HSE Masters Exam

INSTRUCTIONS

- Try to answer all questions. If you cannot answer a question, you can skip it.

- If the degree of accuracy is not specified in the question, and if the answer is not exact, give your answer as a decimal fraction rounded to one decimal place.

1. Andrew wrote 5 binaries: $11000_2, 11110_2, 1111_2, 10011_2, 11011_2$. He converted four of them to decimals (in some order) and obtained $15_{10}, 24_{10}, 27_{10}, 30_{10}$. Write the remaining binary number that Andrew did not convert to decimal.
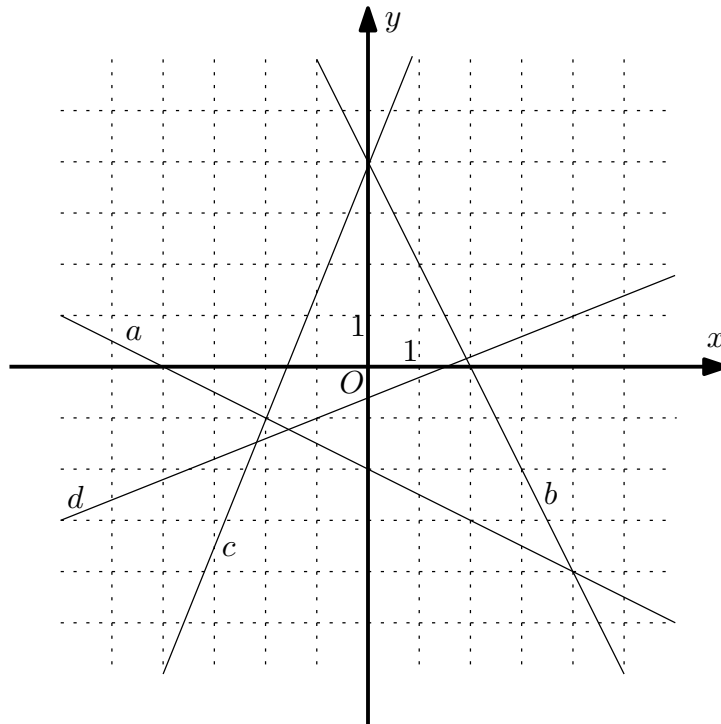(Answer: 10011)

2. Find the remainder when $1234 - 1 - 2 - 3 - 4 + 1 \cdot 2 \cdot 3 \cdot 4$ is divided by 9. Write your answer as a nonnegative integer from 0 to 8.
(Answer: 6)

3. What is the label (a, b, c or d) of the line that is parallel to the line $y = \dfrac{2x}{5} + 4$?
(Answer: d)

4. Find the binary number x such that $100_2 \cdot x = 100001_2 + 111001_2 + 101110_2$. Write your answer as a binary number.
(Answer: 100010)

5. Find y if the following system of equations holds:

$$\begin{cases} 4x - 5y = -1 \\ 7x - 9y = 1 \end{cases}$$

(Answer: -11)

6. Let x_1 and x_2 be the roots of the quadratic equation $-x^2 + 4x + 5 = 0$. Find $\dfrac{10}{x_1} + \dfrac{10}{x_2}$.
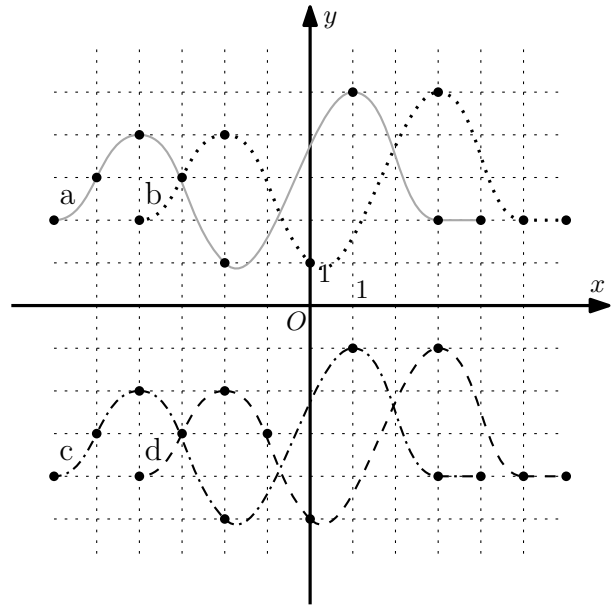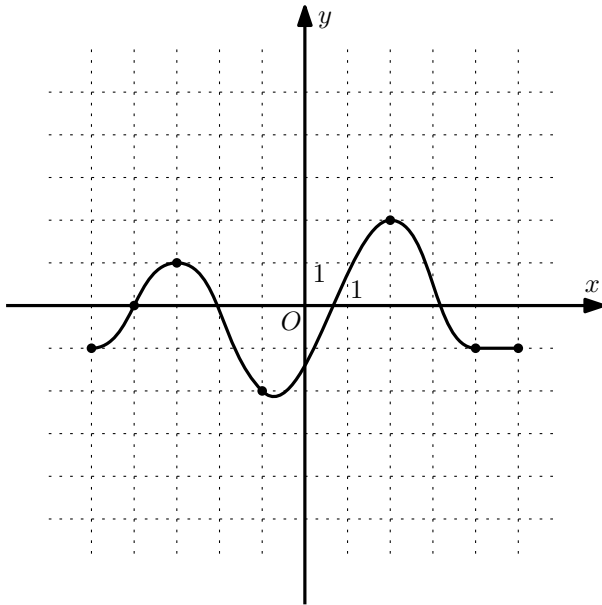(Answer: -8)

