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A dynamic goniometer-spectrometer with angle-to-time conversion

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Abstract

We present a dynamic goniometer–spectrometer for measuring the refractive index (RI) of transparent triangular prisms by converting the beam deviation angles into time intervals. The proposed measuring scheme allows the automatic determination of the RI simultaneously for several different wavelengths by measuring the time intervals between the moments of autocollimation of the reference and refracted rays reflected from a mirror continuously rotating around the object with constant angular speed. This makes it possible to avoid the use of an expensive ring laser or encoder for angle measurements and to use relatively inexpensive instruments for time measurements, which reduces the cost of measuring equipment. To calculate the RI value, we used the minimum deviation technique. A functional diagram of the measuring equipment is given. A reference prism made of NBK-7 Schott optical glass was experimentally studied and the measurement errors were estimated. The proposed measuring scheme can be used for automated RI measurement and to study the dispersion characteristics of triangular prisms made of optically transparent materials and liquid optically transparent substances filling a hollow triangular prism.

Keywords: refractive index, refractive index measurements, dynamic goniometer, spectrometer, angle-to time conversion

1. Introduction

The refractive index n (RI) and its spectral dependence are the most important parameters of optical materials [1]. Accurate and reliable measurements of RI are needed in the optical industry to increase the resolution of lenses and other optical devices [2, 3], and the study of the spectral dependence of RI is needed to determine the dispersion characteristics used for calculations of precision optics with minimal chromatic aberrations [4, 5]. RI measurements are also necessary in the chemical industry to check the composition of substances, in the food industry in the production of sugar, juices, alcoholic beverages, fats, oils, etc [6, 7].

To measure RI, various methods are used [8], including goniometric methods based on measuring the angles of reflection and refraction of light passing through the sample. As samples for goniometric methods, triangular prisms made of the material under study or hollow prisms with plane-parallel transparent portholes for liquid substances are usually used [9, 10]. To measure the angles of light refraction with a prism, goniometers are used [11].

Various goniometric measurements of the RI of triangular prisms are known, including the minimum deviation method [12]. To ensure the high accuracy of RI calculation, the automation of the measuring experiment, and control of the prism temperature and ambient air parameters (temperature, humidity, pressure) are required [13]. Our previous work [14] describes the use of a dynamic goniometer with a ring laser for automated measurements of RI by the minimum deviation

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method. Such an instrument is highly accurate, however, the use of an expensive He–Ne ring laser [15] to determine the angular position of the autocollimation mirror significantly limits its widespread use.

The accuracy of measuring RI using goniometric methods is determined primarily by the errors of angular measurements and depends on the prism method, air and sample temperature, sample manufacturing quality, etc. When using the minimum deviation method to determine RI with an error of the level of $\pm 10^{-5}$, it is necessary to make angular measurements with an error of no more than ± 3–$6''$ [16]. Reference instruments that provide an error of measurement of the RI up to $\pm 10^{-6}$, require angle control with an error of $\pm 0.5''$ [17]. Such accuracy of angular measurements can be achieved using expensive equipment such as a He–Ne ring laser or a precision photoelectric encoder [18]. However, it is possible to achieve high accuracy in determining RI using relatively inexpensive measuring devices for the transition from angular measurements to time and frequency measurements [19].

This work contains a description of a dynamic goniometer–spectrometer with the conversion of beam deviation angles into time intervals, which allows simultaneous automated measurements of RI at several fixed wavelengths to reduce the cost of equipment and time spent on the study of the dispersion characteristics of the sample.

2. Measurement method

The method of measuring RI used for the goniometer–spectrometer is based on determining the angle of the minimum deviation of light passing through an optically transparent triangular prism. By changing the angle of incidence of light on the face of the prism φ, it is possible to position the prism so that the angle of deviation of the beam ε reaches a certain minimum—the angle of minimum deviation ε_{min}. The value of ε_{min} depends on the value of RI, the apex angle of the prism α, and the radiation wavelength λ due to the dispersion properties of the material.

By measuring ε_{min}, the relative RI of the prism material n can be calculated using the formula [20]:

$$n = \sin\left[\left(\alpha + \varepsilon_{\text{min}}\right)/2\right] / \sin\left(\alpha/2\right). \quad (1)$$

To determine the dispersion characteristics [21] (Abbe number, principal dispersion, dispersion coefficients, relative partial dispersion, etc) it is necessary to calculate RI for different wavelengths corresponding to the specific spectral lines—*F* (486.13 nm), *d* (587.56 nm), *C* (656.27 nm) etc. For practical approximation, dispersion formulas are available: we can use the Sellmeier equation [22], Schott equation, or the simpler Cauchy dispersion formula [23]:

$$n = A + B/\lambda^2 + C/\lambda^4, \quad (2)$$

where *A, B, C* are empirical coefficients determined by measuring the RI for three wavelengths.

