ be the number of types of all the access roads to the existing transportation hub at node $i', i' \in \overline{1, N^{exist}}$ that are planned to remain on the transportation network as a result of its development during the planning period,

$s_{ji}^{k\ new}$ be the yearly volume of cargo that is planned to be moved between node $j, j \in \overline{1, M}$ of the transportation network and a new transportation hub at node $i, i \in \overline{1, N^{new}}$ via a new access road of type $k, k \in \overline{1, L_i}$,

$s_{ji'}^{k'\ exist}$ be the yearly volume of cargo that is planned to be moved between node $j, j \in \overline{1, M}$ of the transportation network and the existing transportation hub at node $i', i' \in \overline{1, N^{exist}}$ via the existing access road of type $k', k' \in \overline{1, l_{i'}}$,

$t_{ji}^{k\ new}$ be the (average) cost of transporting a unit volume of cargo between node j and a new transportation hub at node i via a new access road of type k to the hub, which cargo owners and cargo carriers are expected to pay to the operators that will provide transportation services in the framework of the regional transportation infrastructure (which will act under an agreement with the regional transportation authorities or on their behalf) for the access to the new transportation hub at node $i, j \in \overline{1, M}, i \in \overline{1, N^{new}}, k \in \overline{1, L_i}$,

$t_{ji'}^{k'\ exist}$ be the (average) cost of transporting a unit volume of cargo between node j and the existing transportation hub at node i' via the existing access road of type k' to the hub, which cargo owners and cargo carriers are expected to pay to the operators of the regional transportation infrastructure (which will act under an agreement with the regional transportation authorities or on their behalf) for the access to the existing transportation hub at node $i', j, j \in \overline{1, M}, i' \in \overline{1, N^{exist}}, k' \in \overline{1, l_{i'}}$,

$Q_{i\mu}^{k\ new}$ be the yearly capacity of a new access road of type k to a new transportation hub at node i with the hub yearly capacity $d_{i\mu}^{new}$ on the transportation network, $k \in \overline{1, L_i}, \mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}$,

$Q_{i\mu}^{k\ new\ max}$ be the maximal yearly capacity of a new access road of type k to a new transportation hub at node i with the hub yearly capacity $d_{i\mu}^{maxnew}$ on the transportation network, $k \in \overline{1, L_i}, \mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}$,

$Q_{i\mu}^{k\ new\ min}$ be the minimal yearly capacity of a new access road of type k to a new transportation hub at node i with the hub yearly capacity $d_{i\mu}^{min\ new}$ on the transportation network, $k \in \overline{1, L_i}, \mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}$,

$Q_{i'}^{k'exist}$ be the yearly capacity of the existing access road of type k' to the existing transportation hub at node $i', k' \in \overline{1, l_{i'}}, i' \in \overline{1, N^{exist}}$,

$f_{i\mu}$ be the cost of building a new cargo transportation hub of variant μ of the hub yearly capacity at node i, $\mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}$ on the transportation network,

$g_{i\mu}^{k}$ be the cost of building a new access road of type k to a new transportation hub of variant μ of the hub yearly capacity at node i, $\mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}, k \in \overline{1, L_i}$.

$c_{i\mu}^{new}$ be the yearly maintenance cost of a new cargo transportation hub of variant μ of the hub yearly capacity at node i, $\mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}$ on the transportation network,

$q_{i\mu}^{k\ new}$ be the yearly maintenance cost of a new access road of type k to a new transportation hub of variant μ of the hub yearly capacity at node i, $\mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}, k \in \overline{1, L_i}$.

$c_{i'}^{exist}$ be the yearly maintenance cost of the existing cargo transportation hub at node i', $i' \in \overline{1, N^{exist}}$ on the transportation network, and

$q_{i'}^{k'\ exist}$ be the yearly maintenance cost of the existing access road of type k' to the existing transportation hub at node $i', k' \in \overline{1, l_{i'}}, i' \in \overline{1, N^{exist}}$.

Further, let

$y_{i\mu}$ be a binary (Boolean) variable that equals 1 if a new transportation hub of variant μ of the hub yearly capacity will be chosen to be built at node i and equals 0, otherwise, $\mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}$,

$z_{i\mu}^{k}$ be a binary (Boolean) variable that equals 1 if a new access road of type k to a new transportation hub of variant μ of the hub yearly capacity will be chosen to be built at node i and equals 0, otherwise, $k \in \overline{1, L_i}, \mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}$, and

ψ be the number of years in the planning period of time for which the regional administration is interested in estimating the economic effectiveness of developing a new regional freight transportation infrastructure.

Basic assumptions

(1) The cost of building a new transportation hub of variant μ of the hub yearly capacity at node i is a piecewise linear function of the hub yearly capacity so that for each segment $d_{i\mu}^{min\ new} \leq d_{i\mu}^{new} \leq d_{i\mu}^{max\ new}$, this cost is a linear function

$$f_{i\mu} = a_{i\mu} + \gamma_{i\mu} d_{i\mu}^{new}, \mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}.$$

where $a_{i\mu}, \gamma_{i\mu}$ are positive, real numbers $\mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}$, and the inequalities

$d_{i\mu}^{new\ max} \le d_{i(\mu+1)}^{new\ min}$ hold, $\mu \in \overline{1, \epsilon_i - 1}, \epsilon_i \ge 2$, $i \in \overline{1, N^{new}}$.

(2) The cost of building a new access road of type k to a new transportation hub of variant μ of the hub yearly capacity at node i is a piecewise linear function of the capacity of this road (which depends on the hub yearly capacity) so that for each segment $Q_{i\mu}^{k\ new\ min} \le Q_{i\mu}^{k\ new} \le Q_{i\mu}^{k\ new\ max}$, this cost is a linear function

$$g_{i\mu}^{k} = b_{i\mu}^{k} + \beta_{i\mu}^{k} Q_{i\mu}^{k\ new}, \mu \in \overline{1, \epsilon_i},$$

$$i \in \overline{1, N^{new}}, k \in \overline{1, L_i}.$$

where $b_{i\mu}, \beta_{i\mu}$ are positive real numbers $\mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}$, and the inequalities $Q_{i\mu}^{k\ new\ max} \le Q_{i(\mu+1)}^{k\ new\ min}$ hold, $\mu \in \overline{1, \epsilon_i - 1}, \epsilon_i \ge 2, i \in \overline{1, N^{new}}$.

One should bear in mind that access roads to a new transportation hub are those connecting this hub to the closest element of the existing regional transportation network (a highway segment, a railroad segment, etc.) rather than any new roads to be built to connect any node j to this hub. These existing elements can certainly be considered as access roads to the corresponding points on the transportation network under consideration. It is assumed that each such point has at least one access road of any particular type $k', k' \in \overline{1, l_{i'}}$. In calculating the cost t_{ji}^{knew}, the total length of the road between node j and transportation hub i, the length of the access road of type k to hub i, and other parameters affecting the cost are taken into account.

Also, the assumption on a piecewise structure of both costs reflects two features of this type of approximation. First, it allows one to approximate any particular continuous function of one variable that may appear in transportation practice with any needed degree of accuracy (by increasing the number of segments on each of which the function is approximated by a linear one). Second, it helps remain within linear or mixed programming with the systems of constraints and goal functions having a linear structure in formulating both standard and robust optimization problems formalizing practical problems under consideration in this article, which makes a difference in calculating solutions to these problems.

(3) The expected minimum and maximum yearly demands for (the volumes of) cargo transportation services at node j of the regional transportation network are strictly positive, real numbers $\forall j \in \overline{1, M}$ so that the inequalities

$$0 < s_j^{min} \le s_j \le s_j^{max}, \forall j \in \overline{1, M}$$

hold.

(4) The inequalities $\sum_{i'=1}^{N^{exist}} d_{i'}^{exist} < S^{min} < S^{max}$, and $Q_{i\mu}^{k\ new\ max} \gg L_i, \mu \in \overline{1, \epsilon_i}, k \in \overline{1, L_i}, i \in \overline{1, N^{new}}$ hold. No new access roads to already existing transportation hubs will be built, and no modernization construction work will be done there in the planning period.

(5) The number of types of new access roads that can be built to a new transportation hub at node $i, i \in \overline{1, N^{new}}$ to choose from does not depend on the hub yearly capacity. At the same time, the capacities of the new access roads to a new transportation hub chosen to be built may depend on the hub yearly capacity.

(6) Cargo flows may originate inside every new transportation hub and inside every existing transportation hub, and they may go to any node of the transportation of network consisting of M nodes under consideration.

(7) The amount of the cash flow formed by the taxes to be charged for providing access to the transportation infrastructure of the region is calculated as a particular percentage (v) of the corresponding transportation tariffs. These tariffs are those expected to be paid by the cargo owners to the transportation carriers based on the situation in the market of transportation services. This percentage is considered to be the same for the whole planning period of time ψ (in years), where $\psi \ge 1$.

(8) In negotiations with potential private sector partners, the regional administration chooses an arbitrary length of the planning period ψ for which it estimates the expenses associated with developing the regional freight transportation infrastructure. It proceeds from the yearly capacities of the new transportation hubs and new access roads to them to be built during that period. However, the planning period starts once all the new elements of this infrastructure or any particular elements of it (selected by the regional administration) have been built and start functioning.

(9) The functioning of the regional freight transportation infrastructure generates revenue, and to the regional administration, this revenue comes in the form of taxes. These taxes start coming in once all the facilities (new transportation hubs and new access roads to them) expected to be built in the planning period have been built.

In Table A1–A6, reflecting the results of testing the proposed tool on the model data (see Appendix 2), both this revenue and the profit/loss that the potential partnership may receive/sustain are calculated for different periods of time. These time segments begin from the moment at which all the above facilities start their cargo operations (i.e., for different $\psi, \psi \geq 1$).

Let the regional administration determine that the existing transportation infrastructure cannot meet the expected demand for moving cargoes in the region in principle.

Situation 1

Based upon this determination, the administration then intents

- to find out what new transportation hubs should be built, where these hubs should be located, what types of access roads to each of them, how many, and of what yearly capacities should be built,
- to analyze the expediency of keeping the existing distribution scheme for at least some of the cargo flows between the M nodes on the regional transportation network and the existing freight transportation hubs (which is done by estimating the results of possibly redistributing the existing cargo flows by switching portions of these flows to new transportation hubs that are planned to be built), and
- to analyze the expediency of possibly directing parts of the expected new cargo flows to some of (or to all) the existing transportation hubs.

These estimates and this analysis should be done to determine an economic strategy of developing the regional freight transportation infrastructure. This strategy much depends on the ability of the regional administration to obtain federal funds to support this project. It also depends on the administration's ability to convince private investors to contribute to this project on acceptable (to them and to the administration) conditions in the framework of, for instance, a public–private partnership.

Let the regional administration know the values that the parameters $t_{ji'}^{k' exist}$, $t_{ji}^{k\ new}$, $f_{i\mu}$, $g_{i\mu}^{k}$, $c_{i\mu}^{new}$, $q_{i\mu}^{k\ new}$, $c_{i'}^{exist}$, and $q_{i'}^{k'\ exist}$ may assume in the planning period. Then it can estimate the expected total expenses associated with developing the regional freight transportation infrastructure and redistributing the existing cargo flows between the existing transportation hubs and those to be built. This can be done by minimizing the function describing these expenses on the set of feasible solutions to the system of constraints binding the variables $s_{ji}^{k\ new}$, $s_{ji'}^{k' exist}$, $y_{i\mu}$, and $z_{i\mu}^{k}$.

