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Protocol of measuring hot-spot correlation length for SNSPDs with near-unity detection efficiency

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***Abstract*—We present a simple quantum detector tomography protocol, which allows, without ambiguities, to measure the two-spot detection efficiency and extract the hot-spot interaction length of SNSPDs with unity intrinsic detection efficiency. We identify a significant parasitic contribution to the measured two-spot efficiency, related to an effect of the bias circuit, and find a way to rule out this contribution during data post-processing and directly in the experiment. From the data analysis for waveguide-integrated SNSPD, we find signatures of the saturation of the two-spot efficiency and hot-spot interaction length of order of 100 nm.**

***Index Terms*—Nanowire single-photon detector, Superconducting materials measurements, Algorithms**

I. Introduction

Superconducting nanowire single photon detectors (SNSPDs) functionality is based on the local suppression of the superconducting order parameter upon photon absorption, causing a change in resistance which can be converted into a recordable electrical voltage signal. The hot-spot (HS), produced by photon absorption, is the key player in the operation of SNSPDs. Size and profile of the order parameter suppression along the wire, and their dependence on material parameters and temperature, are widely discussed in the literature, but it a direct measurement of these properties is experimentally challenging – HS is too small and too short-living. Because of this, there still exist large spread of opinions about these properties, even in widely used NbN. For instance, the size of HS, estimated as being from 20 to 80 nm, the dependence of the size and the depth of the spot on the superconducting material and substrate properties, on the photon energy, on temperature etc., are under debates [1-5].

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The most direct way to measure the HS size, or rather hot-spot interaction length *s*, is the method known as quantum detector tomography (QDT) [6]. For SNSPDs, it was introduced by Renema et al. [7]. Recently, this group used this method to extract s in NbN [8] and reported a value of 23±2 nm. Unfortunately, because of essentially non-uniform geometry of their samples and the necessity to account for linear losses separately, ambiguous interpretation might arise. Close approach was formulated in [9], but use of long meanders with insufficient homogeneity didn't allow to extract unambiguous information about *s*. In [10], a technic close to QDT was used, but the main goal was to determine HS lifetime rather than *s.*

In this paper, we propose a simple protocol to extract the hot-spot interaction length from detector tomography data, i.e, set of dependencies of count probability vs mean number of photons in a laser pulse. The proposed method allow to study the dependence of *s* on any parameter. The protocol requires accurate measurement of the number of absorbed photons. This requirement is fulfilled for waveguide integrated SNSPD (WSNSPD) [11], for which the amount of power absorbed in the detector can be measured directly [12]. We analyze data for several WSNSPD which demonstrate saturation of the detection efficiency at high bias current and estimate the hot-spot interaction length.

II. Protocol for extracting the hot-spot interaction length from SNSPD tomography data.

To introduce the hot-spot interaction length, we consider the situation when exactly two photons are absorbed simultaneously in the uniform strip of an SNSPD at two points with coordinates along the strip x_1 and x_2. The probability that they produce a "double-spot count" is $P_2(x_1, x_2)$. The term "double-spot count" means that if one of the spots is removed, the probability turns to zero. On the contrary, if photon count is generated in presence of only one spot, we adopt the term "single-spot counts" and, in experimental data processing, the corresponding probability $P_1(x)$ should be subtracted. The probability to have a double-spot count, if exactly two photons are absorbed anywhere in the strip, i.e. the double-spot detection efficiency, is defined as:

$$\eta_2 = \frac{\int_0^L \int_0^L dx_1 dx_2 \rho(x_1)\rho(x_2) P_2(x_1, x_2)}{2\left(\int_0^L dx \rho(x)\right)^2} \quad (1)$$

with ρ(x) the probability density of absorbing photon at x, $\int dx\rho(x)$ is the total probability of absorption, and L the length of the strp. If the strip is uniform, P_2 depends only on the difference $\Delta x=x_1-x_2$. Assuming that the scale at which P_2 falls to zero is small compared to the strip length, and that ρ(x) varies slowly at this scale, one can integrate over x_1-x_2 and obtain $\eta_2=BL_c/L$, or