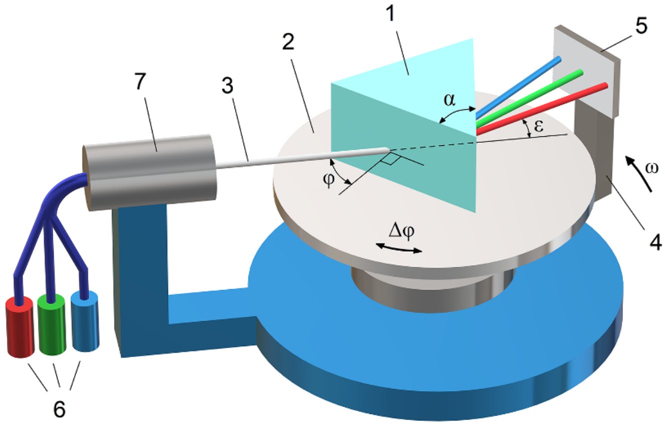

Figure 1. A dynamic goniometer–spectrometer. 1—triangular prism; 2—object table; 3—radiation beam; 4—rotating console; 5—two-sided mirror; 6—laser diode modules; 7—zero indicator with photodetector.

3. A dynamic goniometer–spectrometer

To measure the desired angle of the minimum deviation at different wavelengths, a dynamic goniometer–spectrometer is used as part of the measuring equipment (figure 1) [24].

The prism (1) is fixed on an object table (2) in such a way that the radiation beam (3) falls on the input working face at some angle φ. The console (4) with a mounted autocollimation mirror (5) is rotated at a constant speed by an electric motor. A two-sided autocollimation mirror is fixed in a position where the normal to the mirror is perpendicular to the axis of rotation. To determine the angle of rotation of the console, the measurement of the time intervals between the moments of the beam autocollimation is used.

As radiation sources for the goniometer–spectrometer, three laser diode modules (6) (*Laserscom* LDI series) with wavelengths of 455 nm (blue), 520 nm (green) and 638 nm (red) are used simultaneously. The radiation of laser diode modules is combined into a common beam (3) using fiber-optic couplers. After passing through the prism, this beam is separated due to the dispersion properties of the prism material. The difference in the value of ε_{min} for the laser diodes used is on the level of units of angular degrees, which corresponds to the delay between consecutive signals on the photodetector (*Hamamatsu* G10899-03K InGaAs photodiode) in units of milliseconds at a console rotation speed of about 40 rpm. We used a non-contact direct current motor (DB32 type) to rotate a console.

To determine the moments of autocollimation, a zero indicator (7) is used, based on an autocollimator with thin slit diaphragms in the lighting and receiving parts (figure 2). Therefore, the photodetector generates a signal close in shape to a triangle pulse at those moments of time when the outgoing beam is perpendicular to the surface of the rotating autocollimation mirror. The first pulse from the zero indicator, which sets the beginning of the angular measurements, occur when the beam is reflected from the outer surface of the mirror. The second, third and subsequent pulses, which set the deviation

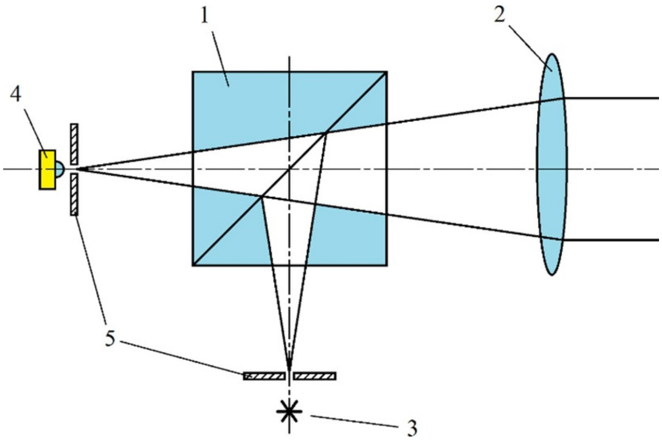

Figure 2. Optical scheme of the autocollimator. 1—beam splitter cube; 2—lens; 3—radiation source; 4—photodetector; 5—slit diaphragms.

