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Equidistant Arrangement of Agents on Line: Analysis of the Algorithm and Its Generalization

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Abstract—Consideration was given to generalization of one of the formation control algorithms, that of equidistant arrangement of agents over a fixed interval. In distinction to the earlier approaches that are based on the equations of the first order, a second-order algorithm was proposed. It was proved to be stable and with proper selection of the adjusted parameter able to provide a higher rate of convergence in comparison with its first-order counterparts. Relation was demonstrated between the problem of arrangement over an interval and the classical problem of consensus. Examples of the results of modeling were presented.

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1. INTRODUCTION

Decentralized/distributed formation control may be characterized as a generation of geometrical patterns in space by a group of agents using the local exchange of information such as the coordinates of the neighbor agents or the relative distances to them. Formation of certain geometrical patterns is often encountered in the nature as behavior of the flocks of birds of passage making up V-shaped formations, fish shoals, and animal herds. For the commercial and military reasons, formation control is one of the most important lines of research in the theory of up-to-date multiagent systems. This problem is rooted in the elementary geometry. As early as in 1878 G. Darboux studied behavior of polygons by applying to the vertices a simple averaging rule at each iteration of his algorithm [1]. In this way a special case of the well-known problem of consensus for the discrete systems was studied in other terms without using the vector-matrix description and studying the system matrix spectrum. As was indicated in [2], the problem of formation control of agents possessing information about the relative arrangement of their neighbors is that of consensus. At the same time, there are algorithms enabling one to determine the geometrical patterns with the use of other approaches. The simple linear algorithm of equidistant arrangement of agents over an interval with fixed endpoints provides an example. The discrete and continuous cases of this algorithm were first considered in [3] within the framework of the control of the multiagent systems. Later on a similar continuous algorithm was proposed and some of its generalizations were obtained in [4] independently of [3]. A discrete version of the arrangement algorithm over the interval and some other algorithms were studied in [5]. In the aforementioned publications, all algorithms obey the first-order equations. Modifications of the cyclic pursuit algorithm with the double integrators used as the agent models were obtained and studied in [6, 7]. Various algorithms with second-order integrators were also studied in [8].

The present paper relies on [6–8] to propose a new, second-order algorithm of equidistant arrangement over an interval and analyzes in detail its stability and rate of convergence as compared with the first-order algorithm. Stability of the system is studied using the criterion for stability of the multiagent systems [9, 10]. Moreover, it was demonstrated that after certain transformations the problem of equidistant arrangement over an interval can be reduced to that of consensus.

2. FORMULATION OF THE PROBLEM

A linear law for moving the agents so as to arrange them uniformly over an interval with boundaries that are either fixed or vary according to a certain law was proposed in [4]. By the agents are meant indexed points on the line which can change their arrangement. The control law implies that there exists information about the distances between the agent and its neighbors that are nearest in number. Within the framework of the proposed strategy, each agent moves toward the middle of the interval connecting its neighbors. At that, the first and last agents seek to occupy a position between the interval boundaries and their nearest-in-number neighbors. Motion is controlled by varying the agent's velocity, that is, the algorithm obeys differential equations of the first order.

For the case of the interval with fixed endpoints, we describe the algorithm in formal terms. Let $x_i(t)$, $i = 1, 2, \ldots, n$, be the coordinate of the ith agent's position on the line at the time instant $t \geqslant 0$. We denote by x_b and x_e the coordinates of the interval's beginning and end, respectively. Let the motion of each agent follow the first-order integrator

$$\dot{x}_i = u_i, \quad i = 1, 2, \ldots, n. \tag{1}$$

The following law of control is proposed [4]:

$$\begin{aligned} u_1 &= \frac{x_2 + x_b}{2} - x_1, \\ u_i &= \frac{x_{i+1} + x_{i-1}}{2} - x_i, \quad i = 2, \ldots, n-1, \\ u_n &= \frac{x_e + x_{n-1}}{2} - x_n. \end{aligned} \tag{2}$$

