Optik - International Journal for Light and Electron Optics 292 (2023) 171400

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

The modified minimum deviation method for measuring the refractive index

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ARTICLE INFO

Keywords:
Refractive index
Dynamic goniometer
Minimum deviation method
Refractive index measurements

ABSTRACT

We present a modified method for measuring the refractive index (RI) based on the minimum deviation technique, which allows us to determine the RI of transparent triangular prisms with unknown apex angles. In the proposed method, the angles of light deviation are measured on three faces of the prism, and RI of the material and prism angles are determined from the solution of a system of equations. A high precision dynamic goniometer was used to implement the proposed method. A set of reference prisms made of optical glass were experimentally studied and the measurement errors were estimated. It is shown that the modified method can be used for high-precision measurements of the RI in cases when the prism angles are unknown, or their measurement is associated with technical difficulties. The method of measuring RI proposed in the article was implemented using the State Primary Standard of the Refractive Index Unit GET 138–2021 of the All-Russian Research Institute for Optical and Physical Measurements. This standard is intended for storing, reproducing, and transmitting the unit size of the RI of solid and liquid substances.

1. Introduction

The refractive index (RI) is an important characteristic of a substance in any of the three states of aggregation - solid, liquid, or gaseous, and is equal to the ratio of the light speed in vacuum to the light speed in the substance under study. Performing high-precision and reliable measurements of the RI is necessary in the optical industry [1–3], in the chemical industry to control the composition of substances [4], in the food industry for product quality control [5,6], in medicine [7], biology [8], astronomy [9,10] etc.

RI is one of the few physical quantities that can be measured with high accuracy, quickly, and with only a small amount of material. To measure the RI, various methods are used, based on measuring the angle of refraction of light by a substance (refractometric methods) [11], measuring the phase delay of a light wave (interference methods) [12], laser spectroscopy methods [13], digital holography [14] etc. When implementing these methods, knowledge of the basic laws of optics is required, based on which the calculation of the parameters of light is carried out during its reflection, refraction, and absorption by the substance.

Most often, for measuring the RI, refraction is used, which consists in changing the direction of the propagation of an

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https://doi.org/10.1016/j.ijleo.2023.171400
Received 11 April 2023; Received in revised form 5 September 2023; Accepted 18 September 2023
Available online 20 September 2023
0030-4026/

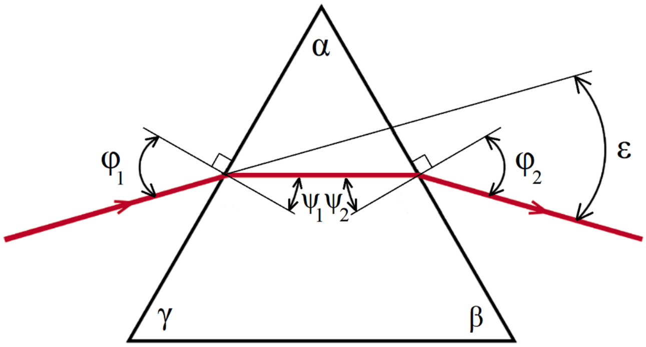

Fig. 1. Refraction of a light beam at the faces of a prism.

electromagnetic wave (or light) at the interface between two substances, which obeys the law of light refraction discovered in 1621 by the Dutch mathematician, physicist, and astronomer Snellius (*Willebrord Snel van Royen*, 1580–1626) [15]. The spread of refractometry as one of the most important physical methods of analysis was facilitated by the valuable combination of high accuracy, technical simplicity, and accessibility.

Goniometric methods based on measuring the angles of refraction of radiation passing through a sample have become widespread due to their versatility, high accuracy, and ease of measurement [16]. These methods can be applied to both solids and liquids. As samples, triangular prisms are most often used, made of the material, or filled with the substance under study [17].

In this article, we propose a new high-precision modified method for measuring the RI of a triangular prism based on the minimum deviation technique. Existing methods require separate measurements of the prism angles. The proposed method allows the simultaneous measurement of RI and prism angles, thereby simplifying the measurement procedure.

2. Measurement methods

Well-studied and widely used for measuring RI are some variations of the prism methods – autocollimation [18,19], minimum deviation [20,21], constant deviation, and grazing incidence (critical angle) [22]. The highest measurement accuracy is achieved by using the most popular minimum deviation method [23]. The application of this method implies to find such a position of the prism at which the minimum change in the direction of the incident beam is achieved.