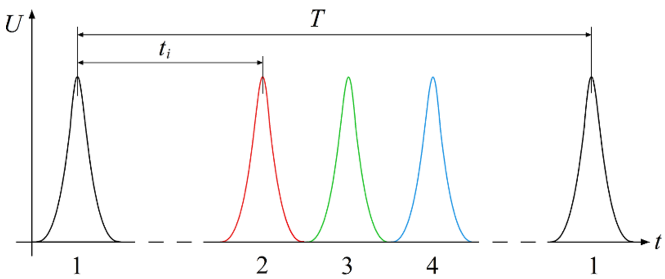

Figure 3. The time diagram of the voltage on the photodetector $U(t)$. 1—signal from the outer side of the rotating mirror; 2, 3, 4—signals from the inner side of the rotating mirror for the red, green, and blue laser diodes respectively.

angles ε_i of the refracted rays with respect to the incident, occurs when reflected from the inner surface of the mirror.

Since the beam is formed simultaneously by several laser modules, for one complete rotation of the console, several sequential signals (triangular pulses) containing information about the deviation angles ε_i for each wavelength are received on the photodetector (figure 3). Since the console rotates at a constant angular speed ω, the deviation angles for each wavelength ε_i are proportional to the time intervals between pulses:

$$\varepsilon_i = 2\pi t_i/T - \pi\,, \tag{3}$$

where t_i is the time interval between the first and ith pulses; T is the console rotation period.

As relative measurements of time intervals are implemented, it is sufficient to ensure the short-term stability of the time scale, the unevenness of which is less than $1 \cdot 10^{-9}$ in 1 s, even for relatively inexpensive measuring devices.

To measure the time intervals between pulses, various instruments can be used—a frequency counter, an oscilloscope, or an analog-to-digital converter (ADC). The disadvantage of using a frequency counter is that it has a certain level of response, and thus the measured time interval will depend on the ratio of the amplitudes of the triangular pulses. To equalize the intensity of the reflected radiation, the coating on the outer surface of the mirror is made with a lower reflective coefficient. Since the signals are received from different sources, there are technical difficulties of ensuring the same amplitude of the pulses, so we used a high-precision ADC (*Rudnev-Shilyaev* LAn10 series) for automated measurements. The data from the ADC are transmitted to a personal computer for processing. The vertex of each pulse is approximated using a polynomial of the second degree, the maximum point is determined, and the time intervals T and t_i are calculated. It was obtained by numerical modeling that a polynomial of the second degree would be sufficient to solve the problem. Approximation of data by polynomials of the third and higher orders, considering the possible amplitude noise, can lead to false maximum and not required in this case.

To find $\varepsilon_{\min}$, a series of deviation angle measurements are carried out at different angular positions of the prism (i.e. at different angles of incidence φ), which are obtained by rotating the object table at a certain fixed angle $\Delta\varphi$ using a stepper motor. The dependencies $\varepsilon_i(\varphi)$ for each wavelength are also approximated by a polynomial of the second degree and the corresponding values of $\varepsilon_{\min}$ are calculated.

Using formula (1), we can find the value of RI for each of the wavelengths, and then calculate the coefficients of the dispersion formula. For the Cauchy formula (2), it is sufficient to use three wavelengths, but for the most accurate Sellmeier equation with six coefficients, it is necessary to use six different laser diodes.

4. Measuring equipment

The functional diagram of the measuring equipment can be represented as six modules (figure 4):

(1) zero indicator module (laser diodes, slit diaphragms, beam splitter cube, lens, photodetector);
(2) optical-mechanical module (object table, rotating console with two-sided mirror, stepper motor);
(3) meteorological module (barometer, thermometer, hygrometer) for monitoring the ambient air parameters;
(4) electronic module for transmitting the signal from the photodetector and controlling the stepper motor;
(5) instrumental module (oscilloscope for monitoring the presence and the level of the photodetector signal, high precision ADC for time intervals measurements, contact thermometer for prism temperature measurements);
(6) computing and control module (a personal computer with software for processing the measurement data and transmitting stepper motor control commands).

To reduce random errors, the results obtained over several console rotations for each angular position of the prism are averaged.