$$\eta_2 = \frac{1}{B}\frac{L_c}{L} \tag{2}$$

Here, we defined the hot-spot correlation length L_c

$$L_c = \frac{1}{2}\int_{-\infty}^{+\infty} d\Delta x P_2 \tag{3}$$

If P_2 falls abruptly from 0 to 1 at some distance between the spots, L_c simply equals this distance, which justifies the term used to call it. In this case, (2) has a simple interpretation – to have a double-spot click, the two photons must be absorbed within the nanowire segment of the length L_c, and the probability of this event is L_c/L. The "form-factor"

$$B = \frac{(\int dx\rho(x))^2}{L\iint dx\rho^2(x)} \tag{4}$$

If the absorption is uniform along the strip, $\rho(x)=a/L$, with a the coefficient of absorption, then B=1. Otherwise, B<1.

Generally, the emergence of the voltage response after the appearance of the hot-spot or of the two hot-spots is a probabilistic process, P_2 can be much less than unity even at $x_1=x_2$. For uniform nanowire SNSPD, the single spot intrinsic efficiency η_1, i.e. the probability of response to a single hot-spot, approaches 1 at sufficiently large bias current I. This same behavior is expected for $P_2(x_1=x_2)$. In this case, L_c characterizes the interaction length of the hot-spots, as previously discussed. Hence, to measure the hot-spot interaction lengths s, one needs to observe saturation of $\eta_2(I)$ with the increase of I, at some level η_2^{sat}, which can then be interpreted as s/BL, and calculate s as

$$s = BL\,\eta_2^{sat} \tag{5}$$

Then, one can study the dependence of s on different parameters - the strip width, the sheet resistance, temperature, etc.

In a typical experiment, the number of absorbed photons cannot be controlled. The mean number of incoming photons, hence, the mean number of absorbed photons M are instead used to probe the device response. To find η_2, one can use the technic known as the quantum detector tomography. The detector is irradiated by a pulsed laser and M is assumed controllable and known. By measuring the dependence of photon-count probability on M, one can find single- and double spot efficiencies $\eta_{1,2}$.

The probability of obtaining a single-spot count with the mean number of photons absorbed in the detector M can be calculated as follows. Having, on average, M absorbed photons, the average number of hot-spots producing clicks, here named "resultant spots", is $\eta_1 M$. Because photons are absorbed independently, the statistics of this number is Poissonian. In particular, the probability of having zero resultant spots, i. e. the probability that there will be no click, is $\exp(-\eta_1 M)$. Then the probability that there will be a click is $P^{(1)} = 1-\exp(-\eta_1 M)$. Expanding this in series, we get the approximate formula for the case when this probability is low:

$$P^{(1)} \approx \eta_1 M - \frac{1}{2}(\eta_1 M)^2 \tag{6}$$

Similarly, the probability to have double-spot count, in the limit of low probability, is:

$$P^{(2)} \approx \frac{1}{2}(\eta_2 M^2) \tag{7}$$

The factor 1/2 appears here because of Poissonian statistics. Higher order absorption (i.e. triple, quadruple,… hot spots) can also be considered, although their corresponding efficiencies scales with s/L, high photon number is needed thus increasing the probability of introducing measurement artifacts, which will be discussed in the following.

In most of the experiments, it is not possible to distinguish counts generated from single- or double spots, but only register the presence or absence of the click. Hence, the total click probability in response to the laser pulse is

$$P \approx \eta_0 + \eta_1 M - \frac{1}{2}(\eta_1 M)^2 + \frac{1}{2}\eta_2 M^2 \tag{8}$$

Here, η_0 accounts for dark counts. Measuring dependencies of P vs. M and fitting them with (8), one can find η_1 and η_2. Repeating this fit for data at different bias currents, one can obtain $\eta_1(I)$ and $\eta_2(I)$ – which is the quantity of interest.