For $\psi \geq 1$, this function takes the form

$$\psi\left(\sum_{i'=1}^{N^{exist}} c_{i'}^{exist} + \sum_{i'=1}^{N^{exist}} \sum_{k'=1}^{l_{i'}} q_{i'}^{k' exist}\right) + \sum_{i=1}^{N^{new}} \sum_{\mu=1}^{\epsilon_i} \left(f_{i\mu} + \psi c_{i\mu}^{new}\right) y_{i\mu}$$
$$+ \sum_{i=1}^{N^{new}} \sum_{k=1}^{L_i} \sum_{\mu=1}^{\epsilon_i} \left(g_{i\mu}^{k} + \psi q_{i\mu}^{k\ new}\right) z_{i\mu}^{k}$$
$$- \nu\psi\left(\sum_{i'=1}^{N^{exist}} \sum_{j=1}^{M} \sum_{k'=1}^{l_{i'}} t_{ji'}^{k'\ exist} s_{ji'}^{k'\ exist} + \sum_{i=1}^{N^{new}} \sum_{j=1}^{M} \sum_{k=1}^{L_i} t_{ji}^{k\ new} s_{ji}^{k\ new}\right)$$

For the sake of simplifying the reasoning on mathematically modelling the problem under consideration, it is assumed that for every year of the planning period, each of the parameters $t_{ji'}^{k'\ exist}$, $t_{ji}^{k\ new}$, $f_{i\mu}$, $g_{i\mu}^{k}$, $c_{i\mu}^{new}$, $q_{i\mu}^{k\ new}$, $c_{i'}^{exist}$, and $q_{i'}^{k'\ exist}$ assumes the same value. This assumption, however, is not restrictive, and one can consider any values of these parameters for each particular year and add corresponding terms into all the three components in the above formula.

Situation 2

A deal with potential investors on a public–private partnership associated with developing a regional freight transportation infrastructure is the major priority for the regional administration. However, the version of this infrastructure may substantially depend on what the private sector investors may be interested in considering as the investment subject. That is, depending on what the providing of transportation services may bring to the service providers, the investors may become interested in both developing the infrastructure and providing these services. Thus, the potential investors may also be interested in signing, for instance, a concession agreement with the regional administration on operating the transportation network that is to be built thanks to their investment. Then the situation changes compared with the one in which the development of the regional freight transportation infrastructure is considered as the only subject of the private–public partnership with the investors. That is, the taxes expected to be paid by the providers of transportation services to the regional administration will no longer be considered by the administration as its financial contribution to this partnership. As a signee to the concession agreement, the investors will receive a license to provide transportation services by hiring transportation and other companies to work with both cargo owners and cargo recipients. In this capacity, the private investors will be responsible for paying taxes to the regional administration. In exchange, they will be entitled to receive a profit from providing transportation services (provided this profit may exist in principle).

Certainly, the administration is interested in having a decision-support tool that would allow it to estimate the investor expenses in both situations. It is obvious that the second state of affairs (in which paying taxes becomes the responsibility of the private investors contributing to the development of the regional freight transportation infrastructure) is covered by the previous reasoning. That is, the goal function in the optimization problems to be solved to this end will differ from the ones to be solved in *Situation 1* and *Situation 2*, described in Section I, only by the sign before its third term (which will be plus instead of minus; see in the above how this function looks in the optimization problems to be solved in both cases of *Situation 1*, described in Section I).

Remark 1

For the sake of uniformity of definitions, two different cases in *Situation 2*, considered in the above—that is, the case in which the potential investors make their investments only in the development of the regional transportation infrastructure and the case in which these investors also sign a concession agreement with the regional administration on operating the transportation network that is to be built thanks to their investment—are further on called *Subcase 1* and *Subcase 2* instead of *Case A* and *Case B*, as this was done in considering two different cases in Section I.

The reason to consider *Case A* and *Case B* in *Situation 1* is associated with the difference in the mathematical models underlying the formulations of the optimization problems to be solved by the decision-support system in these cases. That is, in *Case A*, the values of all the (listed in (a)–(g) in Section I) parameters in the mathematical model that is used in *Situation 1* are considered to be the know numbers by the decision-makers. In contrast, in *Case B*, the values of only a part of these parameters, listed in (d)–(g) from the list (a)–(g) in Section I, are considered to be the known numbers by the decision-makers, whereas for the parameters listed in (a)–(c) from the list of (a)–(g) in Section I, only the areas to which their values belong are known to the decision-makers. However, in both *Case A* and *Case B* of *Situation 1*, both *Subcase 1* and *Subcase 2,* associated with signing or not signing a concession agreement, are to be considered in just the same way this takes place in *Situation 2*.

In *Situation 2*, only the areas to which the values of all the parameters listed in (a)–(g) from Section I belong are known to the decision-makers, so one and the same system of constraints in the mathematical model is used in the formulations of the optimization problems to be solved by the decision-support system in this situation. Thus, *Subcase 1* and *Subcase 2* are to be considered for the only case in *Situation 2*.

One should, however, bear in mind, that though the mathematical models are presented in this article only for *Subcase 1* in both *Situation 1, Case A* and *Situation 1, Case B* and only for *Subcase 1* in *Situation 2*, the calculations on model data (see Section V) are conducted for *Subcase 1* and *Subcase 2* in both *Situation 1, Case A* and *Situation 1, Case B* and for *Subcase 1* and *Subcase 2* in *Situation 2*.

Finding the minimum of the goal function, considered in "*Situation 1, Case B, Subcase 1,*" requires solving a mathematical programming problem with mixed variables. The problem to be solved takes the form

$$\sum_{i=1}^{N^{new}}\sum_{\mu=1}^{\epsilon_i}\left(f_{i\mu}+\psi c_{i\mu}^{new}\right)y_{i\mu} + \sum_{i=1}^{N^{new}}\sum_{k=1}^{L_i}\sum_{\mu=1}^{\epsilon_i}\left(g_{i\mu}^k+\psi q_{i\mu}^{k\ new}\right)z_{i\mu}^k -$$

$$\nu\psi\left(\sum_{i'=1}^{N^{exist}}\sum_{j=1}^{M}\sum_{k'=1}^{l_{i'}} t_{ji'}^{k'\,exist} s_{ji'}^{k'\,exist} + \sum_{i=1}^{N^{new}}\sum_{j=1}^{M}\sum_{k=1}^{L_i} t_{ji}^{k\ new} s_{ji}^{k\ new}\right) \to \min, \quad (1)$$

$$\sum_{\mu=1}^{\varepsilon_i} y_{i\mu} \le 1, i \in \overline{1, N^{new}}, \quad (2)$$

$$\sum_{\mu=1}^{\epsilon_i} z_{i\mu}^k \le, i \in \overline{1, N^{new}}, k \in \overline{1, L_i}, \quad (3)$$

$$\sum_{k\in L_i} z_{i\mu}^k \le |L_i| y_{i\mu}, \mu \in \overline{1,\epsilon_i}, i \in \overline{1, N^{new}} \quad (4)$$

$$y_{i\mu} \le \sum_{k=1}^{L_i} z_{i\mu}^k \le \left(\sum_{k=1}^{L_i} Q_{i\mu}^{k\ new\ max}\right) y_{i\mu}, \quad i \in \overline{1, N^{new}}, \mu \in \overline{1, \varepsilon_i}, \quad (5)$$

$$\sum_{j=1}^{M}\sum_{k=1}^{L_i} s_{ji}^{k\ new} \leq \sum_{\mu=1}^{\epsilon_i} y_{i\mu} d_{i\mu}^{new\ max}, i \in \overline{1, N^{new}}, \quad (6)$$

$$\sum_{j=1}^{M} s_{ji}^{k\ new} \leq \sum_{\mu=1}^{\varepsilon_i} z_{i\mu}^{k} Q_{i\mu}^{k\ new\ max}, \quad i \in \overline{1, N^{new}}, k \in \overline{1, L_i}, \quad (7)$$

$$\sum_{j=1}^{M} s_{ji'}^{k'\ exist} \leq Q_{i'}^{k'\ exist}, k' \in \overline{1, l_{i'}}, i' \in \overline{1, N^{exist}}, \quad (8)$$

$$\sum_{i=1}^{N^{new}}\sum_{k=1}^{L_i} s_{ji}^{k\ new} + \sum_{i'=1}^{N^{exist}}\sum_{k'=1}^{l_{i'}} s_{ji'}^{k'\ exist} = s_j, j \in \overline{1, M}, \quad (9)$$

$$s_j^{min} \leq s_j \leq s_j^{max}, j \in \overline{1, M}, \quad (10)$$

$$S^{min} \leq \sum_{j=1}^{M} s_j \leq S^{max} \quad (11)$$

$$\sum_{j=1}^{M}\sum_{k'=1}^{l_{i'}} s_{ji'}^{k'\ exist} \leq d_{i'}^{exist}, i' \in \overline{1, N^{exist}} \quad (12)$$

$$s_{ji}^{k\ new} \geq 0, i \in \overline{1, N^{new}}, j \in \overline{1, M}, k \in \overline{1, L_i}, \quad (13.1)$$

$$s_{ji'}^{k' exist} \geq 0, i' \in \overline{1, N^{exist}}, j \in \overline{1, M}, k' \in \overline{1, l_{i'}}, \quad (13.2)$$

$$y_{i\mu} \in \{0, 1\}, \mu \in \overline{1, \epsilon_i}, i \in \overline{1, N^{new}}, \quad (14.1)$$

$$z_{i\mu}^{k} \in \{0, 1\}, \mu \in \overline{1, \epsilon_i}, k \in \overline{1, L_i}, i \in \overline{1, N^{new}}. \quad (14.2)$$

The constraints in Problems (1)–(14) have the following meaning:

(2) no more than one new transportation hub (of any variant of the hub yearly capacity) can be located at each place from the set $\overline{1, N^{new}}$,

(3) no more than one new access road of each of L_i types to a new transportation hub at place $i, i \in \overline{1, N^{new}}$ (whose yearly capacity corresponds to the hub yearly capacity) can be built,

(4) new access roads of all L_i types to a new transportation hub at place $i, i \in \overline{1, N^{mew}}$ can be built for every chosen variant of the yearly capacity of this hub,

(5) for any variant of the yearly capacity $\mu \in \overline{1, \epsilon_i}$ of a new hub that is to be built at place $i, i \in \overline{1, N^{new}}$, an access road to the hub of at least one type (corresponding to the yearly capacity of this hub) is to be built,

(6) a yearly cargo flow via every new transportation hub that is to be built at place $i, i \in \overline{1, N^{new}}$ cannot exceed the yearly capacity of this hub,

(7) a yearly cargo flow via a new transportation hub at place $i, i \in \overline{1, N^{new}}$ that goes via a new access road to this hub of any particular type of this road (which yearly capacity corresponds to the chosen variant of the hub yearly capacity) cannot exceed the yearly capacity of this road,

(8) a yearly cargo flow via every existing access road of type $k' \in \overline{1, l_{i'}}$ to existing transportation hub $i', i' \in \overline{1, N^{exist}}$ cannot exceed the yearly capacity of this road,

(9) an expected yearly cargo flow via location $j, j \in \overline{1, M}$ equals the sum of the yearly cargo flows that go via this location via both the existing transportation hubs and new ones to be built,

(10) each yearly cargo flow volume $s_j, j \in \overline{1, M}$, can vary within certain known limits,

(11) the total yearly volume of the cargo flow via all the M locations is to remain within certain known limits,

(12) a yearly cargo flow via every existing transportation hub $i', i' \in \overline{1, N^{exist}}$ cannot exceed the yearly capacity of this hub,

(13.1), (13.2) all the continuous variables in the problem are non-negative,

(14.1), (14.2) all the integer variables in the problem are Boolean.