Let us consider the passage of a beam through a triangular prism (Fig. 1). The angle between the two working faces of the prism α is the prism angle. A beam of light passing through a prism will be considered strictly monochromatic. Using the law of light refraction and the geometric properties of angles, we can write down the system of equations [15]:

$$\begin{cases} \sin\varphi_1 = n\sin\psi_1 \\ \sin\varphi_2 = n\sin\psi_2 \\ \psi_1 + \psi_2 = \alpha \\ \varphi_1 + \varphi_2 = \varepsilon + \alpha \end{cases} \tag{1}$$

where φ_1, ψ_1 are the angles of incidence and refraction on the first facet, φ_2, ψ_2 are the angles of incidence and refraction on the second facet, ε is the beam deviation angle, α is the prism angle, n is the relative RI of the prism material.

The deviation angle ε reaches an extremum if

$$d\varepsilon/d\varphi_1 = 0. \tag{2}$$

Note that this condition is satisfied if the rays pass through the prism symmetrically and the angle of incidence is equal to the exit angle, i.e. $\varphi_1 = \varphi_2$ and $\psi_1 = \psi_2$.

The solution of the system (1) is the expression (3):

$$\varepsilon(\varphi_1) = \varphi_1 - \alpha + \arcsin[n\ \sin(\alpha - \arcsin(\sin(\varphi_1)/n))]\cdot \tag{3}$$

Thus, the deviation angle for a fixed wavelength and temperature is a function of three parameters: φ_1, α and n. As the angle of incidence increases, the deflection angle changes and reaches a minimum ε_{min} at a certain value of φ_1.

The angle of minimum deviation ε_{min} is determined as the minimum of function (3) by the angle φ_1:

$$\varepsilon_{min} = 2\arcsin[n\ \sin(\alpha/2)] - \alpha\cdot \tag{4}$$

From the expression (4) we obtain a simple formula for calculating RI with a minimum deviation [19]:

$$n = \sin[(\alpha + \varepsilon_{min})/2]/\sin(\alpha/2)\cdot \tag{5}$$

Thus, to calculate RI by the minimum deviation method, in addition to measuring the angle ε_{min}, it is first necessary to measure the prism angle α, which is not always possible. An urgent task is to develop methods for measuring RI that do not require separate measurements of prism angles. Earlier, a group of authors proposed a new variation of the minimum deviation method that simultaneously measures the RI and prism angles, thereby relinquishing the need for prism angle measurements [24]. Compared to the classical minimum deviation method, this variation has two limitations. The first limitation is that in the automated form the prism angles must be close to 60 degrees. The second limitation is that there is not one but six measurements of the angle of minimum deviation. Our modified method requires only three automated measurements and fits any prism angles (for prism angles not exceeding 2acrsin$(1/n)$ because of total internal reflection).

3. The modified method

The essence of the modified minimum deviation method proposed in this paper is as follows. To determine RI, the following operations are performed:

1. The angles of minimum deviation $\varepsilon_{\min\,\alpha}$, $\varepsilon_{\min\,\beta}$, $\varepsilon_{\min\,\gamma}$ are measured on all three faces of the prism with apex angles α, β and γ.
2. A system of equations based on (4) is compiled:

$$\begin{cases} \varepsilon_{\min\alpha} = 2\arcsin(n\sin(\alpha/2)) - \alpha \\ \varepsilon_{\min\beta} = 2\arcsin(n\sin(\beta/2)) - \beta \\ \varepsilon_{\min\gamma} = 2\arcsin(n\sin(\gamma/2)) - \gamma \\ \alpha + \beta + \gamma = \pi \end{cases} \tag{6}$$

3. Calculations of the n (relative RI) and the values of the prism angles α, β and γ from the solution of system (6) are made.

Thus, using the modified method there is no need to pre-measure the prism angles, which is typical for the classical minimum deviation method.

4. Implementation

A high-precision RI measurements with an error of $\pm 1{\cdot}10^{-6}$ requires primarily the measurement of the prism angles and deviation angle at the level of tenths of an angular second [25]. We used a high-precision RI measurement system based on a dynamic goniometer containing a He-Ne ring laser for angle determination [26], which provides the necessary accuracy (with root mean square better than $\pm$ 0,03″). Measurements with such a goniometer are made under dynamic conditions with the autocollimation mirror rotating continuously. Firstly, we perform a series of automatic measurements of the deviation angle ε with various angular positions of the prism relative to the immobile beam, i.e., for various angles of incidence φ_i, $i = 0, 1, 2,., K$, where K is the total number of measurements. Then the experimental $\varepsilon(\varphi_i)$ is fitted to a polynomial of the second degree and $\varepsilon_{\min}$ is calculated.