The important assumption of this procedure is, that all the parameters which η_i can depend on, are fixed, i.e. do not vary with M. Otherwise, the dependence on M enters the coefficients η_i and the fit (8) does not allow to extract them correctly. One known effect of this kind is the so-called AC-biasing [13]. A parasite current due to AC coupling with the readout circuits adds to the detector bias current causing artifacts in the detector response, including an increase of the count rate, introducing a dependence of $\eta_{1,2}$ with the count rate, and hence on M. To account for this effect, we replace $\eta_{1,2}(I)$ in (8) by $\eta_{1,2}(I+\Delta I)$, where $\Delta I=I\varphi P(M)$, with $\varphi=f\tau$ the product of the repetition rate of the laser pulses f and the duration of the SSPD response pulse τ. Expanding all the quantities in powers of M and keeping terms up to the second order, one comes to the corrected version of (8):

$$P(M) \approx \eta_0 + \eta_1 M + \frac{1}{2}(a_2 M^2) \tag{9}$$

Here, $a_2=\eta_2+a_2^{stat}+a_2^{bias}$, with $a_2^{stat}=-(\eta_1)^2$ and $a_2^{bias}=2I\varphi\eta_1(d\eta_1/dI)$. We neglected the analogous corrections arising from $\eta_0(I)$, because η_0 is very small in our data. After extracting a_2 from the fit (9), one has to subtract the systematic errors a_2^{stat} and a_2^{bias} to obtain $\eta_2(I)$. If the factor φ is not known accurately, the way to find a_2^{bias} is to vary repetition rate f.

Another effect to take care is optical heating of the detector, which can reduce the detector critical current and introduce an increase of the count rate. This contribution might become significant only when the repetition rate f is comparable with the cooling rate, i.e. the energy relaxation rate of the superconducting film, which does not exceed 1 ns even for the 'slowest' material, WSi [14]. Hence, for the repetition rates of tens of MHz, heating doesn't seem to cause a systematic error, independently on the amount of energy contained in the laser pulse.

Based on the above consideration, we formulate the following conditions to unambiguously determine *s*.

1) The detector is uniform enough, because uniformity of the strip is assumed in the derivation of the upper boundary on $\eta_2(I)$, *s/L*.
2) One knows *M* exactly. Together with the requirement 1), this means that it is desirable to have detector which exhibits saturation of $\eta_1(I)$ at large *I*. The saturation by itself indicates uniformity of the strip, and interpreting the saturation as $\eta_1=1$, one can find the proportionality coefficient between *M* and the mean number of photons in the incident pulse.
3) One expects not too small ratio *s/L* in the detector, to be able to extract $\eta_2(I)$ against the background of single-spot counts. This means that the length *L* should be small, but not too small, to keep the condition 1). Because we expect that *s* is of order of the estimated hot-spot size, tens of nm, or of the strip width, of order of 100 nm, it seems that the reasonable length is of order of one or several μm.

Up to our knowledge, one of the best realizations of SNSPD for this goal, is the waveguide-integrated SNSPD. While all the above conditions are satisfied for the best WSNSPD devices, also, they provide very efficient coupling of photons to subwavelength-sized nanowire, and an opportunity to measure the amount of absorbed photons directly, by controlling the power which passes through the detector [12].

WSNSPD has non-uniform absorption of photons: the power decays exponentially while propagating along the strip, and so does the probability of absorption of a photon: $\rho(x)=(1/L_0)\exp(-x/L_0)$, with L_0 the exponential decay length. Calculating the formfactor for this law of absorption using (4), one comes to

$$B_{WSNSPD} = -\frac{2a}{(2-a)\ln(1-a)} \qquad (10)$$

III. Estimation of Hot-Spot Interaction Length in NbN, Using WSNSPD

We apply the protocol, described in the previous section, with the experimental data obtained on several WSNSPDs, made of disordered 5 nm (nominal) thick NbN film on SiN optical waveguide. Details on fabrication and measurements can be found in [12]. Parameters of the studied samples are listed in Table I. We coupled light on-chip at 1550nm wavelength. WSNSPDs demonstrated clear saturation of $\eta_1(I)$ at large *I*, see the example on the inset in Fig.1 for the sample I33. Their total length varied from 80 to 280 um, which is greater than the optimal value mentioned at the end of the previous section but significantly shorter than a standard meandered SNSPD. At low *I*, data demonstrated clear non-linear dependences of *P* on *M*, as shown in Fig. 1.

