Conformal Field Theory

Based on M. Alfimov, G. Ferrando, V. Kazakov and E. Olivucci, arXiv:2311.01437

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Motivation

- There exists a multitude of CFTs in spacetime dimensions $d < 4$.
- For $d \geqslant 4$, however, the list of known CFTs is very short (Banks, Zaks'82), unless supersymmetry enters the game.
- Allowing for non-unitary, logarithmic CFTs (Gurarie'93), an ample class of integrable theories in $d = 4$ has been discovered in as a special double-scaling limit of γ-deformed $\mathcal{N} = 4$ super Yang–Mills theory (Gürdoğan, Kazakov'16), later generalized to any dimension d (Kazakov, Olivucci'18).
- In the planar 't Hooft limit, the perturbation theory of such CFTs is dominated by a very limited number of Feynman diagrams represented by the regular square lattice, called fishnets.
- Apart from these Fishnet CFTs, a vast class of so-called Loom FCFTs was proposed in (Kazakov, Olivucci'22).
- This construction relies on the existence, for each such diagram, of an associated Baxter lattice (Zamolodchikov'80) – a collection of straight lines parallel to M directions that we dub *slopes*.
- In this work we study a class of Loom FCFTs with $M = 4$ slopes that feature only quartic scalar vertices.

Definition of Checkerboard CFT

- The Lagrangian of the theory is

$$\mathcal{L}^{(CB)} = N\text{Tr}\left[\sum_{j=1}^{4} \bar{Z}_j(-\partial_\mu\partial^\mu)^{w_j} Z_j - \xi_1^2\, \bar{Z}_1\bar{Z}_2 Z_3 Z_4 - \xi_2^2\, Z_1 Z_2 \bar{Z}_3\bar{Z}_4\right],$$

where $w_1 + w_2 + w_3 + w_4 = d$ and therefore the couplings $\xi_{1,2}$ are dimensionless. The fields Z_k, $k = 1, 2, 3, 4$ are in the adjoint representation of $SU(N)$.

- We introduce the following parametrization

$$w_1 = u + d - \Delta_+\,, \quad w_2 = -u + \Delta_-\,, \quad w_3 = u + \Delta_+\,, \quad w_4 = -u - \Delta_-\,.$$

- It follows that the scaling dimensions of the fields $\bar{Z}_j, Z_j$ are $\Delta_j = \frac{d}{2} - w_j$.

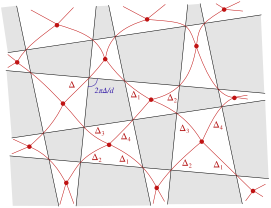
- While for generic w_j's the Lagrangian is UV complete and the theory is finite, there are *special values*

$$\Delta_1 + \Delta_2 = \frac{d}{2} = \Delta_3 + \Delta_4 \quad \text{or} \quad \Delta_1 + \Delta_4 = \frac{d}{2} = \Delta_2 + \Delta_3\,.$$

when double-trace correlators of length-2 operators are divergent and the corresponding counterterm must be added

$$\mathcal{L}^{(CB)}_{\text{dt}} = \alpha(\xi_1, \xi_2)\text{Tr}(\bar{Z}_1\bar{Z}_2)\text{Tr}(Z_3 Z_4) + \bar{\alpha}(\xi_1, \xi_2)\text{Tr}(Z_1 Z_2)\text{Tr}(\bar{Z}_3\bar{Z}_4)\,.$$

Propagators and vertices of Checkerboard CFT



- The propagators of the adjoint fields Z_k are

$$D_i(x) = \langle Z_i(x)\bar{Z}_i(0)\rangle = \frac{\Gamma\left(\frac{d}{2} - w_i\right)}{4^{w_i}\pi^{\frac{d}{2}}\Gamma(w_i)} \frac{1}{(x^2)^{\frac{d}{2}-w_i}},$$

- Such a simple content of Feynman diagrams of the theory is a consequence of the non-Hermiticity, i.e. the *chirality*, because these vertices are absent

$$\text{Tr}[\bar{Z}_i\bar{Z}_{i+1}Z_{i+2}Z_{i+3}]^\dagger = \text{Tr}[\bar{Z}_{i+3}\bar{Z}_{i+2}Z_{i+1}Z_i].$$