In the formulation of Problem (1)–(14), the first term in the expression for the function describing the expected total expenses associated with developing the regional freight transportation infrastructure and redistributing the existing cargo flows between the existing transportation hubs and those to be built (see this expression earlier in this section of the article) is not present in (1). This term, which describes the total maintenance cost associated with the existing transportation hubs and access roads to them, is a positive real number. The absence of this constant in the goal function of Problem (1)–(14) affects only the values of this function while not affecting the feasible solution (or feasible solutions) to this problem at which this value is attained.

Problem (1)–(14) is a mathematical programming one with mixed variables, which corresponds to *Situation 1*, *Case B*, described in Section I. For *Situation 1, Case A* (described in Section I), inequalities (10) are to be replaced by the equalities $s_j = s_j^*$, where a particular value of s_j^* such that the inequalities $s_j^{min} \le s_j^* \le s_j^{max}$ hold can be chosen by the regional administration for any particular calculation, $j \in \overline{1, M}$. Additionally, inequalities (11) are to be excluded from the system of constraints of Problem (1)–(14) (since for $S^* = \sum_{j=1}^{M} s_j^*$, the inequalities $S_j^{min} \le S^* \le S_j^{max}$ must hold). In both *Situation 1, Case B* and *Situation 1, Case A*, this problem can be solved with the use of standard software packages that are currently widely available (see, for instance, (Mittelmann, 2019)).

Since even the average values of the parameters in Problem (1)–(14) may not be known with certainty, the administration may decide to estimate the expected minimal total expenses under consideration in the "worst-case scenario." To this end, the estimates of the areas of the parameter values that these parameter values may belong to (in any particular planning period) should be taken into consideration. This is also the case when (a) the parameters $t_{ji}^{k\ exist}$, $t_{ji}^{k\ new}$, $f_{i\mu}$, and $g_{i\mu}^k$ are the piecewise linear functions described earlier, and (b) the parameters determining the maintenance costs for the new transportation hubs and those for access roads to these hubs may vary.

In such situations, a robust optimization problem should be formulated and solved.

Let

$$x = \left(s_{ij}^{k\ new}, s_{i'j}^{k\ 'exist}\right) \in R_+^{M\sum_{i=1}^{N^{new}} L_i + M\sum_{i'=1}^{N^{exist}} l_{i'}},$$

$$t = \left(t_{ij}^{k\ new}, t_{i'j}^{k\ 'exist}\right) \in R_+^{M\sum_{i=1}^{N^{new}} L_i + M\sum_{i'=1}^{N^{exist}} l_{i'}},$$

$y = (y_{i\mu}) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i}$, $f = (f_{i\mu}) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i}$, $c = \left(c_{i\mu}^{new}\right) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i}$ $g = \left(g_{i\mu}^{k}\right) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i L_i}$, $q = \left(q_{i\mu}^{knew}\right) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i L_i}$, $z = \left(z_{i\mu}^{k}\right) \in R_+^{\sum_{i=1}^{N^{new}} \epsilon_i L_i}$ be vector variables, and let the inclusions

$$t \in \Lambda = \{t \ge 0 : tI \le l\}, f \in \Theta = \{f \ge 0 : fF \le r\},$$

$$g \in \Gamma = \{g \ge 0 : gG \le e\},$$

$$c \in \Delta = \{c \ge 0, cW \le \lambda\}, q \in \Upsilon = \{q \ge 0, q\Psi \le \eta\},$$

$$x \in MX = \{x \ge 0 : Ax \ge b\},$$

$$y \in \Omega Y = \left\{ y \ge 0 : By \ge \pi, y \in T_{\sum_{i=1}^{N^{new}} \epsilon_i} \right\},$$

$$z \in HZ = \left\{ z \ge 0 : Kz \ge h, z \in T_{\sum_{i=1}^{N^{new}} L_i \epsilon_i} \right\},$$

$$(x, y, z) \in \Phi = \{(x, y, z) \ge 0 : P(x, y, z) \ge \delta\},$$

where $I, F, G, W, \Psi, P, A, B, K$ are matrices and $l, r, e, \lambda, \eta, \delta, b, \pi, h$ are vectors of corresponding dimensions, $T_{\sum_{i=1}^{N^{new}} \epsilon_i}$ is a unit cube in $R_+^{\sum_{i=1}^{N^{new}} \epsilon_i}$, and $T_{\sum_{i=1}^{N^{new}} L_i \epsilon_i}$ is a unit cube in $R_+^{\sum_{i=1}^{N^{new}} \epsilon_i L_i}$, hold.

Here, it is assumed that (a) the sets $MX, \Lambda, \Theta, \Gamma, \Delta$, and Υ are (nonempty) polyhedra in Euclidean spaces of corresponding dimensions, that is, the systems of linear inequalities describing these sets are compatible, (b) each of the sets ΩY and HZ is a subset of a convex polyhedron in a finite-dimensional space of a corresponding dimension ($R^{\sum_{i=1}^{N^{new}} \epsilon_i}$ and $R^{\sum_{i=1}^{N^{new}} L_i \epsilon_i}$, respectively) and consists of only the vectors from this polyhedron each coordinate of which is either 0 or 1 (i.e., the vectors being the vertices of the unit cubes $T_{\sum_{i=1}^{N^{new}} \epsilon_i}$ and $T_{\sum_{i=1}^{N^{new}} L_i \epsilon_i}$, respectively), and (c) $P(x, y, z) \ge \delta$ determines a subset of the set $MX \times \Omega Y \times HZ$ in which the vectors (x, y, z) with Boolean components forming the vectors (y, z) are located.

Under the assumption made and with the use of this notation, one can formulate a problem that corresponds to *Situation 2* (see Section I) and generalizes Problem (1)–(14). A solution to the generalized problem allows the regional administration to estimate the expected minimal total expenses under consideration in the above-mentioned "worst-case scenario." For the planning period of $\psi \ge 1$ years and under Basic Assumptions 1–9, this problem can be written in the vector-matrix form, for instance, as follows:

$$\max_{(t,f,c,g,q) \in \Lambda \times \Theta \times \Delta \times \Gamma \times \Upsilon} (-\nu\psi\langle t, x\rangle + \langle (f + \psi c), y\rangle + \langle (g + \psi q), z\rangle) \to \min_{(x,y,z) \in (MX \times \Omega Y \times HZ) \cap \Phi}. \quad (15)$$

Let $u = (t, (f + \psi c), (g + \psi q)), v = (x, y, z)$, let $D = \begin{pmatrix} -\nu\psi E_1 & 0_1 & 0_2 \\ 0_3 & E_2 & 0_4 \\ 0_5 & 0_6 & E_3 \end{pmatrix}$ be a quadratic matrix with the number of rows equaling the sum of the numbers of all the vector components belonging to the vectors t, f, and g, where $E_1 E_2, E_3$ are unit matrices of the sizes $\left(M \sum_{i=1}^{N^{new}} L_i + M \sum_{i'=1}^{N^{exist}} l_{i'} \right)$, $\sum_{i=1}^{N^{new}} \epsilon_i$, and $\sum_{i=1}^{N^{new}} \epsilon_i L_i$, respectively, $0_\kappa, \kappa \in \overline{1,6}$, are zero matrices of the corresponding sizes, and let $\Pi = (MX \times \Omega Y \times HZ) \cap \Phi$.

Then Problem (15) can be rewritten as

$$\max_{u \in \Lambda \times \Theta \times \Delta \times \Gamma \times \Upsilon} \langle u, Dv \rangle \to \min_{v \in \Pi}. \tag{16}$$

Remark 2.

One should emphasize the difference between the statement underlying the formulation of Problem (15) and that of another problem that may, eventually, be considered by the regional administration. This other problem statement may appear in an attempt to deal with the expected volumes of cargo to be moved via the transportation network (being part of the regional freight transportation infrastructure) to be built. That is, in Problem (15), the administration chooses both locations for new transportation hubs and types of new access roads to these hubs to be built, along with cargo flows to go via the hub locations for each particular cargo flow. This makes the flow volumes a part of the variables that the regional administration controls. However, (a) the costs of cargo transportation, (b) the costs of building new transportation hubs, (c) the costs of building new access roads to them, and (d) the maintenance costs for both the hubs and the access roads to them are considered as market variables, not controlled by the administration.

In the above-mentioned other statement of the problem of determining optimal locations for new transportation hubs and optimal types of access roads to them, the transport costs are considered to be under the regional administration control. The costs of building new transportation hubs, the costs of building new access roads to them, and the maintenance costs for both the hubs and access roads to them are still considered to be market variables. However, the volumes of cargoes to be moved in particular directions (flow volumes) are considered to be market variables as well. In this case, a different minimax problem is to be formulated.

If $\hat{u} = (x, (f + \psi c), (g + \psi q)), \hat{v} = (t, y, z)$, and $\hat{\Pi} = (\Lambda \times \Omega Y \times HZ) \cap \Phi$, this minimax problem can then be written as

$$\max_{\hat{u} \in MX \times \Theta \times \Delta \times \Gamma \times \Upsilon} \langle \hat{u}, D\hat{v} \rangle \to \min_{\hat{v} \in \hat{\Pi}}. \tag{16'}$$

However, since the vector x is present in the description of both the set MX and the set $\hat{\Pi}$, this minimax problem turns out to be the one with connected variables. That is, while the maximization of the function $\langle \hat{u}, D\hat{v} \rangle$ is done over the vector variables that include the vector x, the minimization of the maximum function is done over the vector variables $\hat{v} = (t, y, z)$. However, the vector x is present in the description of the set $\hat{\Pi}$ (via the description of the set Φ), binding the variables y, z and x, which makes Problem (16′) a problem with connected variables. Even when all the variables are continuous (which is not the case in Problem (16′)), problems with connected variables are more complicated than Problem (15) (Belenky, 1997). In any case, Problem (16′) is not a subject of considerations in the present article.

Remark 3.

The goal function in Problem (15) can also be rewritten as follows:

$$\max_{(t,f,c,g,q) \in \Lambda \times \Theta \times \Delta \times \Gamma \times \Upsilon} (-\nu\psi \langle t, x \rangle + \langle f, y \rangle + \psi \langle c, y \rangle + \langle g, z \rangle + \psi \langle q, z \rangle) \to \min_{(x,y,z) \in (MX \times \Omega Y \times HZ) \cap \Phi}. \tag{17}$$

Let now $(f, c) = \tilde{f}, (g, q) = \tilde{g}$, and let

$$\tilde{f} \in \tilde{\Theta} = \left\{ \tilde{f} = (f, c) \geq 0 : fF \leq r, cW \leq \lambda \right\},$$

$$g \in \tilde{\Gamma} = \{ \tilde{g} = (g, q) \geq 0 : gG \leq e, q\Psi \leq \eta \}.$$

Here, $\tilde{\Theta}$ and $\tilde{\Gamma}$ are polyhedra, and Problem (15) can be rewritten in the form

$$\max_{(t,\tilde{f},\tilde{g}) \in \Lambda \times \tilde{\Theta} \times \tilde{\Gamma}} \left(-\nu\psi \langle t, x \rangle + \left\langle \tilde{f}\tilde{E}_2, y \right\rangle + \left\langle \tilde{g}\tilde{E}_3, z \right\rangle \right) \to \min_{(x,y,z) \in (MX \times \Omega Y \times HZ) \cap \Phi} \tag{17'}$$

where $\tilde{E}_2 = \begin{pmatrix} E_2 \\ \psi E_2 \end{pmatrix}$, $\tilde{E}_3 = \begin{pmatrix} E_3 \\ \psi E_3 \end{pmatrix}$, and E_2, E_3 are unit matrices of corresponding sizes (see (15) and (16)). Thus, Problem (16) can be rewritten as

$$\max_{\tilde{u} \in \Lambda \times \tilde{\Theta} \times \tilde{\Gamma}} \left\langle \tilde{u}, \tilde{D}v \right\rangle \to \min_{v \in \Pi}, \tag{18}$$

where $\tilde{u} = \left(t, \tilde{f}, \tilde{g}\right)$, $v = (x, y, z)$, and $\tilde{D} = \begin{pmatrix} -v\psi E_1 & 0 & 0 \\ 0_7 & E_2 & 0_8 \\ 0_9 & \psi E_2 & 0_{10} \\ 0_{11} & 0_{12} & E_3 \\ 0_{13} & 0_{14} & \psi E_3 \end{pmatrix}$, and $0_\kappa, \kappa \in \overline{7, 14}$ are zero vectors of corresponding sizes.