TABLE I
Parameters of the Samples

Label	NW width (nm)	NW length (nm)	OCDE[a] (at saturation)
I 33	80	4 x 70	77,55 %
F 34	80	2 x 40	26,68 %
C 30	100	2 x 70	72,63 %
E 33	80	2 x 50	60,14%

[a]OCDE is the on-chip detection efficiency at saturation of $\eta_1(I)$. This value is equal to the absorption coefficient of the nanowire.

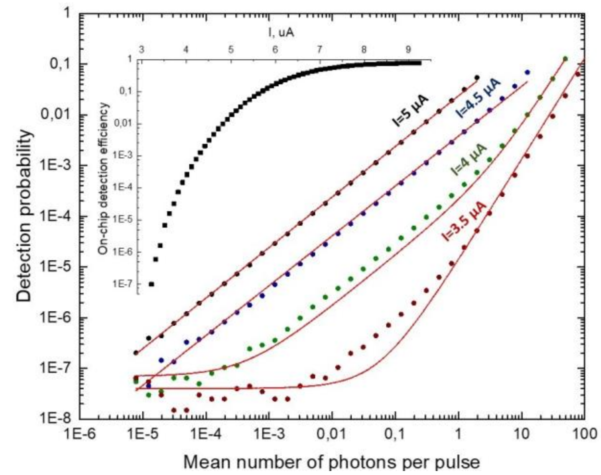

Fig. 1. Dependences of count probability on mean number of photons per pulse for the sample I33, and polynomial fits. Inset – detection efficiency for the same sample vs. bias current, demonstrating clear plateau at large current.

To find $\eta_2(I)$, we proceeded as follows. First, we calculated mean number of absorbed photons *M*, just multiplying the known mean number of incident photons by the on-chip detection efficiency (OCDE) at saturation, assuming, that each absorbed photon produces a click (see condition 2). Then, we fitted the dependencies *P*(*M*) by the 2-nd order polynomial (4). To be able to restrict fitting by only second order in *M*, we considered only data far from the saturation of *P*(*M*), below 10^{-1}, such that *P*(*M*)<<1. We repeated the fit for data acquired at different bis current, such that the accuracy of the fitting was enough to extract a_2.

We found for all the analyzed samples, that after some signatures of saturation, a_2 starts to rise with *I*, reaching values in the order of 10^{-1}. An example is presented on the Fig. 2. This is unreasonable for η_2, because this corresponds to *s* of order of 10 μm. We attributed this behavior to the effect of AC-biasing. Quantitative comparison clearly shows that this is a right explanation. An example for the sample I33 is given on the Fig. 2. Red dots is a_2, and black dots is $a_2^{stat}+a_2^{bias}$, calculated from $\eta_1(I)$ (empty dots), with φ the only fitting parameter (hence it is impossible to fit more than one arbitrary point). One sees that the increase of a_2 at the largest currents is nicely fitted by $a_2^{stat}+a_2^{bias}$. Similar behavior has been observed for all other samples. We also notice, that the fitting parameter φ depends systematically on the sample length – the shorter the sample, the smaller the φ, i.e. the shorter the time $\tau=\varphi/f$. Finally, the absolute values of τ for all samples were close to the expected fall time of the voltage pulse, estimated as the ratio of kinetic inductance to the load impedance [12].

To subtract the contribution $a_2^{\ stat} + a_2^{\ bias}$ from a_2 to obtain $\eta_2(I)$, we either discarded points as unrelated to η_2, if they belonged to $a_2^{stat}+a_2^{bias}$ curve; or kept them, i.e. assume $\eta_2=a_2$, if $(a_2^{stat}+a_2^{bias})<<a_2$ and the correction is not important, as mentioned on Fig.2. The bias current dependency of η_2, determined considering only the selected datapoints, is shown in the Fig. 3 for all 4 analyzed samples. We rescaled the data, multiplying each η_2 to the respective nanowire length. In presence of saturation of $\eta_2(I)$, the corresponding value would be the hot-spot correlation length (The formfactor *B*, calculated using (10), is from 0.84 to 0.99 for our samples, and we don't account for it).