The dynamics of the entire system obeys the vector differential equation of the first order

$$\dot{x} = Ax + b, \tag{3}$$

where

$$x = \begin{bmatrix} x_1, & x_2, & \ldots, & x_n \end{bmatrix}^\top, \tag{4}$$

$$A = \begin{bmatrix} -1 & 0.5 & 0 & \ldots & 0 \\ 0.5 & -1 & 0.5 & \ldots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & \ldots & 0.5 & -1 \end{bmatrix} \in \mathbb{R}^{n \times n}, \tag{5}$$

$$b = [0.5x_b, 0, \ldots, 0.5x_e]^\top \in \mathbb{R}^n. \tag{6}$$

It is also indicated in [4] that the eigenvalues of the matrix A are given by

$$\lambda_k = -2\sin^2 \frac{k\pi}{2(n+1)}, \quad k = 1, \ldots, n, \tag{7}$$

that is, system (3) is stable. By introducing the change of variables,

$$z = x - x^*, \tag{8}$$

where

$$x^* = -A^{-1}b, \tag{9}$$

it is possible to go from (3) to the uniform system

$$\dot{z} = Az, \tag{10}$$

for which the following is true [4]:

—system (10) is stable;

—system $z(t) \to 0, t \to \infty$ and $x_i \to x_b + \dfrac{i}{n+1}(x_e - x_b)$ for $t \to \infty, i = 1, 2, \ldots, n$,
that is, from any initial position all points seek to line up over an interval with fixed ends x_b and x_e at equal distances from each other;

—the estimate $\|z(t)\| \leqslant e^{\sigma_1 t}\|z(0)\|$, where

$$\sigma_1 = \max_k \mathrm{Re}\lambda_k = -2\sin^2\frac{\pi}{2(n+1)} \tag{11}$$

is valid.

With provision for (8), we obtain that $\|x(t) - x^*\| \leqslant e^{\sigma_1 t}\|x(0) - x^*\|$. Therefore, the convergence rate of algorithm (3) follows (11). For greater n, the approximate estimate [4]

$$\sigma_1 \approx -\frac{\pi^2}{2n^2} \tag{12}$$

is valid.

As can be seen from (1) and (2), the motion of agents is controlled by an instantaneous change of velocity. This algorithm is of interest from the standpoint of studying the kinematics of agents, but for the mechanical systems it is *unrealizable physically*. According to the second Newton law, by applying force to an object one can vary its acceleration, but not the immediate velocity. A new algorithm of equidistant arrangement over an interval with dynamic models of agents in the form of the second-order integrators is suggests in what follows. It is more realistic because enables one to vary the agents' velocity indirectly through acceleration. Consideration is also given to the rate of convergence of the second-order algorithm as compared with the first-order algorithm.

3. SECOND-ORDER ALGORITHM FOR EQUIDISTANT ARRANGEMENT OF AGENTS

We consider models of the agents in the form of the second-order integrators

$$\ddot{x}_i = u_i, \quad i = 1, 2, \ldots, n, \tag{13}$$

and take the control law as

$$\begin{aligned} u_1 &= \frac{x_2 + x_b}{2} - x_1 - \alpha\dot{x}_1, \\ u_i &= \frac{x_{i+1} + x_{i-1}}{2} - x_i - \alpha\dot{x}_i, \quad i = 2, \ldots, n-1, \\ u_n &= \frac{x_e + x_{n-1}}{2} - x_n - \alpha\dot{x}_n, \end{aligned} \tag{14}$$

where α is some adjustable parameter, the vector x has form (4), and x_b and x_e are, respectively, the coordinates of the beginning and end of the interval. We denote by $s = d/dt$ the differentiation operator acting on each element of the vector x. Then, we obtain from (13) and (14) an expression describing the dynamics of n agents