A. Hardware

A high-precision RI measurement system includes the following:

1) a dynamic goniometer with a He-Ne ring laser and autocollimation zero indicator;
2) a climatic chamber with feedback thermal stabilization and a multichannel digital thermometer with separate temperature sensors to measure prism and air temperature;
3) a barometer to measure the atmospheric pressure in the chamber;
4) a hygrometer to measure the humidity of the air in the chamber;
5) a system for acquiring and processing the data based on a personal computer.

To measure the RI on the different wavelengths we apply three different light sources with fiber-optic output - laser diode modules (*LLC LasersCom* LDI series) with wavelengths of 515 nm and 780 nm, as well as 632.8 nm He-Ne laser (*Lumentum* 1145 P). A *Hamamatsu* G10899–03 K InGaAs photodiode was used as a radiation detector.

B. Software

The original RefractiveIndexMeter software is used to carry out all measurement operations and it consists of two parts: software for angle measurements and software for RI calculation. Angle measurements are carried out in the automatic mode.

C. Samples

We used a set of RI measures №01, №02, №03 [27] and a hollow prism №04 filled with distilled water as an objects to study. The set consists of triangular prisms made of optical colorless glasses N-BAF10, N-BK7 and N-SF1 accordingly manufactured by Schott AG (Germany). In the standard equipment, the prism pyramidality should not exceed 2″. In that case it has no effect on the measurement accuracy with our method. Nonplanar working faces leads to the angle of the prism being different from the nominal value. Consequently, the angle of deviation will also differ from the calculated value. Nonplanarity tests have shown that a

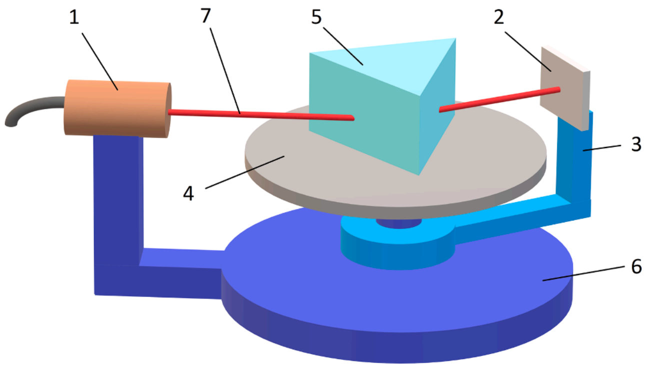

Fig. 2. The scheme of the dynamic goniometer: 1 – zero indicator; 2 – two-sided mirror; 3 – rotating console; 4 – specimen stage; 5 – prism; 6 – base; 7 – light beam.

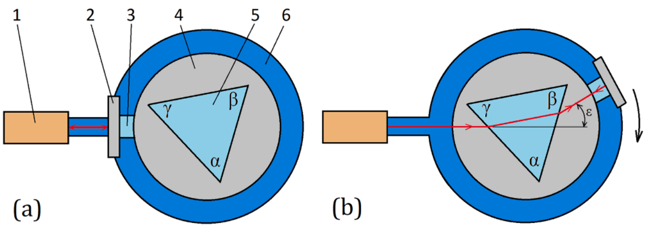

Fig. 3. A light beam reflection scheme: (a) – reflection from the external side of the two-sided mirror; (b) – reflection from the inner side of the two-sided mirror. 1 – zero-indicator; 2 – two–sided mirror; 3 – rotating console; 4 – specimen stage; 5 – prism; 6 – base.

deviation of λ/10 leads to the angle changing by tenths of an angular second, so deviation from planarity of the working faces of the prisms should not be greater than λ/18 [28].

D. **Environmental conditions**

The air and sample temperatures should be kept at constant values during the measurement, while the temperature of the prism and the surrounding parameters (air temperature, atmospheric pressure, and humidity) must be measured with high accuracy. Using a further feature of such a goniometer is that it can perform measurements automatically without the presence of the operator in the measurement zone, i.e., the measurements can be made remotely. This also serves to resolve the problems of thermal stabilization for the measurement volume. The necessary part of the measuring equipment is placed in an insulated chamber, while the other units, which produce the main heat (recording equipment, power supplies, and so on) are kept outside it.

E. **Measurement scheme**

To measure the desired angles of minimum deviation by the proposed method, we used a dynamic goniometer with He-Ne ring laser (Fig. 2). The ring laser creates an angular scale, the accuracy of which is determined by the wavelength of the laser light.