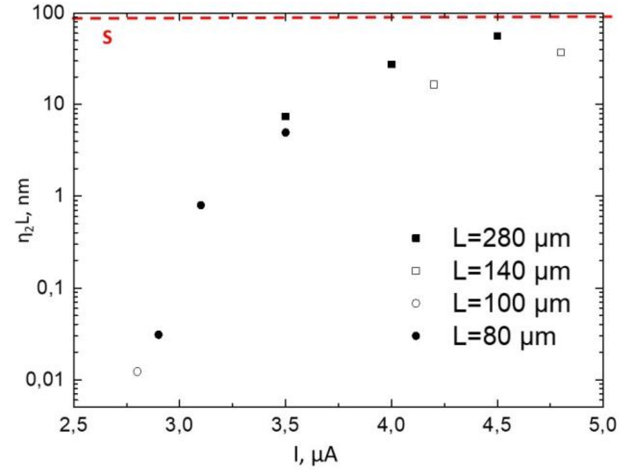

Fig. 3. Double-spot efficiency, rescaled to a length, $\eta_2 L$, vs. current. The limit which $\eta_2 L$ tends to approach with the increase of current, should be equal to the hot-spot interaction length *s*.

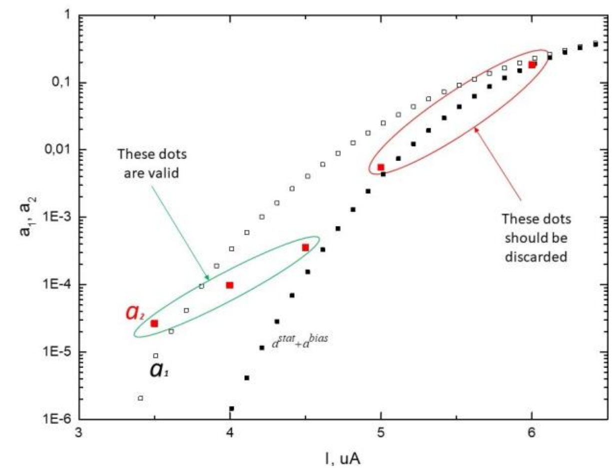

Fig. 2. The coefficients $a_1=\eta_1$ (empty dots; obtained by separate measurement at low power) and a_2 (red dots) as functions of current, extracted from the fit for sample I33. Black dots are for $a_2^{\ stat}+a_2^{\ bias}$, calculated from η_1(I) – see text.

The low number of available datapoints require some precaution in the accuracy of our statements. Nevertheless, it appears that all the points belongs to a universal curve, which suggests a natural explanation. Indeed, all the samples are made of the same film, have the same width (except the sample C30, which is just the most deviated from the common curve), and seems to be highly uniform. So, their strips should be locally identical, and the probability to have double-spot count should has the same dependence on the distance between spots – for all samples.

If, further, we assume that we see the saturation of $\eta_2(I)L$ on this dependence, this yields in *s* approaching 100 nm. While the number of points is too small to state this with confidence, we guess we can say, that, according to our analysis of the data, *s* most likely exceeds the value of 23±2 nm obtained in [15].

IV. Conclusion

In conclusion, we summarize our findings as follows:

- We extracted the quadratic contribution to the photon count probability, even at bias current values in which the linear (single-spot) count response dominates, -- an essential prerogative to see the expected saturation of η_2.
- We found a systematic extra contribution to the quadratic counts, which we correlate, with high level of confidence, to bias coupling with the readout circuitry. We eliminated this contribution to obtain pure η_2 data. A procedure that has never been performed in SNSPD tomography.
- We found a systematic monotonous dependence of $\eta_2 L$ vs. current. We see signatures of the expected saturation of $\eta_2(I)L$ with the increase of *I* and estimate the hot-spot interaction length about 100 nm. This value is close to the width of the WSNSPD strip adopted, which is expectable in the picture where hot-spots interact if they formed within the same square of the strip.

From our findings, we believe that tomography data for several µm long nanowires should allow to extract *s* more accurately. Indeed, reducing *L* should proportionally increase η_2 (but not η_1) and shift-up the current at which the systematic error contribution $a_2^{stat}+a_2^{bias}$ starts being a dominating contribution to a_2. Because with the samples of total length of 140 or 280 µm we see signatures of saturation of $\eta_2(I)$, we judge that samples of 1 or 2 orders of magnitude smaller length shall allow to observe clearly the saturation regime.

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