Физические механизмы спин-токовой модели поляризации мультиферроиков

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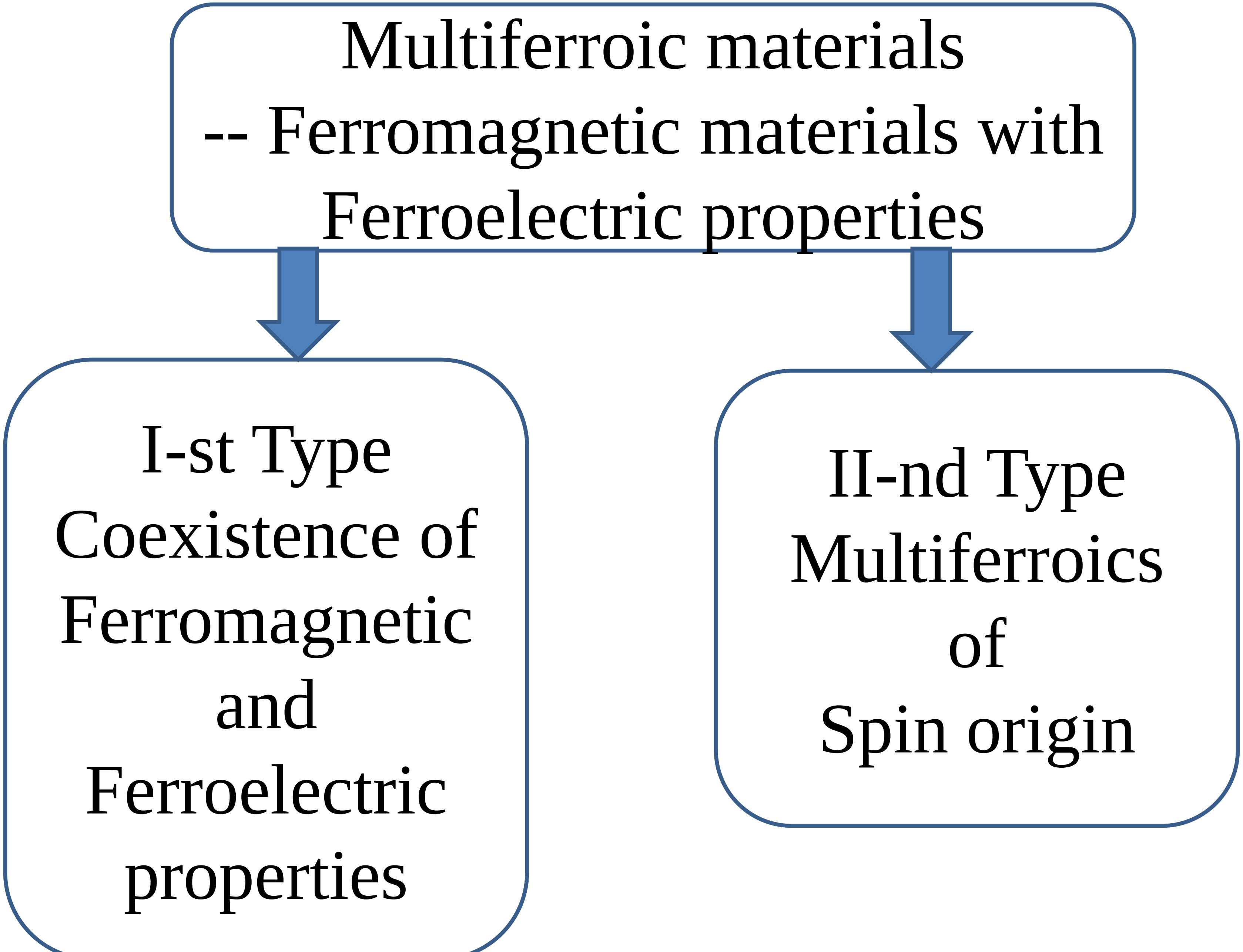
Multiferroic materials
-- Ferromagnetic materials with Ferroelectric properties
I-st Type
Coexistence of Ferromagnetic and Ferroelectric properties
II-nd Type
Multiferroics of Spin origin