- Checkerboard CFT can be viewed as a reduction of the Loom FCFT$^{(4)}$ with $M = 4$ slopes if we keep only

$$\text{Tr}\left[v_1 X_3 Y_2 \bar{u}_1\right] \quad \text{and} \quad \text{Tr}\left[u_1 \bar{v}_1 \bar{X}_3 \bar{Y}_2\right]$$

and then identify the fields as follows

$$u_1 = Z_1\,, \quad \bar{v}_1 = Z_2\,, \quad X_3 = Z_3 \quad \text{and} \quad Y_2 = Z_4\,.$$

Integrability of the correlators

- We will consider a class of $2L$-point functions obtained by complete point-split inside the two traces. A concrete instance of such correlator

$$\langle \mathcal{O}(x_1, \dots, x_L) \widetilde{\mathcal{O}}(x'_1, \dots, x'_L) \rangle ,$$

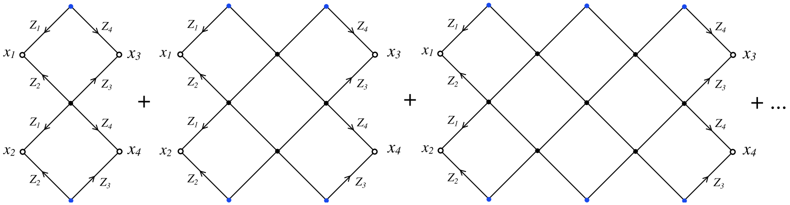
where

$$\mathcal{O}(x_1, \dots, x_L) = \mathsf{Tr}[(Z_1 Z_2)(x_1)\, (Z_1 Z_2)(x_2)\, \dots\, (Z_1 Z_2)(x_L)] ,$$
$$\widetilde{\mathcal{O}}(x'_1, \dots, x'_L) = \mathsf{Tr}[(\bar{Z}_4 \bar{Z}_3)(x'_1)\, (\bar{Z}_4 \bar{Z}_3)(x'_2)\, \dots\, (\bar{Z}_4 \bar{Z}_3)(x'_L)] .$$

- The Feynman diagrams for a given order n can be expressed as a power of a certain integral "graph-building" operator $\widehat{T}$, acting on functions of L variables, say $x_1, \dots, x_L$, in $\mathbb{R}^d$. In practice, one of the L diagrams is expressed as the kernel of $\widehat{T}^{\,n}$, namely

$$T_n(x_1, \dots, x_L | x'_1, \dots, x'_L) =$$
$$= \int \prod_{i=1}^{L} d^d y_i \, T_{n-1}(x_1, \dots, x_L | y_1, \dots, y_L) T(y_1, \dots, y_L | x'_1, \dots, x'_L) .$$

Integrability of correlators II



- The correlator at finite coupling results from the Bethe-Salpeter resummation, namely

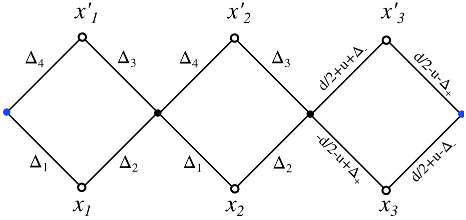
$$\langle \mathcal{O}(x_1,\ldots,x_L)\widetilde{\mathcal{O}}(x'_1,\ldots,x'_L)\rangle = \sum_{j=0}^{L-1} K(x_{1+j},x_{2+j},\ldots,x_{L+j}|x'_1,\ldots,x'_L)\,,$$

with

$$K(x_1,\ldots,x_L|x'_1,\ldots,x'_L) = \xi_2^{2L}\sum_{n=0}^{+\infty}(\xi_1^2\xi_2^2)^{nL}\,T_{n+1}(x_1,\ldots,x_L|x'_1,\ldots,x'_L)\,.$$

- The operator $\widehat{T}$ is the transfer matrix of a non-compact, homogeneous spin chain with $SO(1,d+1)$ symmetry and periodic boundary conditions. Each of the L sites carries the infinite-dimensional representation of a scalar field with scaling dimension $\Delta_1+\Delta_2$.