The absolute RI of the prism material n_{abs} is calculated using the formula [25]:

$$n_{\mathrm{abs}} = n n_{\mathrm{air}} - \beta\,(t - 20^\circ)\,, \tag{4}$$

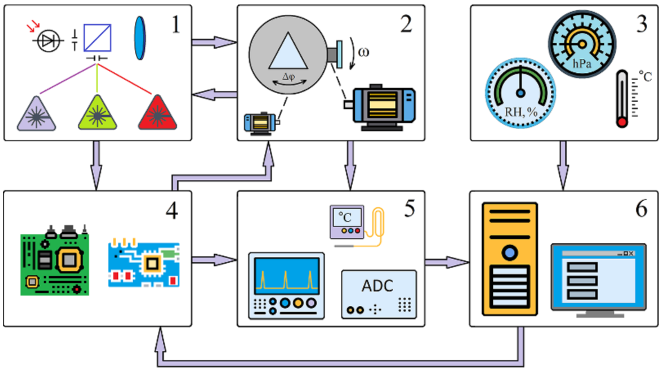

Figure 4. The functional diagram of measuring equipment. 1—zero indicator module; 2—optical-mechanical module; 3—meteorological module; 4—electronic module; 5—instrumental module; 6—computing and control module.

where n_{air} is the RI of air, t is the temperature of the prism during measurement; $\beta = \mathrm{d}n/\mathrm{d}t$ is the temperature coefficient of RI of the prism material.

The RI of the air is calculated using the Edlen formula [26] according to the measured meteorological parameters, the temperature coefficient of RI for the optical glass β can be found from the manufacturer documentation.

Using our goniometer–spectrometer we can perform measurements without the presence of the operator in the measurement zone, i.e. the measurements may be made automatically. All measurement operations carried out using computing and control module (6).

5. Angle measurement accuracy analysis

We evaluate the contribution of the measurement error of time intervals to the error of angular measurements. The error of measuring the time intervals is mainly determined by the following components:

- the timebase stability;
- the time intervals measurements resolution;
- the unevenness of the console rotation speed.

The timebase is provided by a quartz oscillator [19], unevenness at intervals of 1 s is from 10^{-9} to 10^{-12}, so error contribution is negligible.

The resolution error usually does not exceed the unit of the lowest digit and depends on the sampling rate f_S of the ADC used. Since the console rotation speed is about 40 rpm (respectively $T \sim 1.5$ s; $t_i \sim 1.0$ s), and the sampling rate $\sim$100 MHz, expressions can be written for the resolution relative error δT and δt of the time intervals:

$$\delta T = \pm 1/(f_S T) = \pm 1/\left(10^8 \cdot 1.5\right) = \pm 0.7 \cdot 10^{-8}, \quad (5)$$

$$\delta t = \pm 1/(f_S t_i) = \pm 1/\left(10^8 \cdot 1.0\right) = \pm 1.0 \cdot 10^{-8}. \quad (6)$$

Table 1. Minimum deviation angles measurement results.

Instrument	$\varepsilon_{455\ \mathrm{nm}}$ (°)	$\varepsilon_{520\ \mathrm{nm}}$ (°)	$\varepsilon_{638\ \mathrm{nm}}$ (°)	σ (°)
Frequency counter	34.501 439	34.137 684	33.765 844	$2 \cdot 10^{-4}$
Oscilloscope	34.489 887	34.152 550	33.754 189	$1 \cdot 10^{-4}$
ADC	34.496 818	34.147 976	33.758 851	$1 \cdot 10^{-5}$
Ring laser	34.497 511	34.147 705	34.758 842	$2 \cdot 10^{-6}$

Table 2. Absolute RI measurement results.

Instrument	$n_{455\ \mathrm{nm}}$	$n_{520\ \mathrm{nm}}$	$n_{638\ \mathrm{nm}}$
Frequency counter	1.525 615	1.520 717	1.515 694
Oscilloscope	1.525 459	1.520 917	1.515 536
ADC	1.525 553	1.520 855	1.515 599
Ring Laser	1.525 562	1.520 852	1.515 599
Reference value	**1.525 559**	**1.520 847**	**1.515 597**

Note: The reference values for comparing the results are highlighted in bold.

Thus, the main contribution to the error of time intervals measurements is made by the unevenness of the console rotation speed, which can be estimated through the coefficient of irregularity δ, determined by the formula:

$$\delta = (\omega_{max} - \omega_{min}) / \omega_n, \quad (7)$$

where $\omega_{max}, \omega_{min}, \omega_n$ are the maximum, minimum and average values of the angular speed for one cycle.

This coefficient depends on the type of electric motor rotating the console, and typical values of δ are 0.01–0.001, which is not enough to ensure the required angle measurement accuracy. To achieve the angular errors $\pm$3–6$''$, it is necessary to provide $\delta < 0.0001$, for example, by installing a flywheel [27], and to average the measurement results for several rotations of the console. Using these techniques, rotation speed unevenness error can be reduced to $\pm 10^{-6}$, which corresponds to the error of angular measurements in tenths of a second.