$$(s^2 + \alpha s)x = Ax + b, \tag{15}$$

where A and b follow (5) and (6). System (15) is that of linear differential equations of the second order. To analyze it for stability, one can pass to a system of first-order equations and examine its characteristic polynomial. In the general case, this approach is inconvenient because the size of matrix A and the order of the operator in the left-hand side of the equality may be high. There exists a criterion enabling one to explore systems like (15) from the eigenvalues of the matrix A and the properties of the operator. A frequency criterion for analysis of the multiagent systems for stability was proposed in this formulation in [9], and later on a similar criterion was formulated in [10]. Stability of systems like $\phi(s)x = Ax$ is equivalent to the lack of right zeros of the characteristic function $G(s) = \det(A - \phi(s)I_n)$, where $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix. A special notion of the H-domain of the function $\phi(s)$ on the complex plane which simplifies stability analysis was introduced in [9]. This set is constructed using the principle of D-decomposition. If the H-domain exists and the eigenvalues of the matrix A lie within it, then the system is stable. The following statement was formulated and proved using the criterion of [9].

Statement 1 [11]. *System* (15) *is stable for all* $\alpha > 0$, *and the zeros of the characteristic function* $G(s) = \det\left(A - (s^2 + \alpha s)I_n\right)$ *are given by*

$$\lambda_k^{\pm} = -0.5\alpha \pm \sqrt{(0.5\alpha)^2 - 2\sin^2\frac{k\pi}{2(n+1)}}. \tag{16}$$

The proof of stability relies on the fact that for $\alpha > 0$ the H-domain is bounded by the function $\phi(j\omega) = -\omega^2 + j\alpha\omega$ and represents the interior of the parabola lying on the left half-plane. The criterion conditions [9] are satisfied because the eigenvalues of the matrix A are real and negative. We compare the rate of convergence of the algorithms (3) and (15) and analyze the eigenvalues (16). By reasoning like in (7)–(11), we can readily establish that the following is valid for $\alpha > 0$ (stable system (15)): the rate of convergence of algorithm (15) is given by

$$\sigma_2 = \max_k \operatorname{Re}\lambda_k^{\pm}, \tag{17}$$

at that the agents also seek to arrange themselves uniformly over the interval. The following decomposition with respect to α is obtained depending on the sign of the radical expression and with regard for $\alpha > 0$:

$$\sigma_2 = \begin{cases} -0.5\alpha, & 0 < \alpha \leqslant 2\sqrt{2}\sin\dfrac{\pi}{2(n+1)} \\ -0.5\alpha + \sqrt{0.25\alpha^2 - 2\sin^2\dfrac{\pi}{2(n+1)}}, & \alpha > 2\sqrt{2}\sin\dfrac{\pi}{2(n+1)}. \end{cases} \tag{18}$$

Let us compare the convergence rates of the algorithms of the first and second orders over different intervals assuming that the number of agents $n > 1$.

(1) The interval $0 < \alpha \leqslant 2\sqrt{2}\sin\dfrac{\pi}{2(n+1)}$:

(a) the convergence rate of algorithm (15) is higher than (3) if $-0.5\alpha < -2\sin^2\dfrac{\pi}{2(n+1)}$, that is, for $4\sin^2\dfrac{\pi}{2(n+1)} < \alpha \leqslant 2\sqrt{2}\sin\dfrac{\pi}{2(n+1)}$;

(b) the convergence rate is the same for $\alpha = 4\sin^2\dfrac{\pi}{2(n+1)}$;

(c) algorithm (15) converges slower for $0 < \alpha < 4\sin^2\dfrac{\pi}{2(n+1)}$.