7. What is the largest value of a for which the line $y = -1$ intersects the graph of the function $y = x^2 + 4x + a$?
   (Answer: 3)

8. One kilogram (1000 gram) of a baking mixture consists of 40% rye and 60% wheat flour. How much rye flour needs to be added to get a mixture consisting of 50% rye and 50% wheat flour? Write your answer in grams.
   (Answer: 200)

9. The complete graph of the function $y = f(x)$ is shown in the left picture of xy-plane below. What is the label (a, b, c or d) of the graph of the function $y = f(x+1) - 3$?

   (Answer: c)

10. Some university has 500 students. Each student attends exactly six courses. It is known that the number of students in each course is 300. What is the number of courses in this university?
   (Answer: 10)

11. Given $f(x) = x^3 + 7x^2 + ax - 4$, find the value of a whenever $f'(-1) = 7$.
   (Answer: 18)

12. Find the value of x if $\dfrac{18^{3x-4} \cdot 3^{2x-3}}{2^{x+3} \cdot 9^{3x-2}} = 6$.
   (Answer: 4)

13. Tom Sawyer and Huck Finn have to paint a board fence. Tom would take 7 hours to paint the fence alone, and Huck would take 8 hours to paint it alone. When they work together they talk a lot, and their combined output is decreased by 3 boards per hour. Working together, they can paint the fence in 4 hours. How many boards are in the fence?
   (Answer: 168)

14. Hannah produces hand-made jewelry and sells it. The production costs $10 per item. She makes 24 items per month. What is the minimal possible sale price per item if Hannah wants to have a profit of $360 per month? (Answer: 25)

15. A robot is placed in the maze below in A1 position. Consider the program ROBOT:

   • the robot moves right while it is possible;

   • then the robot moves down while it is possible;

- then the robot moves left while it is possible;

- then the robot moves up while it is possible.

Where will the robot stop after executing the program ROBOT 20 times? Give your answer as a pair of a letter and a number, e.g., C6.
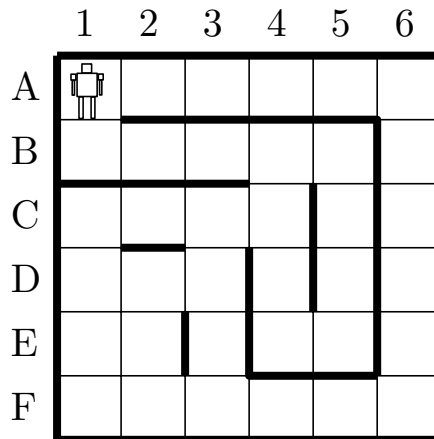
(Answer: B4)

16. George has \$100 in his bank account and no other money. In the bank there are only two possible transactions: withdraw \$35 from an account or add \$14 to an account. What is the maximal sum George can take from the bank after a sequence of such transactions?
   (Answer: 98)

17. Non-zero real numbers a_1, a_2, a_3, a_4 form a geometric progression. Find $a_1 \cdot a_2 \cdot a_3 \cdot a_4$ if

$$a_1 + a_2 + a_3 + a_4 = 15, \quad \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \frac{1}{a_4} = 3.$$

   Note that the general form of a geometric progression is $a, aq, aq^2, aq^3, \ldots$, where a is the first term of the progression and q is its common ratio.
   (Answer: 25)

18. How many 5-letter words can be formed using letters A, B, C if repetitions of letters are allowed but the number of A's in the word is even?
   (Answer: 122)

19. Find the minimal integer x such that at most one of the following conditions is false:

- $4x - 4 > 4$;

- $2x + 1 \geqslant x + 3$;

- $15 < 4x + 10 \leqslant 50$;

- $\dfrac{x}{3} < 6$.

(Answer: 2)

20. Five integers 2, 4, 6, 8, 10 are written on the blackboard. There are two possible operations:

- erase any two numbers a, b, and replace them with one number equal to $ab - a - b + 2$ (e.g., if this operation applied to the numbers 2 and 4, then these two numbers are erased and the number $2 \cdot 4 - 2 - 4 + 2 = 4$ is added);

- erase any three numbers a, b, c, and replace them with one number $a \cdot b \cdot c$.

After a sequence of such operations there is the only one number left on the blackboard. What is the smallest possible value of this number?
   (Answer: 946)