6. Experimental results

We used ten angular positions of the prism to obtain $\varepsilon_i(\varphi)$ dependences and ten rotations of the console for each position to reduce random errors.

Table 1 shows the results of the measurements of the angle of minimum deviation for three laser diode wavelengths (455 nm, 520 nm, 638 nm), obtained with four different instruments—frequency counter, oscilloscope, ADC and He–Ne ring laser, as well as typical standard deviation σ levels for angular measurements. We used a reference NBK-7 Schott glass prism [25] with well-known RI and Sellmeier coefficients as a sample.

Then the relative RI n was calculated using formula (1), n_{air} was calculated according to environmental conditions and wavelength by the Edlen formula, absolute RI n_{abs} was calculated by formula (4). Table 2 shows the results of absolute RI calculation for each wavelength and instrument.

We used Sellmeier coefficients of prism A_i and B_i obtained with State Primary Standard of the RI Unit GET 138-2021 of

Table 3. RI measurement errors results.

Instrument	$\Delta n_{455\text{ nm}}$	$\Delta n_{520\text{ nm}}$	$\Delta n_{638\text{ nm}}$
Frequency counter	$6 \cdot 10^{-5}$	$-1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$
Oscilloscope	$-1 \cdot 10^{-4}$	$7 \cdot 10^{-5}$	$-6 \cdot 10^{-5}$
ADC	$-6 \cdot 10^{-6}$	$8 \cdot 10^{-6}$	$2 \cdot 10^{-6}$
Ring laser	$3 \cdot 10^{-6}$	$5 \cdot 10^{-6}$	$2 \cdot 10^{-6}$

Table 4. Absolute RI calculation results.

Wavelength	486.13 nm	587.56 nm	656.27 nm
Calculated results	1.523 079	1.517 474	1.515 021
Reference value	1.523 062	1.517 490	1.515 002
Error	$1.7 \cdot 10^{-5}$	$-1.6 \cdot 10^{-5}$	$1.9 \cdot 10^{-5}$

the All-Russian Research Institute for Optical and Physical Measurements to calculate the reference values of RI for the wavelengths used and estimate measurement errors Δn_i (table 3) using:

$$n^2 = 1 + \frac{A_1\lambda^2}{\lambda^2 - B_1^2} + \frac{A_2\lambda^2}{\lambda^2 - B_2^2} + \frac{A_3\lambda^2}{\lambda^2 - B_3^2} \quad (8)$$

where $A_1 = 1.039\,652\,63$; $A_2 = 0.235\,792\,13$; $A_3 = 1.283\,407\,00$; $B_1 = 0.004\,406\,73$; $B_2 = 0.024\,594\,55$; $B_3 = 105.892\,630$.

Table 3 shows the error obtained with ADC is comparable to the error of a ring laser, which cannot be achieved using less accurate instruments like a frequency counter or an oscilloscope. The calculated absolute error Δn does not exceed $\pm 1 \cdot 10^{-5}$ when compared with the reference value, which is a very good result and proves the prospect of using the presented scheme for high-precision measurements of the RI of optically transparent materials.

Using an ADC measurement results, it is possible to find the Cauchy coefficients ($A = 1.504\ 877$; $B = 0.004\ 451$; $C = -0.000\ 035$) and calculate the RI at any wavelength. So, we calculated the value of RI at Fraunhofer lines n_F (486.13 nm), n_d (587.56 nm) and n_C (656.27 nm) required to determine the main dispersion characteristics of the prism material (principal dispersion and Abbe number) [23]. Table 4 shows the calculation results compared to the reference value.

According to table 4, the RI calculation error level is $\pm 2 \cdot 10^{-5}$, so for more accurate dispersion characteristics determination, 6 wavelengths and the Sellmeier equation should be used.