Multiferroic materials, II-nd Type -- Classification

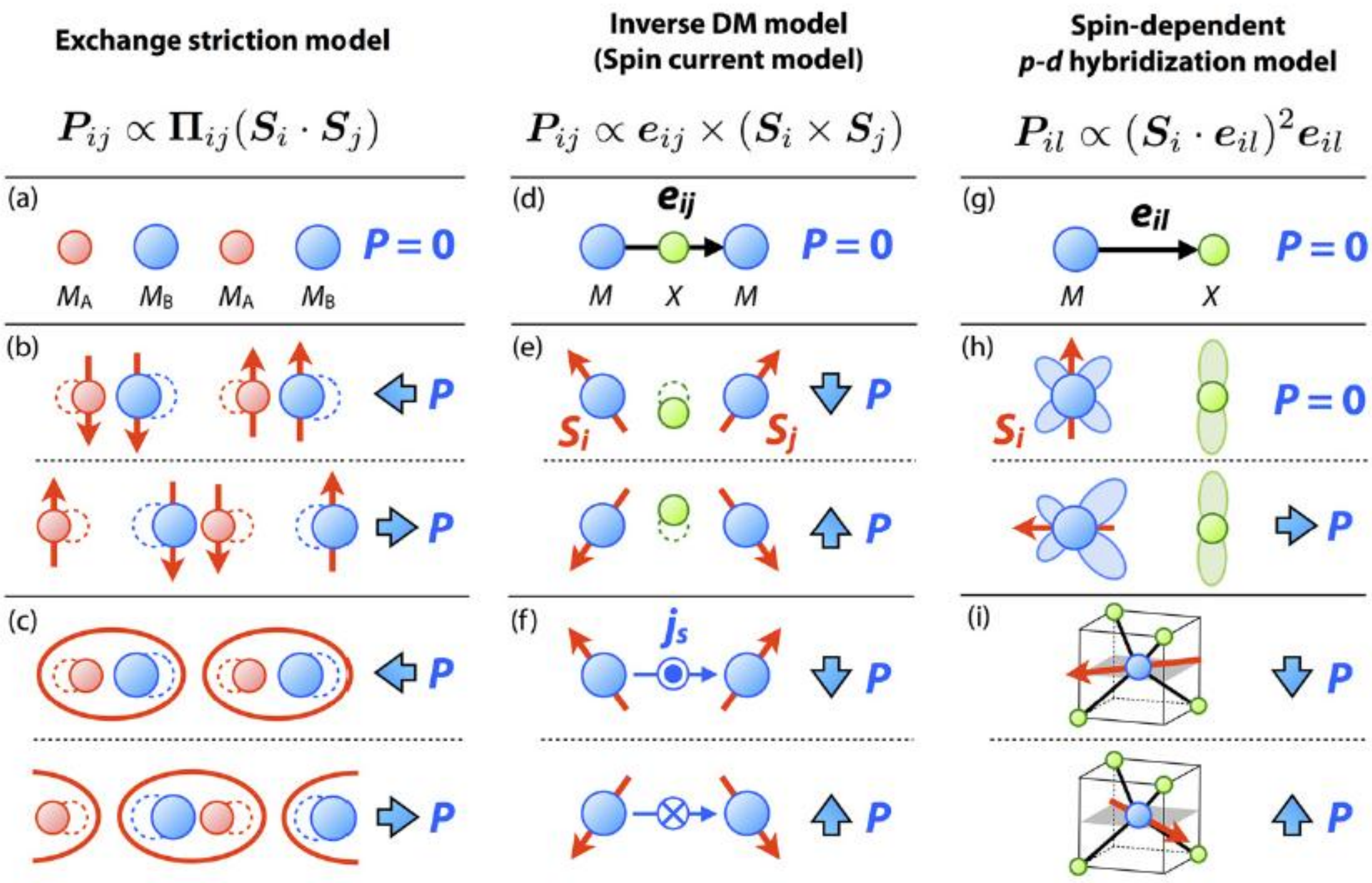

Figure 2. Three major mechanisms of ferroelectricity of spin origin: (*a*)–(*c*) exchange-striction mechanism arising from the symmetric spin