A solution to minimax Problem (17′) provides the estimate of only a part of the expenses of the potential public–private partnership associated with developing a regional freight transportation infrastructure with newly built transportation hubs and access roads to them. As mentioned in considering Problem (1)–(14), this estimate does not take into consideration the expenses associated with the maintenance of the already existing transportation hubs and access roads to them (during the planning period of ψ years). To take these expenses into consideration in estimating the economic effectiveness of the regional freight transportation infrastructure for ψ years, $\psi \geq 1$, one should add either the number

$$\psi\left(\sum_{i'=1}^{N^{exist}} c_{i'}^{exist} + \sum_{i'=1}^{N^{exist}} \sum_{k'=1}^{l_{i'}} q_{i'}^{k'\ exist}\right)$$

or the number

$$\sum_{\kappa=1}^{\psi}\left(\sum_{i'=1}^{N^{exist}} c_{i'}^{exist\ \kappa} + \sum_{i'=1}^{N^{exist}} \sum_{k'=1}^{l_{i'}} q_{i'}^{k'\ exist\ \kappa}\right)$$

to the minimax value to be obtained as a result of solving Problem (17′). Here, $c_{i'}^{exist\ \kappa}$ and $q_{i'}^{k'\ exist\ \kappa}$ are the values of the corresponding parameters during year $\kappa, \kappa \in \overline{1, \psi}$. Finally, one can, of course, consider these two costs to be as uncertain as are the costs $c_{i\mu}^{new}$ and $q_{i\mu}^{k\ new}$ and include corresponding vector variables into the formulation of the minimax problem. This can be done in just the same way this takes place for the variable vectors c and q.

One should also notice that, for the sake of simplicity of describing the ideas underlying the mathematical models presented in this section, no modernization construction works with respect to the existing transportation hubs and the existing access roads to them are reflected in these models. However, one can easily incorporate them in each of these models. To this end, one should consider the existing transportation hubs also as the points where new transportation hubs and access roads to them can be developed with all the parameters associated with both building and providing the maintenance to these "new" hubs. Here, only those types of access roads to each of these "new" hubs that already exist should be considered as the ones that can be built (with all the parameters associated with both building and providing maintenance services to these "new" access roads). Finally, the capacities of both "new" transportation hubs and "new" access roads to them, found as the result of solving one of the corresponding optimization problems considered in the above should be "combined" with the capacities of the existing transportation hubs and with those of the existing access roads to these hubs in determining the needed (if any) volume of the above-mentioned modernization construction works.

The basic assertion

For the sake of definiteness, the Basic Assertion is formulated with respect to Problem (18) assuming that the parameters $c_{i'}^{exist}$ and $q_{i'}^{k'\ exist}$, $i' \in \overline{1, l_{i'}}$, $i' \in \overline{1, N^{exist}}$ are not variables over the planning period of ψ years.

Basic Assertion

The equality

$$\min_{v \in \Pi} \max_{\tilde{u} \in \Lambda \times \tilde{\Theta} \times \tilde{\Gamma}} < \tilde{u}, \tilde{D}v >= \min_{v \in \Pi, Jw \geq \tilde{D}v} \langle \omega, w \rangle$$

holds, where J is a matrix, and ω is a vector of corresponding dimensions.

Proof is presented in Appendix 1.

Corollary 1.

Problem (18) is reducible to a mixed programming problem with the system of constraints having a linear structure.

Corollary 2.

Let $Y = \{y \geq 0 : By \geq \pi\}$, and $Z = \{z \geq 0 : Kz \geq h\}$. Then the number

$$\min_{v \in (MX \times Y \times Z) \cap \Phi, Jw \geq \tilde{D}v} \langle \omega, w \rangle$$

is the lower bound for the number

$$\min_{v \in \Pi, Jw \geq \tilde{D}v} \langle \omega, w \rangle,$$

and this lower bound can be found by solving a linear programming problem

$$\langle \omega, w \rangle \to \min_{v \in (MX \times Y \times Z) \cap \Phi,\ Jw \geq \tilde{D}v}.$$

Testing the proposed tool on model data

The proposed tool was tested on several sets of model data collected by the authors using the open source (see https://github.com/ggfedin/Model_dataset). The aim of the testing was to demonstrate how the proposed decision-support tool can be used in negotiations between a regional administration and potential investors. That is, the aim was to demonstrate that for any set of the model data, which the negotiating parties may change many times, calculation results obtained with the use of this tool can be presented in the form of easy-to-read tables and observable illustrative pictures, helpful for the negotiating parties. Examples of such tables are presented in Appendix 2, and such illustrative pictures are presented in Appendix 3.

The authors would like to make it clear that the testing was conducted to demonstrate no more than promising opportunities offered by the proposed decision-support tool, and it did not intent to present any examples of its practical application. This approach to presenting the testing results is associated with certain difficulties in obtaining real data related to the functioning of real regional transportation systems. For obvious business and security reasons, regional administrations prefer to have at their disposal a tool that would allow them themselves to conduct calculations with any real data that they may decide to put in. This is especially so if negotiations with potential investors from the private sector on developing, particularly, regional freight transportation infrastructures are in progress or are planned.

A model region with 32 cargo origin/destination points was "designed" based on the information taken from open sources. It was assumed that two already functioning transportation hubs ($i' \in \{1, 2\}$) and eight locations for potentially allocating new transportation hubs to be built ($i \in \overline{3, 10}$) were to be considered. Two types of access roads (railways and highways) to both the new and the existing transportation hubs were considered ($k, k' \in \{1, 2\}$), and it was assumed that each type of the access roads could have two capacities to choose from. Three different tax rates were included in the transportation tariffs (that constituted v percents of these tariffs, $v \in \{1, 7, 13\}$), and three particular planning periods (with the length of ψ years, $\psi \in \{1, 3, 5\}$) were considered.

Picture A1 from Appendix 3 shows the geographic locations of the cargo origin/destination points in the "designed" region, possible locations for new transportation hubs in this region, and the locations of the existing transportation hubs in the region in some conditional geographic coordinates that are used in open sources.

Calculations for four variants of the mixed programming problem ("*Situation 1, Case A, Subcase 1*," "*Situation 1, Case A, Subcase 2*," "*Situation 1, Case B, Subcase 1*," and "*Situation 1, Case B, Subcase 2*") and for two variants of the robust (minimax) optimization problem ("*Situation 2, Subcase 1*" and "*Situation 2, Subcase 2*," see Remark 1 in Section III) were conducted.

For "*Situation 1, Case A, Subcase 1*" and for "*Situation 1, Case A, Subcase 2*," described in Sections I and III, the following data were attributed and given particular values: (a) The yearly capacity of a new transportation hub for each of the two variants of the hub yearly capacity at each of the eight potential locations of new transportation hubs ($i \in \overline{3, 10}$), (b) the yearly capacities of the existing two transportation hubs ($i' \in \{1, 2\}$), (c) the yearly capacity of each new access road to each new transportation hub (which corresponds to the hub yearly capacity) for each of the two variants of the hub yearly capacity, (d) the yearly capacities of each access road to each of the two existing transportation hubs, (e) the expected yearly demand for cargo services at each of the above 32 cargo origin/destination points, (f) the expected total yearly demand for cargo services in the region, (g) the cost of building a new transportation hub for each of the two variants of the hub yearly capacity, (h) the cost of building a highway to a new transportation hub for both variants of the highway yearly capacity, (i) the cost of building a railway to a new transportation hub for both variants of the railway yearly capacity, and (j) the maintenance cost for highways for both variants of the highway yearly capacity and for railways for both variants of the railway yearly capacity.

The costs for transporting a unit volume of cargo between each of the 32 cargo origin/destination points and between each of these points and the existing and new transportation hubs were calculated. The calculations of the transportation cost were conducted proceeding from (a) the length of the distance between the above points and the hubs (computed with the use of the Google Maps), (b) the average cost of transporting a unit volume of cargo per kilometer, and (c) a transportation cost discount, which depends on the length of the above-mentioned distance. As mentioned in Section III, *Subcases 2* in *Situation 1* and in *Situation 2* differ from *Subcases 1* in both *Situation 1* and *Situation 2* only by the sign before the third term in the goal functions of the corresponding optimization problems, considered in Section III. That is, in *Situation 1*, for *Subcase 2* in *Case A* and for *Subcase 2* in *Case B*, the goal function takes the form

$$\sum_{i=1}^{N^{new}}\sum_{\mu=1}^{\epsilon_i}\left(f_{i\mu}+\psi c_{i\mu}^{new}\right)y_{i\mu}+\sum_{i=1}^{N^{new}}\sum_{k=1}^{L_i}\sum_{\mu=1}^{\epsilon_i}\left(g_{i\mu}^{k}+\psi q_{i\mu}^{k\ new}\right)z_{i\mu}^{k}$$
$$+\nu\psi\left(\sum_{i'=1}^{N^{exist}}\sum_{j=1}^{M}\sum_{k'=1}^{l_{i'}}t_{ji'}^{k'exist}s_{ji'}^{k'exist}+\sum_{i=1}^{N^{new}}\sum_{j=1}^{M}\sum_{k=1}^{L_i}t_{ji}^{k\ new}s_{ji}^{k\ new}\right)$$
$$\rightarrow \min.(1')$$

Problems (1)–(14) and (1′)–(14) were solved with the assigned and calculated parameters as described above, and the calculation results for "*Situation 1, Case A, Subcase 1*" and for "*Situation 1, Case A, Subcase 2*" are presented in Tables 1 and 2 from Appendix 2, respectively. Two examples of calculating the optimal allocations of the hubs and their yearly capacities by solving Problems (1)–(14) for "*Situation 1, Case A, Subcase 1*" and by solving Problems (1′)–(14) for "*Situation 1, Case A, Subcase 2*" are depicted on Picture A2 and on Picture A3 from Appendix 3, respectively. These examples were calculated for the particular values of the two parameters ($\nu = 7, \psi = 5$), and the calculation results are presented in line 8 of Table 1 and in line 8 of Table 2, respectively.

One should notice that in just for the same reason it was described in formulating Problem (1)–(14), the first term in the expression for the function describing the expected total expenses (associated with developing the regional freight transportation infrastructure and redistributing the existing cargo flows between the existing transportation hubs and those to be built) is not present in (1′) (see Section III).

For "*Situation 1, Case B, Subcase 1*" and for "*Situation 1, Case B, Subcase 2,*" described in Sections I and III, it was assumed that the expected yearly demands on transporting cargo at each of 32 cargo origin/destination points, as well as the expected total yearly demand for cargo services in the region, may vary. It was assumed that for the expected yearly demand on transporting cargo at each of the above 32 locations, the expected maximum yearly demand could not exceed 130% of this expected yearly demand (for cargo services at each of the above 32 origin/destination points), specified in (e) from the above list of (a)–(j) for *Situation 1, Case A*. Further, it was assumed that the expected minimal yearly demand there could not be lower than 80% of this expected yearly demand for *Situation 1, Case A*, whereas the expected total yearly demand for cargo services in the region could not exceed 115% of its expected yearly demand, specified in (f) from the above list of (a)–(j) for *Situation 1, Case A*. Finally, it was assumed that the expected minimum of the total yearly demand for cargo services in the region could not be lower than 95% of the expected total yearly demand, specified in (f) from the above list of (a)–(j) for *Situation 1, Case A*. All the other values, specified in (a)–(d) and (g)–(j) from the above list of (a)–(j) for *Situation 1, Case A*, were the same as in *Situation 1, Case A*.