Integrability of correlators III



- The operator $\widehat{T}$ is the trace over the auxiliary space (infinite-dimensional representation of dimension $\Delta_0 = \Delta_1 + \Delta_4$) of a product of L solutions $\widehat{R}_{0k}$ of the Yang–Baxter equation (Chicherin, Derkachev, Isaev'12), that is

$$\widehat{T} = \mathrm{Tr}_0 \left[\widehat{R}_{01} \widehat{R}_{02} \ldots \widehat{R}_{0L} \right] .$$

- The kernel of each of the operators $\widehat{R}_{0k}$, $k = 1, 2, \ldots, L$ is

$$R(x_1, x_0 | x_{1'}, x_{0'}) =$$

$$= \frac{c}{(x_{10}^2)^{-u-\frac{d}{2}+\Delta_+} (x_{01'}^2)^{u+\frac{d}{2}+\Delta_-} (x_{1'0'}^2)^{-u+\frac{d}{2}-\Delta_+} (x_{0'1}^2)^{u+\frac{d}{2}-\Delta_-}} ,$$

where $\Delta_\pm = (\Delta_0 \pm (\Delta_1 + \Delta_2))/2$, and

$$c = \prod_{j=1}^{4} \frac{\Gamma\left(\frac{d}{2} - w_j\right)}{4^{w_j} \pi^{\frac{d}{2}} \Gamma(w_j)} .$$

Anomalous Dimensions for $L = 2$

- We derive the exact expression for the shortest four-point correlator in the Checkerboard CFT, and extract the anomalous dimension of lightest single-trace operator, which dominates the OPE s-channel,

$$\mathsf{Tr}[Z_1 Z_2 Z_1 Z_2](x)\,.$$

- One has

$$\big\langle \mathsf{Tr}[(Z_1Z_2)(x_1)(Z_1Z_2)(x_2)]\mathsf{Tr}[(\bar{Z}_4\bar{Z}_3)(x_1')(\bar{Z}_4\bar{Z}_3)(x_2')]\big\rangle = \\ = K(x_1,x_2|x_1',x_2') + K(x_1,x_2|x_2',x_1')$$

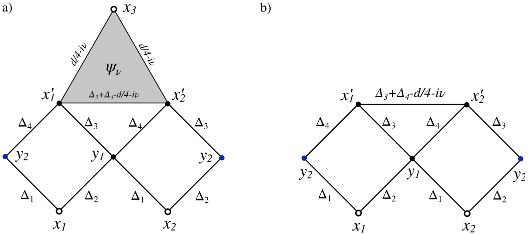
with

$$K(x_1,x_2|x_1',x_2') = \xi_1^4 \sum_{n=0}^{\infty} (\xi_1^2\xi_2^2)^{2n}\, T_{n+1}(x_1,x_2|x_1',x_2')\,.$$

- The kernel of the operator $\widehat{T}$ at $L = 2$ takes the form

$$T(x_1,x_2|x_1',x_2') = \\ = c^2 \iint \frac{d^dx_0 d^dx_{0'}}{(x_{10}^2)^{-u-\frac{d}{2}+\Delta_+}(x_{01'}^2)^{u+\frac{d}{2}+\Delta_-}(x_{1'0'}^2)^{-u+\frac{d}{2}-\Delta_+}(x_{0'1}^2)^{u+\frac{d}{2}-\Delta_-}} \\ \times \frac{1}{(x_{20'}^2)^{-u-\frac{d}{2}+\Delta_+}(x_{0'2'}^2)^{u+\frac{d}{2}+\Delta_-}(x_{2'0}^2)^{-u+\frac{d}{2}-\Delta_+}(x_{02}^2)^{u+\frac{d}{2}-\Delta_-}}\,.$$

Anomalous Dimensions for $L = 2$ II



- The spectral equation reads

$$\iint d^d x_{1'} d^d x_{2'} T(x_1, x_2 | x_1', x_2') \Psi_{\nu,S}(x_{1'}, x_{2'}; x_3) = h(\nu, S)\, \Psi_{\nu,S}(x_1, x_2; x_3)\,,$$

where $\nu \in \mathbb{R}$ is the continuous label of principal series and S is the spin.