Models of Multiferroic materials

Exchange striction model

$$\boldsymbol{P}_{ij} \propto \boldsymbol{\Pi}_{ij}(\boldsymbol{S}_i \cdot \boldsymbol{S}_j)$$

Inverse DM model (Spin current model)

$$\boldsymbol{P}_{ij} \propto \boldsymbol{e}_{ij} \times (\boldsymbol{S}_i \times \boldsymbol{S}_j)$$

Spin-dependent *p-d* hybridization model

$$\boldsymbol{P}_{il} \propto (\boldsymbol{S}_i \cdot \boldsymbol{e}_{il})^2 \boldsymbol{e}_{il}$$

$$P^{\alpha} = g^{\alpha}_{0\Pi}\mathbf{S}^2 + \frac{1}{6} g^{\alpha}_{\Pi}(\mathbf{S} \cdot \triangle \mathbf{S}),$$

Spin current model:

$$P^{\beta} \sim \varepsilon^{\beta\mu\nu} J^{\mu\nu}$$

J is the spin current tensor

Tokura et al , **Multiferroics of spin origin,** Rep. Prog. Phys. **77,** 076501 (2014)
S. Dong et al , Multiferroic materials and magnetoelectric physics: symmetry, entanglement, excitation, and topology, Advances in Physics, 2015

$$P^{\beta} = \alpha\left(M^{\beta}(\nabla \cdot \mathbf{M}) - (\mathbf{M} \cdot \nabla) M^{\beta}\right)$$

Mostovoy M., **Ferroelectricity in Spiral Magnets,** PRL **96,** 067601 (2006)

Model and corresponding spin evolution equation I

$$\partial_t \mathbf{S} = \frac{2\mu}{\hbar}[\mathbf{S}\times\mathbf{B}] + \frac{1}{6}g_u[\mathbf{S}\times\Delta\mathbf{S}] + \frac{2\mu}{\hbar c}\varepsilon^{\alpha\beta\gamma}\varepsilon^{\beta\mu\nu}J^{\gamma\mu}E^{\nu}$$
$$+\frac{1}{3}g_{(\beta)}\left[\left(\mathbf{S}\cdot[\boldsymbol{\delta}\times\nabla]\right)\mathbf{S} - S^{\beta}[\boldsymbol{\delta}\times\nabla]S^{\beta}\right]$$

$$J^{\gamma\mu} \sim S^{\gamma}\cdot v^{\mu} \qquad \mathbf{S}\equiv S^{\alpha}$$

$$\mathbf{S}(\mathbf{r},t) = \int\sum_i \delta(\mathbf{r}-\mathbf{r}_i)\Psi^{\dagger}(R,t)\hat{\mathbf{s}}_i\Psi(R,t)d^{3N}R$$

P. A. Andreev, M. I. Trukhanova, arXiv:2312.16321

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PHYSICS REPORTS (Review Section of Physics Letters) 194, Nos. 3 & 4 (1990) 117-238. North-Holland

MAGNETIC SOLITONS

A.M. KOSEVICH, B.A. IVANOV and A.S. KOVALEV

$$\mathbf{S}(\mathbf{r},t) = \int d^{3N}R \sum_i \delta(\mathbf{r}-\mathbf{r}_i) \cdot \Psi^\dagger(R,t)\hat{\mathbf{s}}_i \Psi(R,t)$$

$$\frac{\partial}{\partial t}\mathbf{S}(\mathbf{r},t) = \frac{i}{\hbar}\int d^{3N}R \sum_i \delta(\mathbf{r}-\mathbf{r}_i) \cdot \Psi^\dagger(R,t)[\hat{H},\hat{\mathbf{s}}_i]\Psi(R,t)$$

average spin of a lattice site,

$$M_n = -\frac{2\mu_0}{a^3}\langle S_n\rangle = -\frac{2\mu_0}{a^3} S(x_n), \quad S_n \to -\frac{a^3}{2\mu_0} M(x_n),$$

where a is the interaction distance.

