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Simulation of Drop Oscillation Using the Lattice Boltzmann Method

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Abstract—We simulate the oscillation of the viscous drop in the viscous liquid. We combine methods of chromodynamics model and Shan-Chen pseudo-potential for the immiscible fluids. We measure the frequency of the first nontrivial eigenmode using the initial ellipsoid form of the drop. Drop oscillates about the equilibrium spherical form of radius R. Computed frequency as a function of the radius R follows to the well known Rayleigh formula. We discuss the simulation setup in the framework of the Lattice Boltzmann method.

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1. INTRODUCTION

The study of the multi-component fluids is an active subject last years with many applications in the manufacturing [1], printing [2], oil recovery [3], and cyber-physical [4] systems, among many others. The most promising method for simulations of the processes is the Lattice Boltzmann method (LBM) [5]. The LBM is well suited to the simulations of complex fluids in complex geometry that is not practically realizable with the direct simulations using the hydrodynamic approach. Another essential advantage of LBM is the possibility of extensive and massively parallel simulations using supercomputer capabilities.

In the paper, we analyze the accuracy of the LBM using as an example the oscillating drop immersed in the fluid. We build a model close to the realistic parameters used in the laboratory experiments [6] and in-vivo medical experiments [7], and compare results with the known analytical solution.

2. DROP OSCILLATION MODEL

Rayleigh [8] has calculated eigenfrequencies of the drop with density ρ_D surrounded by the fluid with density ρ_F

$$\omega^2 = \frac{n(n-1)(n+1)(n+2)}{(n+1)\rho_D + n\rho_F}\frac{\sigma}{R^3}, \tag{1}$$

where σ is the surface tension, and R is the drop radius. The formula was checked experimentally in [9] and it is correct for the small amplitude oscillations.

To generate the first nontrivial eigenmode $n = 2$ we use as an initial condition the ellipsoid with the volume equivalent to the volume of a sphere of radius R. Competition between inertia and surface tension leads to the oscillation of the liquid drop immersed in a second fluid.

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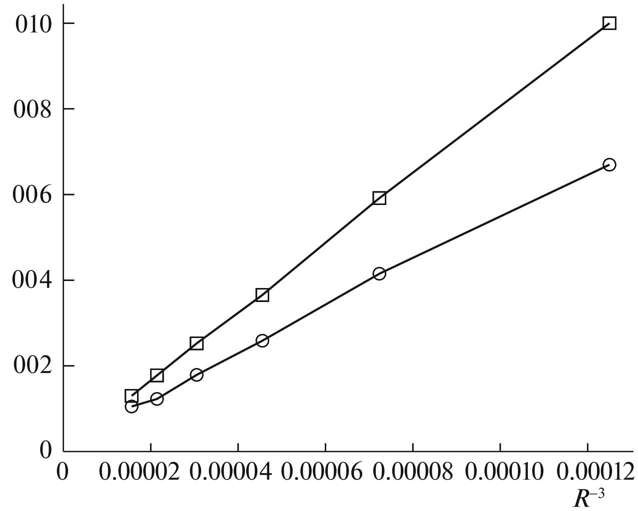

Fig. 1. Dependence of ω^2 from R^{-3} for two values of relaxation time τ. Squares corresponds to $\tau = 1$ and circles corresponds to $\tau = 3/4$. Lines are the guide for the eyes. The vertical axes are scaled with the factor 10^7.