Problems (1)–(14) and (1′)–(14) were solved with thus chosen, assigned, and calculated data, and the calculation results are presented in Tables 3 and 4 from Appendix 2, respectively. Two examples of calculating the optimal hub allocations and their yearly capacities by solving Problem (1)–(14) for "*Situation 1, Case B, Subcase 1*" and by solving Problem (1′)–(14) for "*Situation 1, Case B, Subcase 2*" are depicted on Picture A4 and on Picture A5 from Appendix 3, respectively. These examples were calculated for the particular values of the two parameters ($\nu = 7, \psi = 5$), and the calculation results are presented in line 8 of Table 3 and in line 8 of Table 4, respectively.

For the robust (minimax) problem in *Situation 2*, described in Section I, with *Subcases 1 and 2*, described in Section III, particular values of the following parameters in this problem were used for all the possible new transportation hubs and for all the possible new access roads to them based on the expert estimates of:

- The maximal and the minimal average cost of transporting a unit volume of cargo between node j and a new (or an existing) transportation hub at point i (i') via an access road of type k (k') for each $i \in \overline{3,10}, (i' \in \{1,2\})$, for each $k, k' \in \{1,2\}$, and for both variants of the hub yearly capacity,
- the maximal and the minimal average costs of building a new transportation hub at each place $i, i \in \overline{3,10}$ for both variants of the hub yearly capacity,
- the maximal and the minimal average costs of building a new access road of each of the two types (highways and railways) to a new transportation hub $i, i \in \overline{3,10}$ for both variants of each type of the access road (each variant having the yearly capacity corresponding to one of the two variants of the hub yearly capacity),
- the maximal and the minimal average maintenance costs for every new transportation hub $i, i \in \overline{3,10}$ for both variants of the hub yearly capacity,
- the maximal and the minimal average maintenance costs for highways and railways to every new transportation hub for both variants of the hub yearly capacity, for both variants of each type

of the access road (each variant having the yearly capacity corresponding to one of the two variants of the hub yearly capacity), and

- the maximal and the minimal average maintenance costs for highways and railways to every existing transportation hub.

Also, the following assumptions were made:

a. The unknown cost of transporting a unit volume of cargo between node j and a new (or existing) transportation hub at point i (i') via an access road of type k (k') could not exceed 130% and could not be lower than 70% of the current market value of this cost for the distance between node j and point i (i'), $k, k' \in \{1, 2\}, i \in \overline{3, 10}$, $i' \in \{1, 2\}$,
b. the unknown cost of building a new transportation hub at point $i, i \in \overline{3, 10}$ of both variants of the hub yearly capacity could not exceed 120% and could not be lower than 91% of its current value (i.e., of the one corresponding to the existing market value),
c. the unknown cost of building a highway to a new transportation hub at point $i, i \in \overline{3, 10}$ of both variants of the hub yearly capacity and of both variants of the highway yearly capacity could not exceed 115% and could not be lower than 85% of its corresponding (to one of the two variants of the highway yearly capacity) current value (i.e., of the currently existing market value),
d. the unknown cost of building a railway to a new transportation hub at point $i, i \in \overline{3, 10}$ of both variants of the hub yearly capacity and of both variants of the railway yearly capacity could not exceed 117.5% and could not be lower that 82.5% of its corresponding (to one of the two variants of the railway yearly capacity) current value (i.e., of the currently existing market value),
e. the unknown maintenance cost for a new cargo transportation hub at point $i, i \in \overline{3, 10}$ of both variants of the hub yearly capacity could not exceed 125% and could not be lower than 90% of its corresponding (to one of the two variants of the railway yearly capacity) current value (i.e., of the currently existing market value),
f. the unknown maintenance costs for highways and railways to a new cargo transportation hub at point $i, i \in \overline{3, 10}$ for both variants of the hub yearly capacity, for both variants of each type of the access road (each variant having the yearly capacity corresponding to one of the two variants of the hub yearly capacity), could not exceed 150% and could not be lower than 50% of their corresponding (to one of the two variants of the highway yearly capacity and the railway yearly capacity) current values (i.e., of the currently existing market values), and
g. the maintenance costs for the existing cargo transportation hubs and the maintenance costs for the access roads to the existing cargo transportation hubs of all the existing types were known numbers that did not change during the planning period.

For "*Situation 2, Subcase 1*," the robust (minimax) problem is formulated as Problem (18), and for "*Situation 2, Subcase 2*," the problem takes the form

$$\max_{\tilde{u} \in \Lambda \times \tilde{\Theta} \times \tilde{\Gamma}} \left\langle \tilde{u}, \widetilde{D'} v \right\rangle \to \min_{v \in \Pi}, \quad (18')$$

where

$$\widetilde{D'} = \begin{pmatrix} v\psi E_1 & 0 & 0 \\ 0_7 & E_2 & 0_8 \\ 0_9 & \psi E_2 & 0_{10} \\ 0_{11} & 0_{12} & E_3 \\ 0_{13} & 0_{14} & \psi E_3 \end{pmatrix}.$$

Problems (18) and (18′) were solved with the above-described input data. The calculation results for "*Situation 2, Subcase 1*" are presented in Table 5, and the calculation results for "*Situation 2, Subcase 2*" are presented in Table 6 from Appendix 2, respectively.

Two examples of calculating the optimal hub allocations and their capacities, and optimal assignments of cargo origin/destination points to the hubs by solving Problem (18) for "*Situation 2, Subcase 1*" and by solving Problem (18′) for "*Situation 2, Subcase 2*" are depicted on Picture A6 and on Picture A7 from Appendix 3, respectively. These examples were calculated for the particular values of the two parameters ($v = 7, \psi = 5$), and the calculation results are presented in line 8 of Table 5 and in line 8 of Table 6, respectively.

Both in *Situation 1* (in Problem (1)–(14) for *Case A* and in Problem (1′)–(14) for *Case B*) and in *Situation 2* (in Problem (18) for *Subcase 1* and in Problem (18′) for *Subcase 2*), the systems of constraints and the goal functions of the corresponding optimization problems were formed in accordance with their description, presented in Section III. These problems were solved with the use of the solver *Intlinprog*, being part of the MatLab interactive environment installed on a personal laptop. The laptop was equipped with 2.5-GHz Intel Core i5 CPU and 16-GB RAM based on the Windows platform. The optimal solutions were obtained in less than 0.5 seconds for Problem

(1)-(14) and for Problems (1′)–(14) for all the nine combinations of the values of ν and ψ in each of the two subcases described by each of these two problems. The optimal solutions were obtained in less than three seconds for Problem (18) and Problem (18′) for all the nine combinations of the values of ν and ψ in each of the two subcases described by each of these two problems.

For "*Situation 1, Subcase 1*" and for "*Situation 2, Subcase 1*," under a particular set of the input data, the solutions to Problem (1)–(14) and Problem (18) show that the yearly revenues expected to be received as a result of the functioning of the regional freight transportation infrastructure do not cover the required expenses within one year after the start of the functioning of all the new facilities. However, in more than a year, the functioning of the regional freight transportation infrastructure results in generating a substantial revenue, exceeding the required expenses. Generally, based on such estimates, which are quite expectable for large-scale projects, the regional administration may make at least two strategic decisions (in the model situation corresponding to the model input data):

(1) The regional administration may offer a part of the above-mentioned (substantial) revenue as its financial contribution to the public–private partnership in negotiations with potential partners from the private sector.
(2) The regional administration may decide not to form any partnership with the private sector on the project for providing the functioning of the regional freight transportation infrastructure, which is planned to be developed with or without any private investment in its development. This may happen if (a) the project is expected to generate profit in a relatively short period of time after all the facilities of the new regional freight transportation infrastructure start functioning, and (b) the regional administration can get a loan from, say, a bank under acceptable conditions to cover at least the expenses associated with the functioning of the developed freight transportation infrastructure. (Certainly, what period of time should be viewed as a short one is to be determined.)

According to Tables 1, 3, and 5, in the model situation corresponding to the model input data, the second strategic decision may be the case in line with

(a) the estimates obtained by solving Problem (1)–(14) for "*Situation 1, Case A, Subcase 1*" and for "*Situation 1, Case B, Subcase 1*" for three years and for five years (under 7% and under 13% of the tax value both in *Case A* and in *Case B*),

(b) the estimates obtained by solving Problem (18) for "*Situation 2, Subcase 1*" for three years and for five years (under 13% of the tax value).

In addition to that, depending on the potential loan conditions, there could be certain combinations of both strategies. Further, a determination of how much to borrow and how much to ask the private investors to contribute may require the use of additional mathematical models and methods. Finally, other strategies that are based upon the above estimates of the expected financial results of the project functioning could be formed. At the same time, the regional administration should bear in mind that particular calculation results are always those for a particular set of the data, and changing the data can lead to calculating a strategy that may seem more promising. Also, particular calculation results may suggest that the values of some parameters reflecting the uncertainty conditions should be reconsidered. For instance, the boundaries within which the expected volumes of cargo flows may vary could be such parameters.

Thus, the estimates that can be calculated with the use of the proposed decision-support tool may provide a certain flexibility to the regional administration in choosing its financial strategy for developing a regional freight transportation infrastructure.

Section III describes how private investors participating in negotiations with the regional administration on potential investments in (a) developing a new regional freight transportation infrastructure and (b) in providing transportation services in the framework of the new regional freight transportation infrastructure can determine whether they can benefit from making corresponding investments by using the proposed decision-support tool. In negotiating the option (b), they can do this by solving Problem (1′)–(14) and Problem (18′). To solve Problem (18′), they need, however, to know the function describing their expenses associated with providing transportation services in the framework of the new transportation infrastructure.

It is clear that in (a) "*Situation 1, Case A, Subcase 2,*" (b) "*Situation 1, Case B, Subcase 2,*" and (c) "*Situation 2, Subcase 2,*" the regional administration may affect the total expenses of its potential partners from the private sector by changing the tax value. The calculation results under a particular set of the input data, presented in Appendix 2, show that the share of taxes in total expenses turns out to be relatively small for one year projects. The lowest share for these projects is about 1% (as shows the result of solving Problem (1′)–(14) for "*Situation 1, Case B, Subcase 2*" for the parameter values $\psi = 1$, $\nu = 1$), and the highest share is about 12% (as shows the result of solving Problem (1′)–(14) for "*Situation 1, Case A, Subcase 2*" for the parameter values $\psi = 1$, $\nu = 13$). At the same time, for five year projects, the lowest share is about 3% (as shows the result of solving Problem (18′) for "*Situation 2, Subcase 2*" for the parameter values $\psi = 5$, $\nu = 1$), and the highest share is about 31% (as shows the result of solving Problems (1′)–(14) for "*Situation 1, Case A, Subcase 2*" for the parameter values $\psi = 5$, $\nu = 13$).

Based on these calculation results (which, as once again should be emphasized, were conducted for a particular set of only model data by solving Problem (1′)–(14) and Problem (18′)), one can conclude that long term projects on developing freight transportation infrastructures may eventually be more sensitive to changes in the tax value than short ones.

Discussion

(1) From the authors' viewpoint, the present article makes a contribution to solving large-scale problems that appear in transportation and energy (see the Introduction) economics. Particularly, it suggests how a problem associated with making strategic management decisions on investing in the development of a regional freight transportation infrastructure can be formalized as a solvable mathematical problem. That is, it shows that a substantially nonlinear problem with mixed variables can be solved with the use of the techniques for solving mixed programming problems with constraints and goal functions having a linear structure, which are implemented in the framework of standard software packages, for instance, MILP. Thus, solving this strategic management problem does not, generally, require developing any heuristics or special software for practically reasonable sizes of the problem. This finding distinguishes the authors' approach to economic problems in transportation from those proposed by some other authors, including (Merakli & Yaman, 2016).