If the angle between adjacent spin vectors is small, then the vectors as continuous functions of the x coordinate, and we can write the fol

$$M_{n+n_0} = M(x_n) + x_i(n_0)\frac{\partial M}{\partial x_i} + \tfrac{1}{2}x_i(n_0)x_k(n_0)\frac{\partial^2 M}{\partial x_i\,\partial x_k} + \cdots.$$

Using the procedure prescribed by eqs. (1.9) and (1.10) for the reali characteristics of a discrete spin system to a continuous description of the expression for the energy density of a ferromagnet correspondi writing the energy density, it should be remembered that the exchange and that the values of J_1, J_2, and J_3 are quite close to each other. exchange constants J_α, we may assume that

$$J_\alpha = J_0 + a^2 j_\alpha, \quad \alpha = 1, 2, 3.$$

To order a^2, one can write

$$w = \frac{\alpha}{2}\frac{\partial M}{\partial x_k}\cdot\frac{\partial M}{\partial x_k} + w_{an} - M\cdot H, \quad \alpha = J_0 a^5/(2\mu_0)^2,$$

Model and corresponding spin evolution equation II

$$\partial_t \mathbf{S} = \frac{2\mu}{\hbar}[\mathbf{S}\times\mathbf{B}] + \frac{1}{6} g_u [\mathbf{S}\times\Delta\mathbf{S}] + \frac{2\mu}{\hbar c}\varepsilon^{\alpha\beta\gamma}\varepsilon^{\beta\mu\nu} J^{\gamma\mu} E^{\nu}$$

$$+\frac{1}{3} g_{(\beta)}\Big[\big(\mathbf{S}\cdot[\boldsymbol{\delta}\times\nabla]\big)\mathbf{S} - S^{\beta}[\boldsymbol{\delta}\times\nabla]S^{\beta}\Big]$$

$$J^{\gamma\mu} \sim S^{\gamma}\cdot v^{\mu} \qquad \mathbf{S} \equiv S^{\alpha}$$

$$g_u = \int \xi^2 U(\xi) d^3\xi \qquad g_{(\beta)} = \int \xi^2 \beta(\xi) d^3\xi \qquad \mathbf{D}_{ij} = \beta(r_{ij})\cdot \mathbf{r}_{ij}\times\boldsymbol{\delta}$$

$$\hat{H} = \sum_i \Bigg[-\hat{\mathbf{d}}_i\cdot\mathbf{E}_i - \hat{\boldsymbol{\mu}}_i\cdot\mathbf{B}_i - \frac{1}{m_i c}\big(\hat{\boldsymbol{\mu}}_i\cdot[\mathbf{E}_i\times\hat{\mathbf{p}}_i]\big)$$

$$-\frac{1}{2}\sum_{j\neq i}\big(U_{ij}\hat{\mathbf{s}}_i\cdot\hat{\mathbf{s}}_j + \mathbf{D}_{ij}\cdot[\hat{\mathbf{s}}_i\times\hat{\mathbf{s}}_j]\big)\Bigg]$$

$$i\hbar\partial_t \Psi(R,t) = \hat{H}\Psi(R,t)$$

Same model and corresponding Momentum evolution equation –derivation of the Spin Current model

$$\hat{H} = \sum_i \left[-\hat{\mathbf{d}}_i \cdot \mathbf{E}_i - \hat{\boldsymbol{\mu}}_i \cdot \mathbf{B}_i - \frac{1}{m_i c}\left(\hat{\boldsymbol{\mu}}_i \cdot [\mathbf{E}_i \times \hat{\mathbf{p}}_i]\right) \right.$$

$$\left. - \frac{1}{2}\sum_{j \neq i}\left(U_{ij}\hat{\mathbf{s}}_i \cdot \hat{\mathbf{s}}_j + D_{ij} \cdot [\hat{\mathbf{s}}_i \times \hat{\mathbf{s}}_j]\right) \right]$$