Simulations are based on the Shan–Chen method for multiphase fluid flows [10]. We use three dimensional D3Q27 representation of velocities c_i pointing from the center of the cube to 8 vertices, to the middle of 12 edges, and to the middle of 6 faces, and to 1 center (zero velocity), i.e. $i = 0, 1, 2, \ldots, 26$ (see, f.e., Refs. [11] and [12]). The time and three-dimensional space (D3) is discrete and measured in units of Δt and Δx, correspondingly. Distribution function is defined $f_i^j(\vec{x}, t)$ for each of two components of fluid ($j = 1, 2$) at lattice position $\vec{x}$ and evaluated in time by the equation

$$f_i^j(\vec{x} + \vec{c}_i \Delta t, t + \Delta t) - f_i^j(\vec{x}, t) = \Omega_i^j(f) + S_i^j,$$

with the collision operator $\Omega_i^j(\vec{x}, t)$ and collision term $S_i^j = \vec{F}^j \cdot \vec{c}_i$ controls the strength of the interaction potential between fluid components $\{j\}$ and force F^j is defined through the Shan–Chen potential [10]. Collision operator is written in the BGK form [13]

$$\Omega_i^j(f) = -\frac{f_i^j - \tilde{f}_i^j}{\tau} \Delta t,$$

with equilibrium distribution function

$$\tilde{f}_i^j(\vec{x}, t) = \mathrm{w}_i \rho^j(\vec{x}, t) \left(1 + \frac{\vec{u} \cdot \vec{c}_i}{c_s^2} + \frac{(\vec{u} \cdot \vec{c}_i)^2}{2c_s^4} - \frac{\vec{u} \cdot \vec{u}}{2c_s^2}\right),$$

where sound speed $c_s = \Delta x / (\Delta t \sqrt{3})$.

3. SIMULATION DETAILS

We perform simulations with the set of parameters: $\Delta x = 1$, $\Delta t = 1$, $\rho_{D,F} = 1$, and collision constant $G = 2.7$ (controls the interface region of two fluids). We perform simulations in the cube with bounce-back boundary conditions for two linear sizes $200\Delta x$ and $250\Delta x$ in order to check the effect of the boundary conditions.

The values of the weights w_i and speeds c_i for D3Q27 LBM can be found in the book [12].

The size of the spherical drop is R. The initial state formed as ellipsoid enlarged in z direction with an initial length of z-axis equal to $2R$, and symmetric in (x, y) direction. The volume of the ellipsoid is $4/3\pi R^3$. The density of the drop is ρ_D inside the ellipsoid and $\rho_D/10^4$ outside the ellipsoid. The density of the second fluid outside the ellipsoid is ρ_F, and inside the ellipsoid is $\rho_F/10^4$. The initial speed of both fluids is zero.

Table 1. Estimated frequency of the drop oscillation ω as a function of R for two values of the relaxation time τ

R	$\tau = 1$ ω^2	$\tau = 3/4$ ω^2
20	6.67E-7	1.00 E-6
24	4.15E-7	5.92 E-7
28	2.59E-7	3.66 E-7
32	1.79E-7	2.53 E-7
36	1.23E-7	1.78 E-7
40	1 .05E-7	1.30 E-7

4. SIMULATION RESULTS

We performs simulations for two values of viscosity defined [12] in LBM as $\nu = c_s^2(\tau - 1/2)$, with $\nu = 1/6$ and $\nu = 1/12$ and relaxation time τ, correspondingly. In simulations, the drop experience oscillations with frequency ω, and oscillations dumped in time. The decay is fast and proportional to $1/R^3$ in accordance with the analytical prediction [14].

Table 1 shows values of frequency estimated from the simulations for a number of drop radius R and for two values of relaxation time τ.

5. DISCUSSION

Results shown in the Table are in good agreement with theory. The figure shows a square of the frequency ω as a function of the inverse cube of the radius, for both values of viscosity ν. Firstly, it is visible that frequency follows the straight line in accordance with Rayleigh formula (1). Secondly, the slope of the fitting lines should reflect the surface tension of the drop. The surface tension is proportional to the weight of w, as one can see from the second line of expression (9.36) in the book [12]. Indeed, the ratio of line slopes is very close to 3/4, which is the ratio of corresponding values of τ for two sets of runs.

We can conclude that developed setup can be used for the accurate description of the movement of the ensemble of the drops immersed in the fluid, which is of the very practical interest.

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