7. Conclusion

A dynamic goniometer–spectrometer with angle-to-time conversion proposed in this article can be used to measure RI and study the dispersion characteristics of triangular prisms made of optically transparent materials or liquid optically transparent substances filling a hollow triangular prism with plane-parallel transparent portholes. Such a scheme can significantly reduce the cost of the equipment and minimize the time spent on the study of dispersion characteristics because measurements carried out simultaneously at several wavelengths, while maintaining high measurement accuracy.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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References

[1] Born M and Wolf E 2013 *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light* (Elsevier)
[2] Rathmann C, Mann G and Nordberg M 1968 A new ultralow-expansion, modified fused-silica glass *Appl. Opt.* **7** 819
[3] Wray J and Neu J 1969 Refractive index of several glasses as a function of wavelength and temperature *J. Opt. Soc. Am.* **59** 774–6
[4] Greysukh G, Ezhov E, Levin I, Kalashnikov A and Stepanov S 2012 Modeling and investigation superachromatozation refractive and refractivediffractive optical systems *Comput. Opt.* **36** 395
[5] Zhang S and Shannon R 1995 Effective optical nonlinearity of KNSBN: Ce crystal and its applications to continuous-wave optical limiting *Opt. Eng.* **34** 3435
[6] Aman M, Handoko D, Sudarmaji A, Handoyo and Soedarmawan L 2022 Determination of diffusion coefficient of palm oil in n-hexane using laser deflection method and image processing *IOP Conf. Ser.: Earth Environ. Sci.* **1034** 012014
[7] Shehadeh A, Evangelou A, Kechagia D, Tataridis P and Chatzilazarou A 2020 Effect of ethanol, glycerol, glucose/fructose and tartaric acid on the refractive index of model aqueous solutions and wine samples *Food Chem.* **329** 127085
[8] Werner A 1968 Methods in high precision refractometry of optical glasses *Appl. Opt.* **7** 837
[9] Astrua M and Pisani M 2009 Prism refractive index measurement at INRiM *Meas. Sci. Technol.* **20** 095305
[10] Pagano R, Manhart P and Sherman P 1994. *Gradient Index Optical Systems, Technical Digest Series* vol GWA3 (Optica Publishing Group)
[11] Tilton L 2018 *Prism Refractometry and Certain Goniometrical Requirements for Precision* (Forgotten Books)
[12] Kuiper M, Van de Nes A, Nieuwland R, Varga Z and Van der Pol E 2021 Reliable measurements of extracellular vesicles by clinical flow cytometry *Am. J. Reprod. Immunol.* **85** e13350
[13] Vishnyakov G and Kornysheva S 2005 Provision of measurement uniformity in refractometry of solids, liquids, and gases *Meas. Tech.* **48** 1099
[14] Vishnyakov G, Yurin A, Minaev V and Golopolosov A 2023 A high precision VIS/NIR dynamic goniometer-spectrometer *Metrologia* **60** 045007
[15] Filatov Y, Loukianov D and Probst R 1997 Dynamic angle measurement by means of a ring laser *Metrologia* **34** 343
[16] Tentori D and Lerma J 1990 Refractometry by minimum deviation: accuracy analysis *Opt. Eng.* **29** 160

[17] Vishnyakov G, Levin G and Kornysheva S 2004 The state primary standard for the unit of refractive index *Meas. Tech.* **47** 1039
[18] Pavlov P, Filatov Y and Zhuravleva I 2021 Calibration of rotary encoders with different interfaces by means of a dynamic goniometer *Opt. Eng.* **60** 074105
[19] Lombardi M 2007 Fundamentals of time and frequency *Mechatronic Systems, Sensors, and Actuators: Fundamentals and Modeling* ed R H Bishop (CRC Press)
[20] ISO 21395-1:2020 *Optics and Photonics—Test Method for Refractive Index of Optical Glasses—Part 1: Minimum Deviation Method*
[21] Efimov A 2007 Refractive index of optical glasses versus wavelength: precision approximation with dispersion formulas *Eur. J. Glass Sci. Technol.* B **48** 235
[22] Tatian B 1984 Fitting refractive-index data with the Sellmeier dispersion formula *Appl. Opt.* **23** 4477
[23] Smith W 2000 *Modern Optical Engineering. The Design of Optical Systems* 3rd edn (The McGraw-Hill Companies, Inc.)
[24] Vishnyakov G, Levin G, Kornysheva S, Zyuzev G, Lyudomirskii M, Pavlov P and Filatov Y 2005 Measuring the refractive index on a goniometer in the dynamic regime *J. Opt. Technol.* **72** 929
[25] Vishnyakov G, Fricke A, Parkhomenko N, Hori Y and Pisani M 2016 Report on supplementary comparison COOMET.PR-S3: refractive index *Metrologia* **53** 02001
[26] Edlen B 1966 The refractive index of air *Metrologia* **2** 71
[27] Genta G 2014 *Kinetic Energy Storage: Theory and Practice of Advanced Flywheel Systems* (Butterworth-Heinemann)