(2) While the number of points on the regional cargo transportation network may be quite high, the number of points suitable for locating new transportation hubs there is usually relatively small (does not usually exceed 10). Also, (a) the number of the hub capacity options to choose from does not usually exceed 4, (b) the number of types of new roads that are planned to be built to a new transportation hub does not usually exceed 3, and (c) the road capacity of each type is usually determined by the hub capacity. Thus, the total number of Boolean variables in practical problems formulated, for instance, as Problem (1)–(14) or Problem (18) is relatively small. This allows one to solve these practical problems with the use of standard software packages such as MILP or CPLEX quite quickly, even when these packages are implemented on laptops. (For optimization software packages, see, for instance, (Bixby, 2002) and (Mittelmann, 2019).)

(3) Even if the number of Boolean variables in any practical problem under consideration in this article were high, the lower estimates of the expenses and the profit/loss, could be calculated with the use of linear programming techniques. These techniques are described, particularly, in (Bertstimas & Tsitsiklis, 1997) and in (Yudin & Golshtein, 1965), and their use in solving problems formulated as Problem (18) and Problem (18′) is possible due to the results from (Belenky, 1981). That is, as shown in (Belenky, 1981), calculating the minimax of a bilinear function with continuous variables, for instance, in a continuous analog of Problem (18), is reducible to solving linear programming problems forming a dual pair. Calculating the abovementioned lower estimates requires solving such a continuous analog.

(4) Finally, as is known, in operations management in general and in transportation (and energy) systems in particular, experimental findings obtained in one system can usually be used to improve daily operations in another one, at least for a short period of time. In strategic management, however, the situation is different, particularly, in transportation systems. That is, strategic decisions in transportation systems are not universal. They are unique for every particular system, and they cannot usually be replicated in other systems. Regularities established by researchers based upon any chosen set of data do not matter much to decision-makers involved in developing strategic management decisions. This is the case since such regularities may change dramatically when a different set of data is used, whereas these

decisions are made for a long period of time. In contrast, these decision-makers feel "armed" when they have an easy-to-operate decision-support tool helping them quickly calculate solutions themselves to the problems they face and do this with any set of the data they may decide to consider and analyze.

The "value" of strategic recommendations that are based on regularities drawn from experiments with a particular set (or even with several particular sets) of model and even real data is usually doubtful. This is the case unless these regularities allow researchers (a) to indicate a class of situations in each of which (within this class) these regularities always hold, and (b) to establish verifiable criteria to determine whether a particular situation belongs to this class. Otherwise, not only do such regularities not contribute to any theory, they may be misleading and even damaging to those who apply them in practice. This is especially so with respect to financial decisions that are to be made by regional administrations or by the country governments, since, usually, the taxpayers' money is at stake. When this is the case, any unsubstantiated decisions that are based on experimental data may cause financial troubles at least to the region for which these decisions are made. However, the authors are not aware of such classes of situations in strategic management either in general or in transportation and energy systems. At the same time, finding such classes of situations was not within the goals of the present article.

This, of course, does not mean that all the experimental calculation results related to strategic management decisions in particular systems, including those from transportation and energy, are useless. Nor does this mean that decision-makers responsible for strategic management decisions will always ignore them. These calculation results may eventually reveal extremely helpful strategic business information. However, for that very reason, as mentioned in Section V, one should bear in mind that real data will very unlikely be made available to interested researchers by regional administrations.

In addition to that, as mentioned earlier, experimental calculation results conducted with any set of (real or not) data may change dramatically when a different set of data is used.

Though the proposed tool is mostly intended to help public administrations in their negotiations with the private sector, its use will certainly benefit the other side of the negotiating process, that is, private investors potentially interested in financing such large-scale projects as developing regional transportation (or) energy infrastructures. The tool particularly provides the following opportunities for these private investors: (a) The possibility to estimate the expected volume of the revenue that can be received under different boundaries on variables and parameters of the models presented in Section III, (b) to be better prepared to negotiations with public administrations on possible investments in developing transportation (or energy) infrastructures, and (c) to become more competitive in the market of providing transportation services by operating the regional freight transportation (or energy) infrastructure that could be developed, possibly, even thanks to their investments. The option to use the proposed tool by both negotiating parties reflects the game-theoretic approach to considering negotiations in general and those associated with the subject of this article, which is exercised by the authors. That is, the minimax Problem (18) and Problem (18′), which are considered in Section III and in Section V of the present article, implement this approach. As one can see from publications on noncooperative games and their applications in economics (see, for instance, (Owen, 2013, von Neumann & Morgenstern, 2007; Gibbons, 1992) in which an equilibrium is sought in an antagonistic game), a public administration can be viewed as one party in the game, whereas the private investor (or investors) can be viewed as the other party.

Results

(1) A mathematical model to formalize problems associated with finding quantitative estimates of investments needed from the private sector for developing a regional freight transportation (or energy) infrastructure is proposed.

(2) Depending on the information available to decision-makers, three optimization problems are formulated on the basis of the proposed mathematical model.

Two of these three problems allow one to find the estimates assuming that the information on the values of the parameters of the model is known exactly either for all the parameters or for a part of them. In both cases, the corresponding optimization problems are formulated as mixed programming ones.

A robust optimization problem is formulated on the basis of the same mathematical model under uncertainty on the values of all the parameters of the model. It is proven that this robust optimization problem is reducible to a mixed programming one with the system of constraints and the goal function having a linear structure under natural assumptions on the boundaries within which the values of the parameters can vary.

(3) In the above-mentioned (three) mixed programming problems, all the integer variables are Boolean, and the number of these variables is relatively small. This allows one to use standard software packages like MILP and CPLEX, implemented even on laptops, to find the quantitative estimates of the investment volumes needed from the private sector for developing a regional freight transportation infrastructure. Thus, any of these three problems—which decision-makers from regional administrations responsible for making strategic management decisions may choose to solve—can be solved on laptops.

(4) The proposed model was used to formulate three above-mentioned optimization problems based upon the model "transportation" data taken from open sources. Solutions to these problems are presented in Appendix 2. Possible strategic management decisions that these solutions may suggest are described in Section V.

(5) Two brief surveys of publications relevant to the subject of the present article are offered. One of the surveys is on hub location problems, whereas the second one is on modelling public–private partnership in transportation projects. Both surveys, particularly, help substantiate the need for developing the decision-support tool proposed in the present article.

Concluding remarks

(1) One of the goals of this article is to describe a decision-support tool that may help a regional administration in its negotiations with both the federal government and private investors on developing a regional freight transportation (or energy) infrastructure. By helping estimate the needed volume of investment in this project, the tool may substantiate the need for a public–private partnership if the federal government and the regional administration cannot finance the project in full.

(2) The proposed approach to modelling the problem under consideration in this article by taking into account the uncertainty in the values of all its parameters consists of formulating this problem as a robust (minimax) one on a Descartes product of two sets of vector variables. One of these sets is a polyhedron, and the other is a subset of another polyhedron formed by the vectors each of whose components equals either 0 or 1. The minimax of a bilinear function of these two vector variables is sought, and it is proven that finding this minimax is reducible to solving a mixed programming problem.

(3) Any decision-support system for analyzing the existing regional freight transportation (or energy) infrastructure and/or for developing an optimal one that has a chance to work effectively should meet certain criteria. Particularly, it should allow the administration of a region

a. to find and to estimate variants of this infrastructure (that the administration may consider to be of interest to the region) in an acceptable time, despite the fact that large-scale problems are to be solved to this end,
b. to depict the locations of all the infrastructure elements on a geographic map graphically, in an easy-to-understand form,
c. to input new and to change already existing information relating to the freight transportation (or energy) infrastructure from easy-to-operate interfaces,
d. to obtain solutions (infrastructure variants) based on the available information only, including the data that can be known only approximately in principle, as well as on statistical estimates that can be calculated based on this information, and
e. to be flexible in incorporating both new information and new regularities formalizing relations between variables and parameters in the mathematical models that are in use as this information and regularities become known in the course of developing and analyzing strategic decisions related to a regional freight transportation (or energy) infrastructure.

The described features of the approach presented in this article bear evidence that a decision-support system for the considered purposes can easily be assembled based on the decision-support tool proposed in this article. Such a decision-support system should incorporate (a) software for solving linear and mixed programming problems, (b) a user-friendly interface for reliably uploading the input information used for determining parameters of the mathematical models underlying the formulations of the above-mentioned optimization problems, and (c) an interface for graphically depicting solutions to mathematical problems allowing one to present these results in easy-to-read tables and observable illustrative pictures.

As mentioned earlier, standard software packages for solving mixed programming problems are widely available even on laptops. A geographic information system software applicable, particularly, to transportation problems is described, for instance, in (Abulizi et al., 2016).

(4) One can easily be certain that the proposed approach can be used in estimating the needed volume of investment in developing regional passenger transportation infrastructures, as well as in developing the regional freight and passenger transportation infrastructures concurrently.

(5) Basic Assumptions 1 and 2 consider piecewise linear approximations of the costs of building both new hubs and new access roads to them, which are usually described by convex functions with positive values. A description of known techniques for approximating, particularly, such convex functions by piecewise linear functions can be found in many scientific publications, including (Gavrilovich, 1975).

(6) Basic Assumption 4, which is about not building new access roads to the existing transportation hubs and not doing any modernization construction work there in the planning period, is not restrictive. One can easily be certain that by introducing new variables, one can make a modernization of the existing transportation hubs and access roads to them a part of the activities associated with developing a new regional freight transportation infrastructure. These new variables should be present in the system of constraints of Problem (1)–(14).

(7) Basic Assumption 9, determining that the revenue in the form of regional taxes (that the administration expects to receive as a result of the functioning of the regional freight transportation infrastructure to be developed) is received after all the facilities (new transportation hubs and access roads to them) that are planned to be built start functioning, is not restrictive. That is, the regional administration may solve, for instance, Problem (1)–(14) or Problem (18) several times taking into account the schedule of developing new facilities within any particular period of time. To this end, this period should be divided into a corresponding number of parts. During each of these parts, the regional administration should consider already built new facilities as existing ones while expecting a particular set of facilities to be built and to start functioning.

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Appendix 1.

Proof of the Basic Assertion.

1. Let $v = v^* = (x^*, y^*, z^*) \in \Pi = (MX \times \Omega Y \times HZ) \cap \Phi$. Since $\Lambda \times \tilde{\Theta} \times \tilde{\Gamma}$ is a (non-empty) convex polyhedron, the linear function $\langle \tilde{u}, \tilde{D}v^* \rangle$ is bounded from above on this polyhedron, so it attains its maximum on the set $\Lambda \times \tilde{\Theta} \times \tilde{\Gamma}$. By the duality theorem of linear programming (see, for instance, Yudin & Golshtein (1965)), this means that the set of feasible solutions to the problem that is dual to the problem of maximizing this linear function on the set $\Lambda \times \tilde{\Theta} \times \tilde{\Gamma}$ is nonempty.

Let the problem of maximizing the linear function $\langle \tilde{u}, \tilde{D}v^* \rangle$ on the set $\Lambda \times \tilde{\Theta} \times \tilde{\Gamma}$ be written as

$$\langle \tilde{u}, \tilde{D}v^* \rangle \to \max_{\tilde{u} \in \Lambda \times \tilde{\Theta} \times \tilde{\Gamma}}, \quad (19.1)$$

$$\tilde{u}J \leq \omega, \ \tilde{u} \geq 0, \ (19.2)$$

where $J = \begin{pmatrix} I & 0 & 0 & 0 & 0 \\ 0 & F & 0 & 0 & 0 \\ 0 & 0 & W & 0 & 0 \\ 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & \Psi \end{pmatrix}$, $\omega = (l, r, \lambda, e, \eta)$, and $\tilde{u} = (t, f, c, g, q)$.