$$i\hbar\partial_t \Psi(R,t) = \hat{H}\Psi(R,t)$$

$$\partial_t \mathbf{p} = \partial_t (mn\mathbf{v}) = P^\beta (\nabla E^\beta) + \mu S^\beta \nabla B^\beta + \frac{\mu}{mc}\varepsilon^{\beta\gamma\delta} J^{\delta\gamma} (\nabla E^\beta)$$

$$+ g_{0u} S^\beta \nabla S^\beta + \frac{1}{3} g_{(\beta)} \left[(\boldsymbol{\delta} \cdot \mathbf{S}) \nabla (\nabla \cdot \mathbf{S}) - (\mathbf{S} \cdot \nabla) \nabla (\boldsymbol{\delta} \cdot \mathbf{S}) \right]$$

$$g_{0u} = \int U(\xi) d^3\xi \qquad g_{(\beta)} = \int \xi^2 \beta(\xi) d^3\xi \qquad \mathbf{D}_{ij} = \beta(r_{ij}) \mathbf{r}_{ij} \times \boldsymbol{\delta}$$

Spin Current model
As a condition of absence of currents

$$P^{\beta} = \frac{\mu}{mc}\varepsilon^{\beta\gamma\delta} J^{\gamma\delta}$$

Spin Current: How to get it?

$$\partial_t \mathbf{S} = \frac{2\mu}{\hbar}[\mathbf{S}\times\mathbf{B}] + \frac{1}{6}g_u[\mathbf{S}\times\Delta\mathbf{S}] + \frac{2\mu}{\hbar c}\varepsilon^{\alpha\beta\gamma}\varepsilon^{\beta\mu\nu}J^{\gamma\mu}E^{\nu} + \frac{1}{3}g_{(\beta)}\left[\left(\mathbf{S}\cdot[\delta\times\nabla]\right)\mathbf{S} - S^{\beta}[\delta\times\nabla]S^{\beta}\right]$$

$$\frac{1}{6}g_u\varepsilon^{\alpha\beta\gamma}S^{\beta}\Delta S^{\gamma} = -\partial^{\beta}J^{\alpha\beta}_{HH}$$

$$J^{\alpha\beta}_{HH} = -\frac{1}{6}g_u\varepsilon^{\alpha\mu\nu}S^{\mu}\partial^{\beta}S^{\nu}$$

$$g_u = \int \xi^2 U(\xi)d^3\xi \qquad g_{(\beta)} = \int \xi^2 \beta(\xi)d^3\xi \qquad \mathbf{D}_{ij} = \beta(r_{ij})\mathbf{r}_{ij}\times\boldsymbol{\delta}$$

Polarization for the Spin Current from HH

$$P^{\beta} = \frac{\mu}{mc}\varepsilon^{\beta\gamma\delta}J^{\gamma\delta}$$

$$\frac{1}{6}g_u\varepsilon^{\alpha\beta\gamma}S^{\beta}\Delta S^{\gamma} = -\partial^{\beta}J_{HH}^{\alpha\beta}$$

$$J_{HH}^{\alpha\beta} = -\frac{1}{6}g_u\varepsilon^{\alpha\mu\nu}S^{\mu}\partial^{\beta}S^{\nu}$$

$$P^{\beta}{}_{HH} = \frac{\mu}{mc}\varepsilon^{\beta\gamma\delta}J^{\gamma\delta}{}_{HH}$$

$$= \frac{1}{6}\frac{\mu}{mc}g_u\left(\left(\mathbf{S}\cdot\nabla\right)S^{\beta} - S^{\beta}\left(\nabla\cdot\mathbf{S}\right)\right)$$