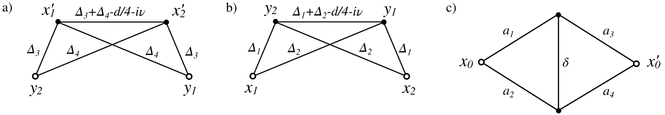
- For $L = 2$ the eigenfunctions of $\widehat{T}$ are entirely determined by its conformal symmetry and $\Psi_{\nu,S}(x_{1'}, x_{2'}; x_3)$ is a conformal 3-point function between two scalars of dimension $\Delta_+ - \Delta_- = \Delta_1 + \Delta_2$ and one symmetric traceless tensor of spin S with dimension in the principal series $\Delta = d/2 + 2i\nu$ (Dobrev et al.'77, Polyakov'70, Fradkin, Palchik'78). The eigenfunction $\Psi_\nu \equiv \Psi_{\nu,0}$ has the form

$$\Psi_\nu(x_1, x_2; x_3) = C(\nu) \left(x_{12}^2\right)^{\frac{d}{4}+i\nu-\Delta_1-\Delta_2} \left(x_{13}^2 x_{23}^2\right)^{-\frac{d}{4}-i\nu} \,.$$

- Sending $x_3^2 \to +\infty$, we obtain the eigenvalue

$$h(\nu) = \left(x_{12}^2\right)^{\Delta_1+\Delta_2-\frac{d}{4}-i\nu} \iint d^d x_1' d^d x_2'\, T(x_1, x_2 | x_1', x_2') \left(x_{1'2'}^2\right)^{\frac{d}{4}+i\nu-\Delta_1-\Delta_2} \,.$$

Anomalos Dimensions for $L = 2$ III



- It is convenient to factor the eigenvalue into the product of two terms $h(\nu) = h_1(\nu)h_2(\nu)$, because $h_1(\nu)$ and $h_2(\nu)$ are in fact the same function (Derkachev, Ivanov, Shumilov'23)

$$B(a_1, a_2, \delta) = \frac{(x_{00'}^2)^{\delta+2a_1+2a_2-d}}{4^{2d-2a_1-2a_2}\pi^{2d}} \left(\frac{\Gamma(a_1)\Gamma(a_2)}{\Gamma\left(\frac{d}{2}-a_1\right)\Gamma\left(\frac{d}{2}-a_2\right)} \right)^2$$

$$\times \iint \frac{d^d x_{1'} d^d x_{2'}}{(x_{1'2'}^2)^{\delta}(x_{01'}^2)^{a_1}(x_{1'0'}^2)^{a_2}(x_{0'2'}^2)^{a_1}(x_{2'0}^2)^{a_2}},$$

evaluated at different values of its parameters, i.e.

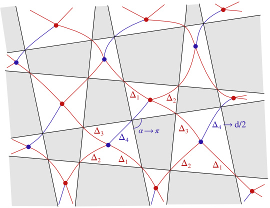
$$h_1(\nu) = B\left(\Delta_1, \Delta_2, \Delta_3 + \Delta_4 - \frac{\Delta}{2}\right), \quad h_2(\nu) = B\left(\Delta_3, \Delta_4, \Delta_1 + \Delta_2 - \frac{\Delta}{2}\right).$$

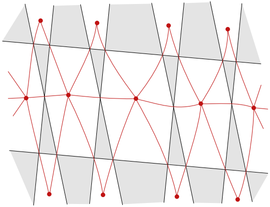
- The insertion of a resolution of the identity reads

$$K(x_1, x_2|x_1', x_2') =$$

$$= \sum_{S=0}^{+\infty} \int d\rho(\nu, S) \int d^d x_3\, \Psi_{\nu,S}(x_1, x_2; x_3)\bar{\Psi}_{\nu,S}(x_1', x_2'; x_3) \frac{\xi_1^4\, h(\nu, S)}{1 - \xi_1^4 \xi_2^4\, h(\nu, S)}.$$

FCFT with regular triangular graphs and ABJM reduction





- An interesting choice for the parameters is given by $w_4 = -u - \Delta_- = 0$.
- The Checkerboard CFT then turns into the anisotropic d-dimensional FCFT

$$\mathcal{L}_d^{(CB)} =$$
$$= N\mathrm{Tr}\Big[\bar{Z}_1(-\partial_\mu\partial^\mu)^{u+d-\Delta_+}Z_1 + \bar{Z}_2(-\partial_\mu\partial^\mu)^{-2u}Z_2 + \bar{Z}_3(-\partial_\mu\partial^\mu)^{u+\Delta_+}Z_3$$
$$-\xi^2\bar{Z}_3\bar{Z}_1\bar{Z}_2Z_3Z_1Z_2\Big] \, ,$$

where the coupling constant is $\xi^2 = \xi_1^2\xi_2^2$.