(2) Interval $\alpha > 2\sqrt{2}\sin\dfrac{\pi}{2(n+1)}$:

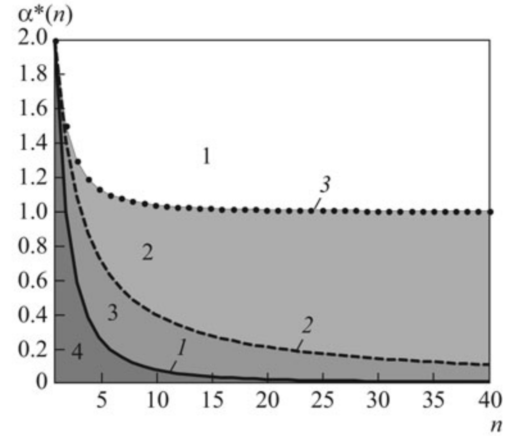

Fig. 1. Graphs of the boundary functions $\alpha(n)$: *(1)* $\alpha_1^*(n) = 4\sin^2\dfrac{\pi}{2(n+1)}$, *(2)* $\alpha_2^*(n) = 2\sqrt{2}\sin\dfrac{\pi}{2(n+1)}$, *(3)* $\alpha_3^*(n) = 1 + 2\sin^2\dfrac{\pi}{2(n+1)}$.

(a) the convergence rate of algorithm (15) is higher than (3) if $\alpha < 1 + 2\sin^2\dfrac{\pi}{2(n+1)}$, that is, for $2\sqrt{2}\sin\dfrac{\pi}{2(n+1)} < \alpha < 1 + 2\sin^2\dfrac{\pi}{2(n+1)}$;

(b) the convergence rate is the same for $\alpha = 1 + 2\sin^2\dfrac{\pi}{2(n+1)}$;

(c) algorithm (15) converges slower for $\alpha > 1 + 2\sin^2\dfrac{\pi}{2(n+1)}$.

By the boundary function is meant the function $\alpha^*(n)$ decomposing the plane $\{\alpha, n\}$ into domains. We illustrate the fact that graphs of the determined dependencies of the boundary functions on the number of agents are disjoint, that is, the decomposition was performed correctly.

As can be seen from Fig. 1, the graphs of the dependencies $\alpha^*(n)$ decompose the plane into four domains. In the first and fourth domains there are those values of the parameter α that provide higher convergence rate of the first-order algorithm; the values of α belonging to the second and third domains provide higher convergence rate of the second-order algorithm. Additionally, it deserves noting that for the optimal selection of α the convergence rate of the second-order algorithm grows with the number of system agents n.

Statement 2. *The second-order algorithm has higher convergence rate for* $\alpha \in \left(4\sin^2\dfrac{\pi}{2(n+1)}, 1 + 2\sin^2\dfrac{\pi}{2(n+1)}\right)$, *the first-order algorithm, for* $\alpha \in \left(0,\ 4\sin^2\dfrac{\pi}{2(n+1)}\right) \bigcup \left(1 + 2\sin^2\dfrac{\pi}{2(n+1)},\ +\infty\right)$, $n > 1$. *The convergence rates of both algorithms are the same for* $\alpha = 4\sin^2\dfrac{\pi}{2(n+1)}$, $\alpha = 1 + 2\sin^2\dfrac{\pi}{2(n+1)}$, $n > 1$. *Additionally, the second-order algorithm has the highest rate of convergence for* $\alpha = 2\sqrt{2}\sin\dfrac{\pi}{2(n+1)} \in \left(4\sin^2\dfrac{\pi}{2(n+1)},\ 1 + 2\sin^2\dfrac{\pi}{2(n+1)}\right)$, $n > 1$.

Therefore, consideration was given to stability of system (15), the convergence rates of the algorithms of first and second orders were compared, and it was established that a higher convergence rate of the second algorithm can be provided with retention of its physical realizability by an appropriate selection of the parameter α.