Then the set of feasible solutions to the problem that is dual to Problem (19.1), (19.2) is determined by the system of linear inequalities $Jw \geq \tilde{D}v^*$, $w \geq 0$, where w is the vector of the (dual) variables in the problem being dual to Problem (19.1), (19.2). This set is non-empty, and the maximum of the function (19.1) in Problem (19.1), (19.2) is attainted.

2. Let now $y = y^*, z = z^*$, and $x \in MX$ be such that $(x, y^*, z^*) \in (MX \times \Omega Y \times HZ) \cap \Phi$. Further, let $v(y^*, z^*) = (x, y^*, z^*) \in (MX \times \{y^*\} \times \{z^*\}) \cap \Phi$, where $(MX \times \{y^*\} \times \{z^*\}) \cap \Phi$ is a convex polyhedron, which is a subset of the set $(MX \times \Omega Y \times HZ) \cap \Phi$. Based on the results from Belenky (1981), one can easily be certain that for every pair of the vectors $(y^*, z^*) : (x, y^*, z^*) \in (MX \times \Omega Y \times HZ) \cap \Phi$, the inequality

$$\min_{v(y^*,z^*) \in (MX \times \{y^*\} \times \{z^*\} \cap \Phi} \max_{\tilde{u} \in \Lambda \times \tilde{\Theta} \times \tilde{\Gamma}} < \tilde{u}, \tilde{D}v(y^*, z^*) >= \\ = \min_{v(y^*,z^*) \in (MX \times \{y^*\} \times \{z^*\} \cap \Phi} \min_{Jw \geq \tilde{D}v(y^*,z^*)} \langle \omega, w \rangle$$

holds.

3. Since the set $(MX \times \{y^*\} \times \{z^*\}) \cap \Phi$ is a subset of a polyhedron for any pair of the vectors (y^*, z^*), and the number of the sets of these pairs for which the inclusion $(x, y^*, z^*) \in (MX \times \{y^*\} \times \{z^*\}) \cap \Phi$ holds is finite, the equalities

$$\min_{v \in \Pi} \max_{\tilde{u} \in \Lambda \times \tilde{\Theta} \times \tilde{\Gamma}} < \tilde{u}, \tilde{D}v >=$$

$$\min_{(y^*,z^*) \in \Omega Y \times HZ} \min_{v(y^*,z^*) \in (MX \times \{y^*\} \times \{z^*\}) \cap \Phi} \min_{Jw \geq \tilde{D}v(y^*,z^*)} < \omega, w >=$$

$$\min_{(x,y,z) \in (MX \times \Omega Y \times HZ) \cap \Phi, Jw \geq \tilde{D}(x,y,z)} < \omega, w >= \min_{v \in \pi, Jw \geq \tilde{D}v} \langle \omega, w \rangle$$

hold. The Basic Assertion is proved

Appendix 2.

Table A1. Problems (1)–(14), *Situation 1, Case A, Subcase 1.*

ψ years	v %	New hubs to build	New highways to build	New railways to build	Revenue USD, mln	Expenses USD, mln	Investments needed/Profit USD, mln
1	1	8(2),9(2),10(2)	8, 9	8, 9, 10	268	4 750	−4 482
1	7	8(2),9(2),10(2)	8, 9	8, 9, 10	1 875	4 750	−2 875
1	13	8(2),9(2),10(2)	8, 9	8, 9, 10	3 482	4 750	−1 268
3	1	8(2),9(2),10(2)	8, 9	8, 9, 10	803	5 450	−4 647
3	7	8(2),9(2),10(2)	8, 9	8, 9, 10	5 624	5 450	174
3	13	3(2),8(2),9(2)	8, 9	3, 8, 9	10 597	5 550	5 047
5	1	8(2),9(2),10(2)	8, 9	8, 9, 10	1 339	6 150	−4 811
5	7	3(2),8(2),9(2)	8, 9	3, 8, 9	9 511	6 250	3 261
5	13	3(2),8(2),9(2)	8, 9	3, 8, 9	17 662	6 250	11 412

Table A2. Problems (1′)–(14), *Situation 1, Case A, Subcase 2.*

ψ years	v %	New hubs to build	New highways to build	New railways to build	Taxes USD, mln (% in total expenses)	Construction expenses USD, mln (% in total expenses)	Total expenses USD, mln
1	1	7(2),8(2),10(2)	7, 10	7, 8, 10	68 (1%)	4 750 (99%)	4 818
1	7	5(2),7(2),10(2)	7, 10	5, 7, 10	357 (7%)	4 850 (93%)	5 207
1	13	5(2),7(2),10(2)	7, 10	5, 7, 10	664 (12%)	4 850 (88%)	5 514
3	1	7(2),8(2),10(2)	7, 10	7, 8, 10	205 (4%)	5 450 (96%)	5 655
3	7	5(2),7(2),10(2)	7, 10	5, 7, 10	1 072 (16%)	5 550 (84%)	6 622
3	13	5(2),7(2),8(2)	5, 7, 8	5, 7, 8	1 837 (24%)	5 680 (76%)	7 517
5	1	7(2),8(2),10(2)	7, 10	7, 8, 10	342 (5%)	6 150 (95%)	6 492
5	7	5(2),7(2),10(2)	7, 10	5, 7, 10	1 787 (22%)	6 250 (78%)	8 037
5	13	3(2),5(2),8(2)	3, 5, 8	3, 5, 8	2 899 (31%)	6 500 (69%)	9 399

Table A3. Problems (1)–(14), *Situation 1, Case B, Subcase 1.*

ψ years	v %	New hubs to build	New highways to build	New railways to build	Revenue USD, mln	Expenses USD, mln	Investments needed/Profit USD, mln
1	1	8(2),9(2),10(1)	8, 9	8, 9, 10	264	4 450	−4 186
1	7	8(2),9(2),10(1)	8,9	8, 9, 10	1 851	4 450	−2 599
1	13	8(2),9(2),10(2)	8, 9,10	8, 9, 10	3 855	4 860	−1 005
3	1	8(2),9(2),10(1)	8,9	8, 9, 10	793	5 150	−4 357
3	7	8(2),9(2),10(2)	8, 9,10	8, 9, 10	6 228	5 580	648
3	13	3(2),8(2),9(2)	3, 8,9	3, 8, 9	11 681	5 680	6 001
5	1	8(2),9(2),10(1)	8,9	8, 9, 10	1 322	5 850	−4 528
5	7	3(2),8(2),9(2)	3, 8,9	3, 8, 9	10 483	6 400	4 083
5	13	3(2),8(2),9(2)	3, 8, 9	3, 8, 9	19 468	6 400	13 068

Table A4. Problems (1′)–(14), *Situation 1, Case B, Subcase 2.*

ψ years	v %	New hubs to build	New highways to build	New railways to build	Taxes USD, mln (% in total expenses)	Construction expenses USD, mln (% in total expenses)	Total expenses USD, mln
1	1	7(2),8(1),10(2)	7, 10	7, 8, 10	47 (1%)	4 450 (99%)	4 497
1	7	5(2),7(2),8(1)	5, 7	5, 7, 8	217 (5%)	4 550 (95%)	4 767
1	13	5(2),7(2),8(1)	5, 7	5, 7, 8	403 (8%)	4 550 (92%)	4 953
3	1	7(2),8(1),10(2)	7, 10	7, 8, 10	140 (3%)	5 150 (97%)	5 290
3	7	5(2),7(2),8(1)	5, 7	5, 7, 8	651 (11%)	5 250 (89%)	5 901
3	13	3(2),5(2),8(1)	3, 5	3, 5, 8	1 091 (17%)	5 350 (83%)	6 441
5	1	7(2),8(1),10(2)	7, 10	7, 8, 10	233 (4%)	5 850 (96%)	6 083
5	7	3(2),5(2),8(1)	3, 5	3, 5, 8	979 (14%)	6 050 (86%)	7 029
5	13	3(2),5(2),8(1)	3, 5	3, 5, 8	1 818 (23%)	6 050 (77%)	7 868

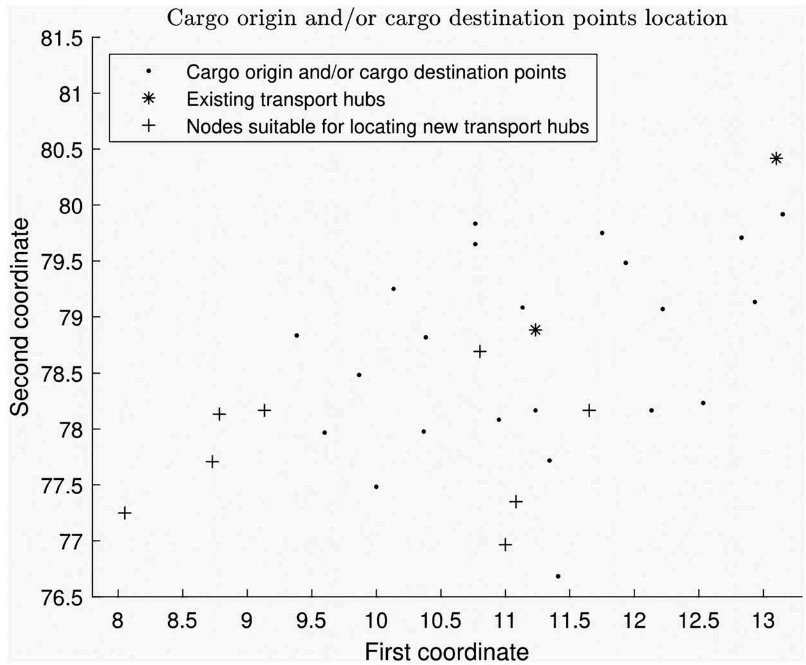
Table A5. Problem (18), *Situation 2, Subcase 1.*

ψ years	v %	New hubs to build	New highways to build	New railways to build	Revenue USD, mln	Expenses USD, mln	Investments needed/Profit USD, mln
1	1	8(2),9(2),10(1)	8, 9	8, 9, 10	185	5 380	−5 195
1	7	8(2),9(2),10(1)	8, 9	8, 9, 10	1 296	5 380	−4 084
1	13	8(2),9(2),10(1)	8, 9	8, 9, 10	2 407	5 380	−2 973
3	1	8(2),9(2),10(1)	8, 9	8, 9, 10	555	6 280	−5 725
3	7	8(2),9(2),10(1)	8, 9	8, 9, 10	3 888	6 280	−2 392
3	13	8(2),9(2),10(2)	8, 9, 10	8, 9, 10	8 096	6 840	1 256
5	1	8(2),9(2),10(1)	8, 9	8, 9, 10	926	7 180	−6 254
5	7	8(2),9(2),10(2)	8, 9	8, 9, 10	7 266	7 770	−504
5	13	3(2),8(2),9(2)	3, 8, 9	3, 8, 9	13 628	7 870	5 758