- This theory is a d-dimensional generalization of FCFT from (Caetano, Gürdoğan, Kazakov'18) stemming from the $3d$ ABJM theory in the double scaling limit.
- This latter theory is recovered at the point $d = 3$, $u = -1/2$, and $\Delta_+ = 3/2$ or, equivalently, $\Delta_1 = \Delta_2 = \Delta_3 = 1/2$.

ABJM $L = 2$ Fishnet

- Here we shall present the explicitly $h(\nu)$, i.e. $h_1(\nu)$ and $h_2(\nu)$ for the ABJM FCFT, namely

$$h_1(\nu) = B\left(\frac{1}{2}, \frac{1}{2}, 2 - \frac{\Delta}{2}\right), \quad h_2(\nu) = B\left(\frac{1}{2}, \frac{3}{2}, 1 - \frac{\Delta}{2}\right),$$

  where $\Delta = 3/2 + 2i\nu$ for $d = 3$.

- By utilizing the result of (Derkachev, Ivanov, Shumilov'23), we are able to calculate

$$h(\Delta) = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

  in the form of rather complicated sums depending on Δ.

- However, we can solve the equation

$$h(\Delta) = \frac{1}{\zeta}, \quad \zeta = (\xi_1 \xi_2)^4$$

  both in the weak coupling and numerically.

- The perturbative expansion suggests solving the equation above with $\Delta = 2 + \gamma$ for small γ.

ABJM $L = 2$ Fishnet II

- After a series of demanding calculations, we obtain the expansion in γ

$$h = -\frac{1}{1024\pi^2\gamma} + \frac{1}{1024\pi^4}\left(\pi^2 + \pi^2\log 2 - \frac{21}{2}\zeta_3\right) - \\ - \frac{1}{1024\pi^4}\left(\pi^2 + \pi^2\log 2 - \frac{21}{2}\zeta_3 + \frac{\pi^4}{40} + \frac{\log^4 2}{2} + 12\mathrm{Li}_4\left(\frac{1}{2}\right)\right)\gamma + \mathcal{O}(\gamma^2)\,.$$

- One may notice a curious observation that the expression

$$\gamma(1+\gamma)h = -\frac{1}{1024\pi^4}\left(\pi^2 - \left(\pi^2\log 2 - \frac{21}{2}\zeta_3\right)\gamma + \right. \\ \left. + \left(\frac{\pi^4}{40} + \frac{\log^4 2}{2} + 12\mathrm{Li}_4\left(\frac{1}{2}\right)\right)\gamma^2 + \mathcal{O}(\gamma^3)\right)$$

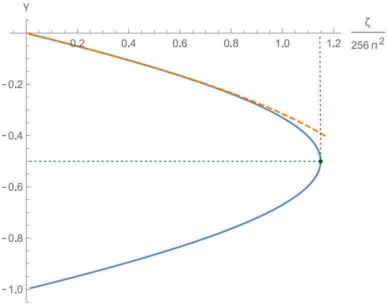
has uniform transcendentality.

- Solving the equation for the spectrum yields

$$\gamma = -\eta - \left(1 + \log 2 - \frac{21\zeta_3}{2\pi^2}\right)\eta^2 - \left(2 + 3\log 2 - \frac{63\zeta_3}{2\pi^2} + \right. \\ \left. + \frac{\pi^2}{40} - \log^2 2 + \frac{\log^4 2}{2\pi^2} - \frac{21\zeta_3\log 2}{\pi^2} + \frac{441\zeta_3^2}{4\pi^4} + \frac{12\mathrm{Li}_4\left(\frac{1}{2}\right)}{\pi^2}\right)\eta^3 + \mathcal{O}\left(\eta^4\right)\,.$$

where $\eta = \zeta/(1024\pi^2)$.

ABJM $L = 2$ Fishnet III



- The perturbative expansion of the anomalous dimension is in complete agreement with the numerical calculation and, moreover, supports the hypothesis that $h(\gamma)$ is even with respect to $\gamma = 1/2$.
- In addition, one can express the following function

$$\gamma(1+\gamma)h = -\frac{1}{1024\pi^4}\Bigg[12\mathfrak{L}_2 + 6\mathfrak{L}_1^2 + \left(12\mathfrak{L}_3 - 2\mathfrak{L}_1^3\right)\gamma + \left(12\mathfrak{L}_4 + \frac{18}{5}\mathfrak{L}_2^2 + \frac{18}{5}\mathfrak{L}_2\mathfrak{L}_1^2 + \frac{7}{5}\mathfrak{L}_1^4\right)\gamma^2 + \mathcal{O}(\gamma^3)\Bigg]$$

solely in terms of the polylogarithms $\mathfrak{L}_j = \mathrm{Li}_j(1/2)$.