4. EQUIDISTANT ARRANGEMENT OF AGENTS AS THE CONSENSUS PROBLEM

4.1. Some Notions of the Graph Theory

It may be noted that since in the limit the agents are arranged at equal distances, that is, the lengths of the intervals between the points tend to become equal, the problem of arrangement over the interval resembles the problem of coordination of characteristics or consensus. We demonstrate how simple transformations enable one to pass to such problem. To make the further presentation understandable, we present some notions from the graph theory and the classical consensus problem.

The apparatus of the graph theory is used to study the interaction of agents making up the networks. We dwell on the simplest case of such networks where the graph becomes undirected.

Definition 1. By the undirected graph is meant an ordered pair $\mathcal{G}(\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{1, 2, \ldots, n\}$ is a nonempty set of vertices and $\mathcal{E} =\subseteq \mathcal{V} \times \mathcal{V}$ is the set of unordered pairs of vertices $\{i, j\}$ called the edges. The undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, a)$ is referred to as ordered if the weight function $a : E \to \mathbb{R}^+$ is defined.

A simple algorithm of consensus between the agents described by the first-order integrators $\dot{x}_i = u_i$, $i = 1, 2, \ldots, n$, may be compactly represented as the following system on a graph of the nth order [2, 12]:

$$\dot{x}_i = \sum_{j=1}^{n} a_{ij}(x_j - x_i), \tag{19}$$

where a_{ij} are the weights of the edges connecting the corresponding vertices of the graph.

Definition 2. The matrix

$$L: \ l_{ij} = \begin{cases} -a_{ij}, & j \neq i \\ \sum\limits_{k \neq i} a_{ik}, & j = i, \end{cases} \tag{20}$$

is called the Laplacian matrix associated with the graph of system (19).

With provision for the introduced definition, Eq. (19) can be readily rearranged in the vector-matrix form

$$\dot{x} = -Lx. \tag{21}$$

The Laplacian matrix of the undirected graph has some important characteristics. It is symmetrical and positive semi-definite, all nonzero eigenvalues are real and positive, and the following statement is true.

Statement 3 [8, 12]. *The process of coordination of* (19), (21) *converges, that is,* $|x_i(t) - x_j(t)| \to 0$, $t \to \infty$ *is satisfied for any* $x_i(0)$ *and* $i, j = 1, 2, \ldots, n$ (*the state of each agent tends to the states of its neighbors*) *if and only if that zero eigenvalue of the matrix* L *is unique. Additionally, these conditions are also equivalent to connectivity of the graph.*

Use of the matrix Laplacian is a well studied means of describing the network multiagent systems. In particular, the spectrum of the Laplacian matrix carries useful information about dynamics of the multiagent system [12]. For example, the least positive eigenvalue L known as the algebraic connectivity or the Fiedler eigenvalue is the measure of network connectivity quality [12].

4.2. Passage to the Consensus Problem

We return to the problem of uniform arrangement over the interval and consider the case where one of the interval boundaries coincides with the origin. One can always pass easily to such description by means of a parallel shift. In this case, (3) is representable as

$$\dot{\tilde{x}} = \tilde{A}\tilde{x}, \tag{22}$$

where $\tilde{x} = \begin{bmatrix} x_1, & x_2, & \dots, & x_n, & x_e \end{bmatrix}^\top$,

$$\tilde{A} = \begin{bmatrix} -1 & 0.5 & 0 & \dots & 0 & 0 \\ 0.5 & -1 & 0.5 & \dots & 0 & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & \dots & 0.5 & -1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{(n+1)\times(n+1)}. \tag{23}$$

As can be seen from (22) and (23), the dimensionality of the state space has increased by one state owing to adding a fixed dummy agent x_e.