Same polarization can be found as average of edm operator

$$\mathbf{P}\left(\mathbf{r},t\right) = \int\sum_i \delta\left(\mathbf{r}-\mathbf{r}_i\right)\Psi^{\dagger}(R,t)\cdot\hat{\mathbf{d}}_i\cdot\Psi(R,t)d^{3N}R$$

$$\hat{\mathbf{d}}_i = \sum_{j\neq i}\alpha_{ij}(r_{ij})[\mathbf{r}_{ij}\times[\hat{\mathbf{s}}_i\times\hat{\mathbf{s}}_j]] \qquad \alpha_{ij} = \frac{\mu}{2mc}U_{ij} \qquad U_{ij} \sim \quad exchange\,\mathrm{int}$$

Polarization for the Spin Current from HH: Comparison with “Tokura and Mostovoy”

$$P^{\beta}{}_{HH} = \frac{\mu}{mc} \varepsilon^{\beta\gamma\delta} J^{\gamma\delta}{}_{HH} = \frac{1}{6}\frac{\mu}{mc} g_u \left((\mathbf{S}\cdot\nabla) S^{\beta} - S^{\beta} (\nabla\cdot\mathbf{S}) \right)$$

Same polarization can be found as average of edm operator

$$\mathbf{P}(\mathbf{r},t) = \int \sum_i \delta(\mathbf{r}-\mathbf{r}_i)\Psi^{\dagger}(R,t)\cdot\hat{\mathbf{d}}_i\cdot\Psi(R,t) d^{3N}R$$

$$\hat{\mathbf{d}}_i = \sum_{j\neq i} \alpha_{ij}(r_{ij})[\mathbf{r}_{ij}\times[\hat{\mathbf{s}}_i\times\hat{\mathbf{s}}_j]] \qquad \alpha_{ij} = \frac{\mu}{2mc} U_{ij} \qquad U_{ij} \sim \quad exchange\ \mathrm{int}$$

Spin Current from DMI

$$P^{\beta} = \frac{\mu}{mc}\varepsilon^{\beta\gamma\delta} J^{\gamma\delta}$$

$$\partial_t \mathbf{S} = \frac{2\mu}{\hbar}[\mathbf{S}\times\mathbf{B}] + \frac{1}{6} g_u [\mathbf{S}\times\Delta\mathbf{S}] + \frac{2\mu}{\hbar c}\varepsilon^{\alpha\beta\gamma}\varepsilon^{\beta\mu\nu} J^{\gamma\mu} E^{\nu}$$

$$+\frac{1}{3} g_{(\beta)}\left[\left(\mathbf{S}\cdot[\delta\times\nabla]\right)\mathbf{S} - S^{\beta}[\delta\times\nabla]S^{\beta}\right]$$

$$g_u = \int \xi^2 U(\xi) d^3\xi \qquad g_{(\beta)} = \int \xi^2 \beta(\xi) d^3\xi \qquad \mathbf{D}_{ij} = \beta(r_{ij})\mathbf{r}_{ij}\times\boldsymbol{\delta}$$

$$-\frac{1}{3} g_{(\beta)} S^{\mu}[\delta\times\nabla]S^{\mu} \equiv -\partial^{\beta} J^{\alpha\beta}_{DM}$$

$$J^{\alpha\beta}_{DM} = -\frac{1}{6} g_{(\beta)}\varepsilon^{\alpha\beta\gamma}\delta^{\gamma}\left(S^{\mu}S^{\mu}\right)$$

$$P^{\beta}{}_{DM} = \frac{\mu}{mc}\varepsilon^{\beta\gamma\delta} J^{\gamma\delta}{}_{DM}$$

$$= -\frac{1}{3}\frac{\mu}{mc} g_{(\beta)}\delta^{\beta}\cdot\mathbf{S}^2$$

Spin Current from DMI: Comparison with "Tokura"

$$P^{\beta} = \frac{\mu}{mc} \varepsilon^{\beta\gamma\delta} J^{\gamma\delta}$$

$$J_{DM}^{\alpha\beta} = -\frac{1}{6} g_{(\beta)} \varepsilon^{\alpha\beta\gamma} \delta^{\gamma} \cdot \mathbf{S}^2$$

$$P^{\beta}{}_{DM} = \frac{\mu}{mc} \varepsilon^{\beta\gamma\delta} J^{\gamma\delta}{}_{DM} = -\frac{1}{3}\frac{\mu}{mc} g_{(\beta)} \delta^{\beta} \cdot \mathbf{S}^2$$