To implement the method of minimum deviation with the goniometer, a two-sided mirror (2) was used, which is mounted on the edge of a constantly rotating console (3) in a position where the normal to the mirror is perpendicular to the axis of rotation. A console is rotated by an electric motor at a constant speed about 40 rpm. The triangular optical transparent prism (5) is placed on a rotary specimen stage (4), which is not mechanically connected to the continuously rotating goniometer console (3). During measurements, the prism and this stage are stationary. Between measurements, the stage with the prism is rotated by a stepper motor at a certain angle.

The first signal from the zero-indicator (1) arises when the light beam is reflected from the external surface of the two-sided mirror with respect to the axis of rotation (Fig. 3(a)). It sets the origin of angular measurements. The second signal arises from the beam that has passed through the prism and is reflected from the inner surface of the mirror (Fig. 3(b)). This signal sets the deviation angle of the refracted beam with respect to the incident beam [29]. The prism rotation angle step was chosen to be the same for all measurements.

To find the angle of minimum deviation ε_{min}, a series of measurements of the deviation angle for various angular positions of the prism were carried out, i.e., for different angles of incidence, after which the dependence of the angle of deviation on the angle of incidence $\varepsilon(\varphi_1)$ was determined, followed by its approximation using a polynomial of the 2nd degree over nine points, from which the minimum value ε_{min} is calculated.

F. **Accuracy analysis**

It is known that the minimum deviation method has the highest accuracy among all prism methods [22]. The error of the classical

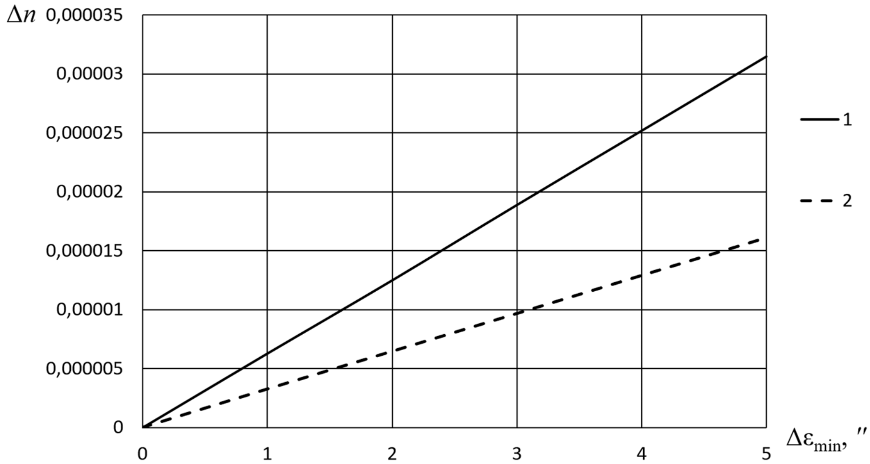

Fig. 4. Dependence of the RI measurement error on the angular measurement error for a prism with $\alpha = 60^\circ$ and $n = 1.5$: 1 – for classical minimum deviation method; 2 – for modified minimum deviation method.

Table 1
Minimum deviation angles measurement results.

Prism №	Wavelength, nm	$\varepsilon_{min\ \alpha}$, °	$\varepsilon_{min\ \beta}$, °	$\varepsilon_{min\ \gamma}$, °
01 (N-BAF10)	515	53.962063	53.958772	53.968987
	632.8	52.947174	52.944425	52.954886
	780	52.294465	52.292588	52.302176
02 (N-BK7)	515	34.170273	44.671426	39.004819
	632.8	33.763033	44.083638	38.521492
	780	33.478064	43.674326	38.182635
03 (N-SF1)	515	47.896424	59.610602	78.282732
	632.8	46.600826	57.782437	74.982965
	780	45.793957	56.656132	73.043462
04 (H_2O)	515	21.789439	23.780425	26.133314
	632.8	21.500302	23.461090	25.776067
	780	21.270314	23.205593	25.491424

Table 2
RI calculation results.