FCFT of BFKL Type From the Checkerboard

- Choosing the scaling dimensions $\Delta_1 = -1-u$, $\Delta_2 = 1+u$, $\Delta_3 = 1-u$, $\Delta_4 = 1+u$, the corresponding Lagrangian reads

$$\mathcal{L}_d = N\mathrm{Tr}\Big[\bar{Z}_1(-\bar{\partial}\partial)^{u+2}Z_1 + \bar{Z}_2(-\bar{\partial}\partial)^{-u}Z_2 + \bar{Z}_3(-\bar{\partial}\partial)^{u}Z_3 + \bar{Z}_4(-\bar{\partial}\partial)^{-u}Z_4 - \xi_1^2\bar{Z}_1\bar{Z}_2Z_3Z_4 - \xi_2^2 Z_1Z_2\bar{Z}_3\bar{Z}_4\Big],$$

- For the selected choice of scaling dimensions, the R-matrix, reduces now to the following operator form

$$\widehat{R}_{10}^{BFKL} = \frac{\Gamma(-1-u)\Gamma(1-u)}{4^{2+2u}\pi^2\Gamma(2+u)\Gamma(u)}\mathbb{P}_{01}(x_{10}^2)^{u+1}(p_0^2)^u(p_1^2)^u(x_{10}^2)^{u-1},$$

- The Taylor-expansion of $\widehat{R}$ around $u=0$ delivers at linear order a differential operator

$$\widehat{R}_{ab}^{BFKL} = \frac{\Gamma(-1-u)\Gamma(1-u)}{4^{2+2u}\pi^2\Gamma(2+u)\Gamma(u)}\mathbb{P}_{ab}(x_{ab}^2)^{u+1}(\hat{p}_b^2)^u(\hat{p}_a^2)^u(x_{ab}^2)^{u-1} = \frac{\mathbb{P}_{ab}}{16\pi^2}\left(1 + u\,\hat{h}_{ab}^{BFKL} + \mathcal{O}(u^2)\right),$$

whose explicit form is (Lipatov'93, Faddeev, Korchemsky'95)

$$\hat{h}_{ab}^{BFKL} = 2\log(x_{ab}^2) + x_{ab}^2\log(p_a^2p_b^2)x_{ab}^{-2} - 4\psi(1) - 4\log 2 - 2 = (p_a^{-2})\log(x_{ab}^2)(p_a^2) + (p_b^{-2})\log(x_{ab}^2)(p_b^2) + \log(p_a^2p_b^2) - 4\psi(1) - 4\log 2 - 2\,.$$

FCFT of BFKL Type From the Checkerboard II

- The eigenvalue in the case of interest is then

$$h(\Delta) =$$
$$= \lim_{\Delta_+ \to 0} B\left(-1-u+\Delta_+, 1+u, 2-\Delta_+ - \frac{\Delta}{2}\right) B\left(1-u-\Delta_+, 1+u, \Delta_+ - \frac{\Delta}{2}\right) =$$
$$= \frac{1}{256\pi^4}\left[1 + 4u\left(-1 - 2\psi(1) + \psi\left(\frac{\Delta}{2}\right) + \psi\left(1-\frac{\Delta}{2}\right)\right) + \mathcal{O}\left(u^2\right)\right].$$

- Writing $\Delta = 1 + 2i\nu$, the latter expression is consistent with the energy of the Pomeron state (Kuraev, Lipatov, Fadin'77, Balitsky, Lipatov'78), obtained in the Regge limit of QCD or in $\mathcal{N} = 4$ SYM theory by

$$\omega(\nu) = 4\left(2\psi(1) - \psi\left(\frac{1}{2} + i\nu\right) - \psi\left(\frac{1}{2} - i\nu\right)\right).$$