Same polarization can be found as average of edm operator

$$\mathbf{P}(\mathbf{r},t) = \int \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \Psi^{\dagger}(R,t) \cdot \hat{\mathbf{d}}_i \cdot \Psi(R,t) d^{3N} R$$

$$\hat{\mathbf{d}}_i = \sum_{j \neq i} \mathbf{\Pi}_{ij}(r_{ij}) \left(\hat{\mathbf{s}}_i \cdot \hat{\mathbf{s}}_j \right)$$

$$\mathbf{\Pi}_{ij} = -\frac{\mu}{3mc} r_{ij}^2 \beta_{ij} \boldsymbol{\delta}$$

$$\frac{\partial \mathbf{\Pi}_{ij}}{\partial r_{ij}} = \frac{\mu}{mc} r_{ij} \beta_{ij} \boldsymbol{\delta}$$

Spin Current from DMI: non-homogeneous contribution

$$P^{\beta}{}_{DM} = \frac{\mu}{mc}\varepsilon^{\beta\gamma\delta} J^{\gamma\delta}{}_{DM}$$
$$= -\frac{1}{3}\frac{\mu}{mc}\delta^{\beta}\left(g_{(\beta)}\mathbf{S}^2 + \frac{1}{5}g_{2(\beta)}\mathbf{S}\cdot\Delta\mathbf{S}\right)$$

Same polarization can be found as average of edm operator

$$\mathbf{P}(\mathbf{r},t) = \int\sum_i \delta(\mathbf{r}-\mathbf{r}_i)\Psi^{\dagger}(R,t)\cdot\hat{\mathbf{d}}_i\cdot\Psi(R,t)d^{3N}R$$

$$\hat{\mathbf{d}}_i = \sum_{j\neq i}\mathbf{\Pi}_{ij}(r_{ij})\left(\hat{\mathbf{s}}_i\cdot\hat{\mathbf{s}}_j\right)$$

$$\mathbf{\Pi}_{ij} = -\frac{\mu}{3mc}r_{ij}^2\beta_{ij}\boldsymbol{\delta}$$

$$\frac{\partial\mathbf{\Pi}_{ij}}{\partial r_{ij}} = \frac{\mu}{mc}r_{ij}\beta_{ij}\boldsymbol{\delta}$$

Spin evolution equation and Polarization evolution equation

$$\mathbf{S}(\mathbf{r},t)=\int\sum_{i}\delta(\mathbf{r}-\mathbf{r}_i)\Psi^{\dagger}(R,t)\hat{\mathbf{s}}_i\Psi(R,t)d^{3N}R$$

$$\mathbf{P}(\mathbf{r},t)=\int\sum_{i}\delta(\mathbf{r}-\mathbf{r}_i)\Psi^{\dagger}(R,t)\cdot\hat{\mathbf{d}}_i\cdot\Psi(R,t)d^{3N}R$$

$$\hat{\mathbf{d}}_i=\sum_{j\neq i}\alpha_{ij}(r_{ij})[\mathbf{r}_{ij}\times[\hat{\mathbf{s}}_i\times\hat{\mathbf{s}}_j]]$$

$$\hat{H}=\sum_{i}(-\hat{\boldsymbol{\mu}}_i\cdot\mathbf{B}_i)$$

$$\partial_t P^{\alpha}=\frac{1}{3}\varepsilon^{\alpha\beta\gamma}\gamma g_{(\alpha)}\cdot\left[S^2\cdot\partial^{\beta}B^{\gamma}+S^{\gamma}S^{\mu}\cdot\partial^{\beta}B^{\mu}+B^{\mu}\left(S^{\gamma}\partial^{\beta}S^{\mu}-S^{\mu}\partial^{\beta}S^{\gamma}\right)\right]$$

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Thank you!