We denote

$$\gamma_1 = x_1 - 0, \quad \gamma_2 = x_2 - x_1, \quad \gamma_3 = x_3 - x_2, \ \dots, \ \gamma_{n+1} = x_e - x_n, \tag{24}$$

where $\gamma_i, \ i = 1, 2, \dots, n+1$, are the distances between the agents. The change of the coordinates in the vector-matrix form is given by

$$\gamma = C\tilde{x}, \tag{25}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ -1 & 1 & 0 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & \dots & -1 & 1 \end{bmatrix} \in \mathbb{R}^{(n+1)\times(n+1)}. \tag{26}$$

One can easily see that the matrix C is nondegenerate. Passing to the new coordinates, we represent (22) as

$$\dot{\gamma} = -L\gamma, \tag{27}$$

where the matrix

$$L = -C\tilde{A}C^{-1} \tag{28}$$

has the form of the Laplacian matrix of the undirected chain graph with the edge weights $a_{ij} = 0.5$:

$$L = \begin{bmatrix} 0.5 & -0.5 & 0 & \dots & 0 \\ -0.5 & 1 & -0.5 & \dots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & \dots & -0.5 & 0.5 \end{bmatrix} \in \mathbb{R}^{(n+1)\times(n+1)}. \tag{29}$$

Analysis of the algorithm also shows that if by the new characteristics-states of the system are meant the lengths of the intervals tending to equality, then simple passage to such new coordinates reduces the problem of uniform arrangement over the interval to the classical problem of consensus for the system whose structure is defined by the undirected chain graph. Additionally, one can readily see that after passing to the consensus problem the greatest eigenvalue of the original matrix of system (5) has the sense of the so-called Fiedler eigenvalue (with opposite sign) which also characterizes the convergence rate of the algorithm.

5. EXAMPLES

Let us consider examples illustrating behavior of the first and second order algorithms and the relation of the problem of equidistant arrangement over the interval with the consensus problem.

Example 1 (comparison of the algorithms of the first and second orders). Consider a system consisting of seven agents seeking to place themselves at equal distances over the interval $[-2, 2]$. The initial coordinates of agent arrangement have the form $x(0) = [0.308,\ 0.496,\ -0.832,\ 0.827,\ 0.652,\ 0.992,\ -0.115]^\top$. Figure 2 shows the results of analyzing the first-order and second-order algorithms. For the latter one, $\alpha = 2\sqrt{2}\sin\dfrac{\pi}{16}$ was selected as providing the maximal rate of convergence.

Example 2 (arrangement over the interval and consensus). We demonstrate by way of a particular example the passage from the problem of equidistant arrangement over the interval to that

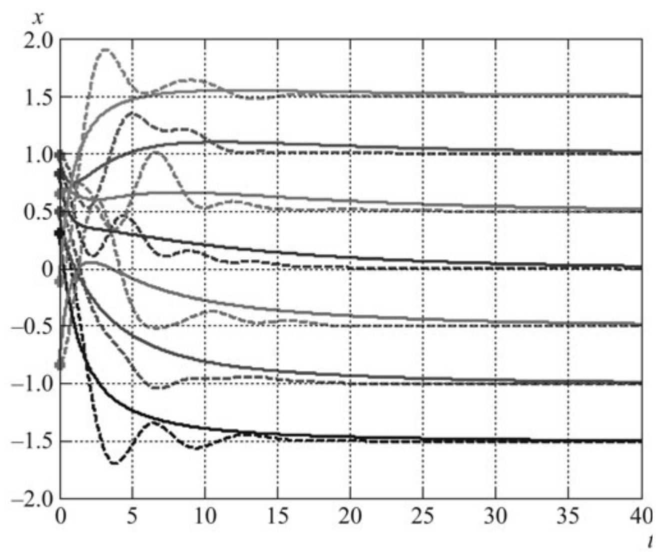

Fig. 2. Trajectories of the agents of the equidistant arrangement algorithm over the interval $[-2,\ 2]$, time interval $[0,\ 40]$ s. The solid line represents the first-order algorithm, the dashed line, the second-order algorithm. The pairs of graphs corresponding to the agents from the first to the seventh are shown in the upward direction.