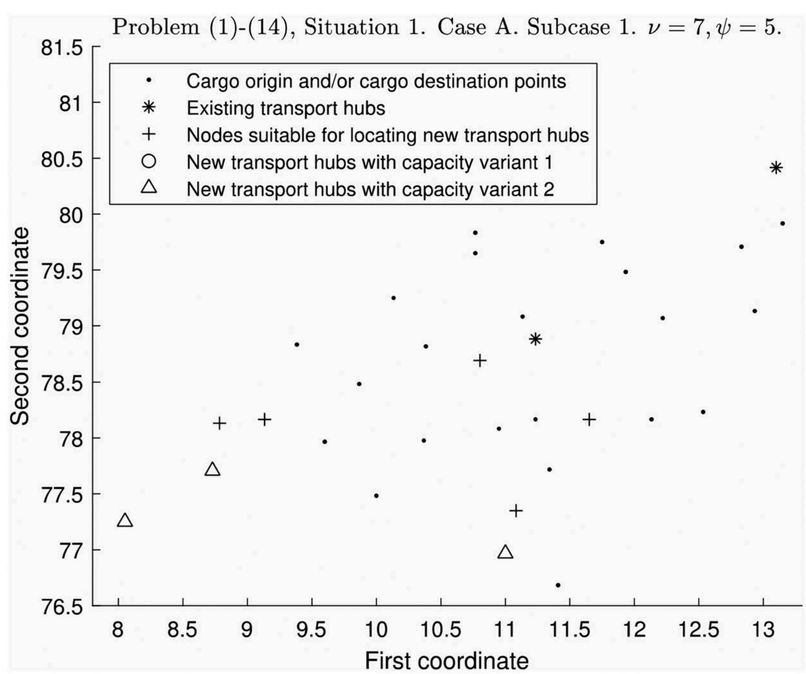
Table A6. Problem (18′), *Situation 2, Subcase 2.*

ψ years	v %	New hubs to build	New highways to build	New railways to build	Taxes USD, mln (% in total expenses)	Construction expenses USD, mln (% in total expenses)	Total expenses USD, mln
1	1	7(2),8(1),10(2)	7, 10	7, 8, 10	61 (1%)	5 380 (99%)	5 441
1	7	5(2),7(2),8(1)	5, 7	5, 7, 8	282 (5%)	5 480 (95%)	5 762
1	13	5(2),7(2),8(1)	5, 7	5, 7, 8	524 (9%)	5 480 (91%)	6 004
3	1	7(2),8(1),10(2)	7, 10	7, 8, 10	182 (3%)	6 280 (97%)	6 462
3	7	5(2),7(2),8(1)	5, 7	5, 7, 8	846 (12%)	6 380 (88%)	7 226
3	13	3(2),5(2),8(1)	3, 5	3, 5, 8	1 418 (18%)	6 480 (82%)	7 898
5	1	5(2),7(2),8(1)	5, 7	5, 7, 8	201 (3%)	7 280 (97%)	7 481
5	7	3(2),5(2),8(1)	3, 5	3, 5, 8	1 273 (15%)	7 380 (85%)	8 653
5	13	3(2),5(2),8(1)	3, 5	3, 5, 8	2 363 (24%)	7 380 (76%)	9 743

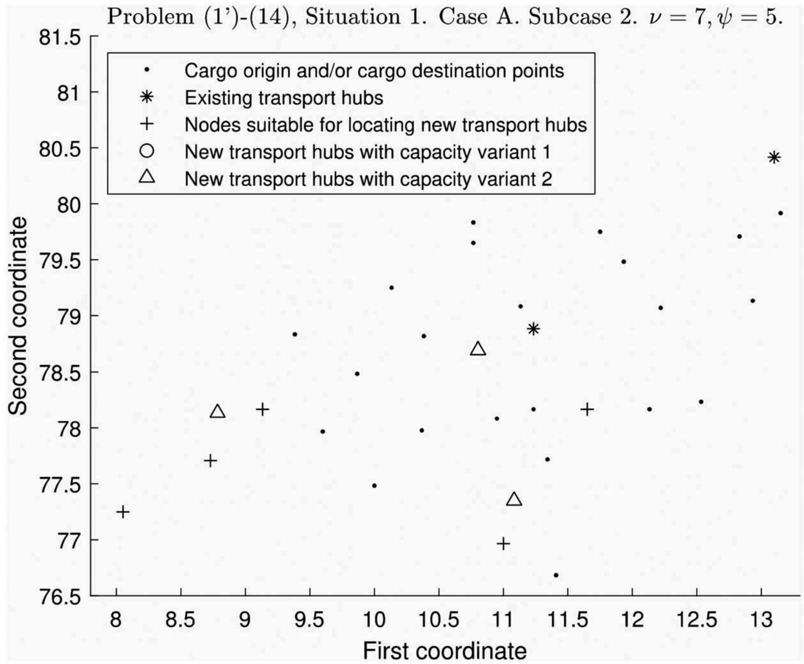
Appendix 3.



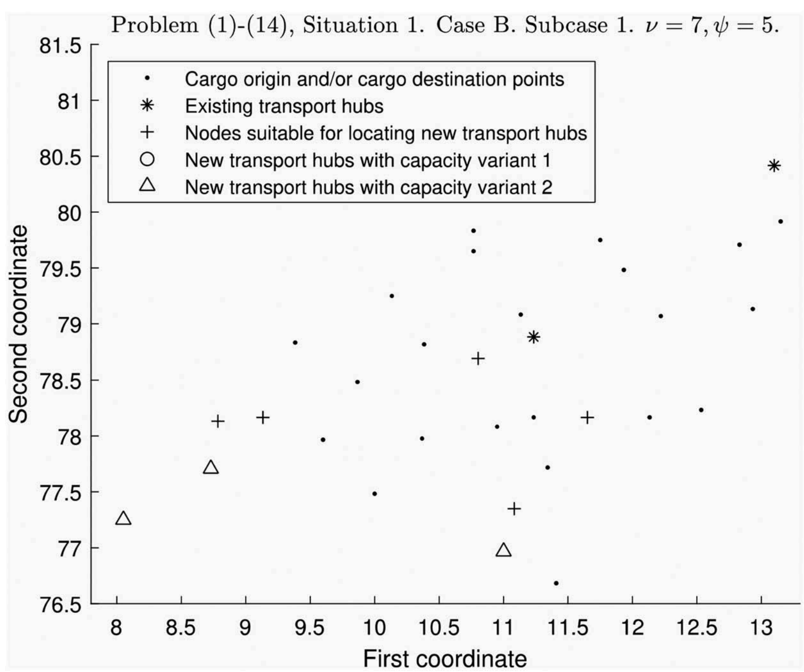
Picture A1



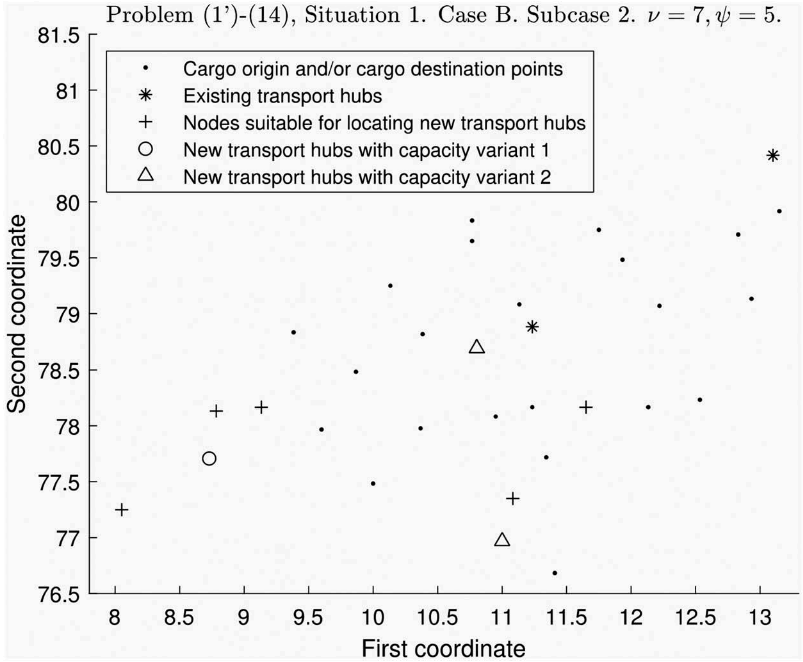
Picture A2



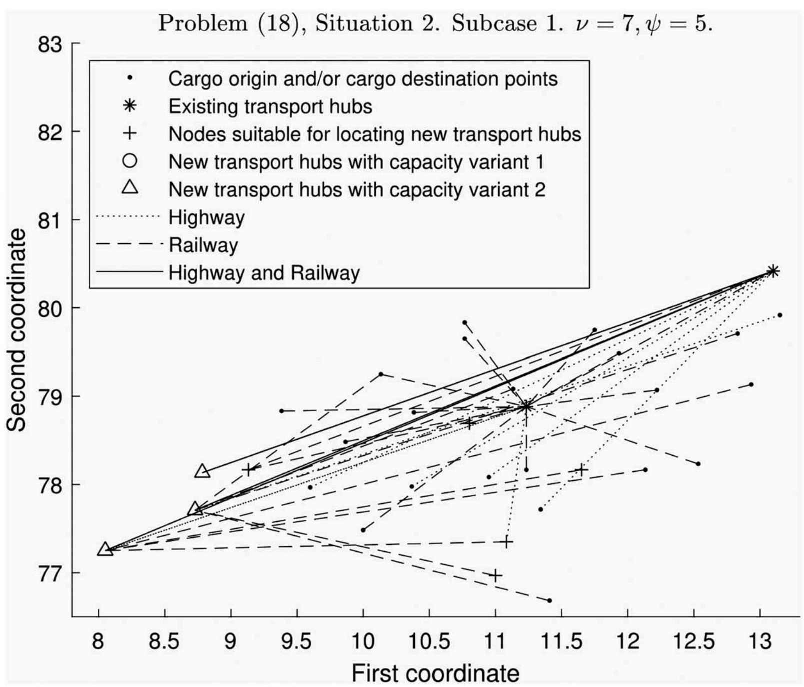
Picture A3



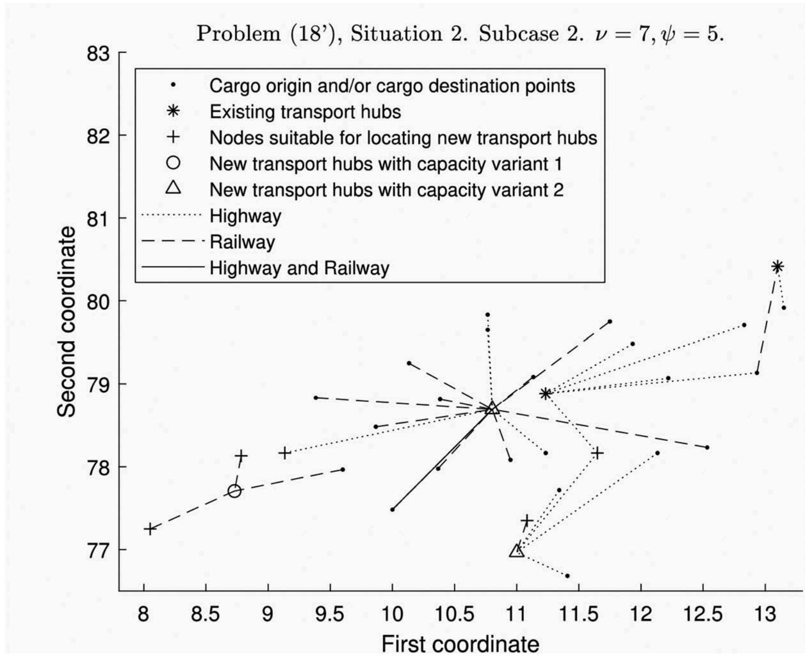
Picture A4



Picture A5



Picture A6


Problem (18'), Situation 2. Subcase 2. ν = 7, ψ = 5.
Cargo origin and/or cargo destination points
Existing transport hubs
Nodes suitable for locating new transport hubs
New transport hubs with capacity variant 1
New transport hubs with capacity variant 2
Highway
Railway
Highway and Railway
Second coordinate
First coordinate
83
82
81
80
79
78
77
8
8.5
9
9.5
10
10.5
11
11.5
12
12.5
13

Picture A7