Prism №	Wavelength, nm	n_0	Δn
01 (N-BAF10)	515	1.676978	$-1\cdot10^{-7}$
	632.8	1.667265	$1\cdot10^{-8}$
	780	1.660949	$1\cdot10^{-8}$
02 (N-BK7)	515	1.520868	$1\cdot10^{-7}$
	632.8	1.515368	$3\cdot10^{-8}$
	780	1.511506	$3\cdot10^{-8}$
03 (N-SF1)	515	1.728640	$1\cdot10^{-7}$
	632.8	1,712378	$2\cdot10^{-8}$
	780	1.702146	$-5\cdot10^{-8}$
04 (H_2O)	515	1.336230	$3\cdot10^{-8}$
	632.8	1.332063	$8\cdot10^{-8}$
	780	1.328732	$3\cdot10^{-8}$

minimum deviation method Δn can be determined analytically [29] if we calculate the partial derivatives of the expression (4):

$$\Delta n = \Delta\varepsilon\ |(\sin[(\alpha + \varepsilon_{min})/2])/[2\ \sin(\alpha/2)]| + \Delta\alpha\ |[\sin(\varepsilon_{min}/2)]/[2\ \sin[2(\alpha/2)]]|, \quad (7)$$

where Δε and Δα are the measurement errors of the corresponding angles.

Since the values of ε and α are often measured using the same methods, we will assume that Δε and Δα are the same and correlated, i.e. $\Delta\varepsilon = \Delta\alpha$.

In the modified method, RI is found by solving systems of equations, therefore, numerical modeling must be used to estimate errors using the worst possible scenario for the distribution of the angular measurements errors. We calculate the error of determining the RI Δn for a prism with $\alpha = 60^\circ$ and $n = 1.5$ depending on the angular measurement error $\Delta\varepsilon_{min}$ using Eq. (7) for classical method and

Table 3
Prism angles calculation results.

Prism №	α, °	β, °	γ, °
01	59.999273	59.997296	60.003430
02	54.946739	65.054430	59.998831
03	52.965740	59.999367	67.034893
04	56.161041	59.884801	63.954158

solving system (6) for our modified method with the values of $\Delta\varepsilon_{min}$ from 0″ to 5″ (Fig. 4).

As it can be seen at Fig. 4, the error $\Delta\varepsilon_{min}$ has less influence on the results of RI measurements in the case of our modified method used.

5. Experimental results

Table 1 shows the results of measurements of the angle of minimum deviation $\varepsilon_{min\alpha}$, $\varepsilon_{min\beta}$, $\varepsilon_{min\gamma}$ for three different faces of the three prisms, from the values of which the relative RI n was calculated by solving the system (6), and n_{air} was calculated according to environmental conditions and wavelength. The system (6) can be solved by using various mathematical software packages (Mathcad, MATLAB, etc.). In our work, n was determined using the Solver add-in of Microsoft Excel.

Table 2 shows the calculated values of relative RI n_0 reduced to the standard atmospheric conditions (101,325 Pa, 20 °C, 50% of relative humidity) according to.

$$n_0 = [n\ n_{air} - \beta(t - 20^\circ)] /\ n_{air0} \quad (8)$$

where β is the temperature coefficient of prism material, t is the temperature of prism, n_{air0} is RI of air under standard conditions.

The Edlen formula [30] was used to consider the RI of air n_{air}.

We used an average RI value, calculated with the classical minimum deviation method for each prism as reference value to estimate measurement errors Δn.

Table 2 shows the absolute measurement error Δn does not exceed $\pm 1\cdot10^{-7}$ when compared with the reference value, which is a very good result and proves the prospect of using the proposed method for high-precision measurements of the RI of optically transparent materials.

It should be noted that when solving the system (6), the prism angles are also determined - α, β, γ, the calculation results for 515 nm laser wavelength are given in Table 3. The absolute error in calculating the angles does not exceed ± 2″ from the reference values. The reference values of the prisms angles were obtained using an automated goniometric system designed to measure the angles between flat surfaces [31] with a photoelectronic encoder providing measurement accuracy of ± 0.25″. That goniometric system was calibrated using a standard reference polygonal prism with traceability to the national measurement standard of a flat angle.

Thus, using the proposed method for measuring the value of RI, it is also possible to determine unknown prism angles with high accuracy, which eliminates the need for the preliminary measurement of these angles.

6. Conclusion

The modified method for measuring RI proposed in this article can be used to study triangular prisms made of optically transparent materials in cases when these prism angles are unknown, or their high-precision measurements are associated with technical difficulties. The method can also be used for liquid optically transparent substances filling a hollow triangular prism. The proposed method was implemented using the State Primary Standard of the Refractive Index Unit GET 138–2021 of the All-Russian Research Institute for Optical and Physical Measurements [32]. This standard is intended for storing, reproducing, and transmitting the unit size of RI of solid and liquid substances.

Disclosures

The authors declare no conflicts of interest.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

Data will be made available on request.

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