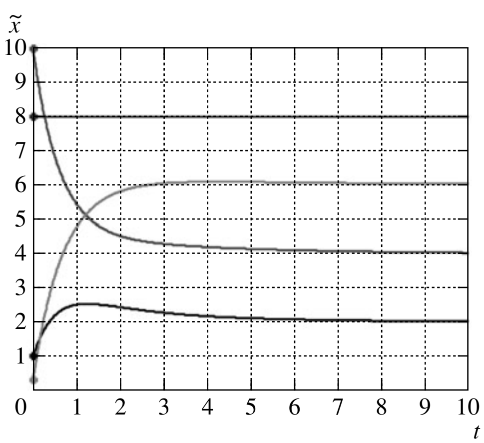

Fig. 3. Trajectories of the agents of the algorithm of equidistant arrangement over the interval.

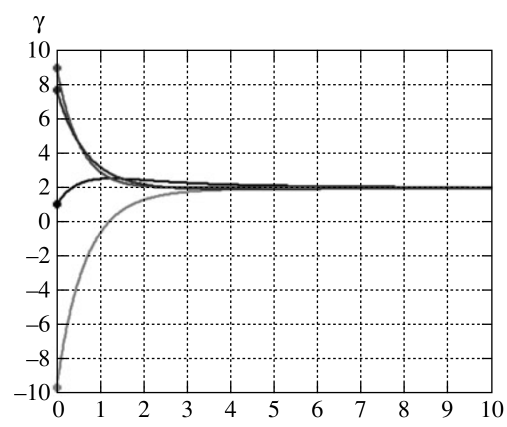

Fig. 4. Trajectories of the “new” agents of the consensus algorithm.

of consensus and consider a system of three agents. Let the initial arrangement of the agents be defined by the vector $x(0) = \begin{bmatrix}1, & 10, & 0.3\end{bmatrix}^\top$ and the interval have the form $[0,\ 8]$. By passing to the description of the system as (22), we get an extended vector of the initial conditions in the form of $\tilde{x}(0) = \begin{bmatrix}1, & 10, & 0.3, & 8\end{bmatrix}^\top$. After the transformation (25), the initial conditions go over to $\gamma(0) = \begin{bmatrix}1, & 9, & -9.7, & 7.7\end{bmatrix}^\top$. Figures 3 and 4 depict the graphs for the algorithm of equidistant arrangement over the interval in the form (22) and the problem of consensus (27).

As can be seen from Fig. 3, one of the agents remains stationary because it is the boundary of the interval. The agents are arranged at identical distances equal to two. After passing to the problem of consensus, the new agents having the sense of the distances between the initial agent points try to come to the common state $\gamma^* = 2$ (see Fig. 4). At that, the rates of convergence remains the same.

6. CONCLUSIONS

The present paper proposed an algorithm for equidistant arrangement of the agent over an interval. In distinction to the earlier approaches, consideration is given to the double integrators as agent models. The developed second-order algorithm enables one to pass from the agent models in the form of single integrators, that is, kinematic models, to the second-order, that is, dynamic, models. The algorithm was shown to be stable for the positive values of the adjusted parameter α. The algorithm's rate of convergence vs. α was considered. The resulting decomposition on the basis of the values of α demonstrated that its appropriate selection provides a higher rate of convergence than the first-order algorithm. It deserves noting that algorithms of higher orders (higher degree of the operator $\phi(s)$) can be proposed, but then the study of stability and convergence rate becomes much more difficult. The algorithm under consideration implies that the agents are arranged equidistantly, but a minor modification of the algorithm similar to [5] can arrange the agents at distances related by certain relationships. The paper also considered the relation between the problem of equidistant arrangement over the interval and the classical consensus problem. The passage to a common state in the new coordinates was shown to mean that the interval takes an equal length, that is, the problem of equidistant arrangement is solved. Therefore, the initial problem of equidistant arrangement over a fixed interval may be reduced to that of consensus.

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