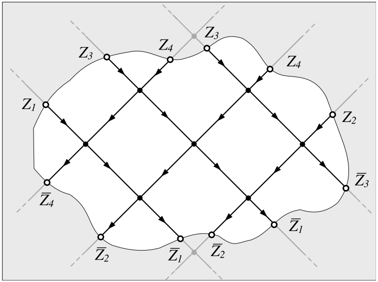
- Let us introduce an effective coupling η through

$$\xi_1\xi_2 = 4\pi(1 - u\eta),$$

which we keep finite in the limit $u \to 0,\ \xi_1\xi_2 \to 4\pi$. We obtain in the limit $u \to 0$ the equation for the spectrum of conformal dimensions $\Delta(\eta)$ of exchange operators in BFKL FCFT (at $L = 2$)

$$\eta = \psi\left(\frac{\Delta}{2}\right) + \psi\left(1 - \frac{\Delta}{2}\right) - 2\psi(1) - 1 + \mathcal{O}(u).$$

Single-trace correlators



- We consider single-trace correlators featuring a number $m_1 + m_2 + \cdots + m_n$ of external fields grouped into n coinciding positions, which have the general form:

$$\frac{1}{N} \langle \mathrm{Tr}[(\Phi_{1,1} \cdots \Phi_{1,m_1})(x_1)(\Phi_{2,1} \cdots \Phi_{2,m_2})(x_2) \cdots (\Phi_{n,1} \cdots \Phi_{n,m_n})(x_n)] \rangle \ .$$

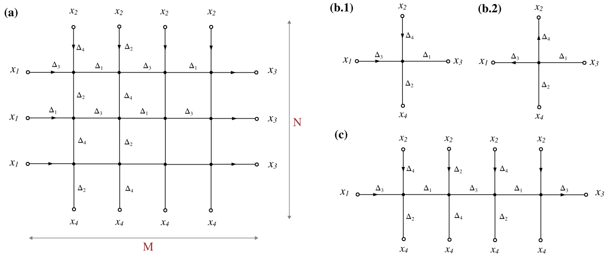
- The fields $\Phi_{n,m}$ are chosen among $Z_k, \bar{Z}_k$ and each pair of brackets $(\dots)$ delimits a product of fields located at the same point and with open $SU(N)$ indices, e.g.

$$(Z_1 Z_2 \bar{Z}_3 Z_1 Z_1 \cdots \bar{Z}_2)_{ij}(x) = \sum_{a_1 \dots a_L} (Z_1)_{ia_1}(x)(Z_2)_{a_1 a_2}(x) \cdots (\bar{Z}_2)_{a_L j}(x) \, .$$

Rectangular and Diamond correlators

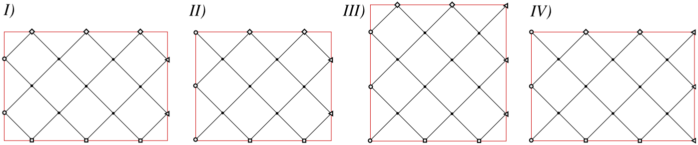
- For a rectangle of size $2n \times 2m$ the corresponding correlator reads

$$I_{2n,2m} = \frac{1}{N} \left\langle \text{Tr}[(Z_1 Z_3)^n(x_1)(\bar{Z}_2\bar{Z}_4)^m(x_4)(\bar{Z}_3\bar{Z}_1)^n(x_3)(Z_4 Z_2)^m(x_2)] \right\rangle ,$$



- We realise the four-point Diamond correlator by labelling with x_i the position of external fields in clockwise order,

$$G^{(I)}_{m,n} = \frac{1}{N} \left\langle \text{Tr}[(\bar{Z}_4 Z_1)^m(x_1)(\bar{Z}_1\bar{Z}_2)^n(x_4)(Z_2\bar{Z}_3)^m(x_3)(Z_3 Z_4)^n(x_2)] \right\rangle .$$



Conclusions and open problems

Summing up what was done:

- The Checkerboard Fishnet CFT introduced and studied in this work is one representative of a huge family of generalised, Loom FCFTs of arbitrary dimension.
- We presented a few analytic calculations of non-trivial physical quantities based on integrability and conformality of the Checkerboard CFT.
- We also showed that the graph-building operator in 2D, at a certain limit of the spectral parameter, reduces to Lipatov's Hamiltonian for reggeized gluons.

There are still many interesting questions related to the Checkerboard CFT:

- Using the techniques of quantum integrability one could try to compute these correlation functions exactly at any L, at all orders, close to (Derkachev, Korchemsky, Manashov'01, Lipatov, De Vega'01).
- It would be interesting to obtain the sigma-model representation for the Checkerboard fishnets with cyllindric topology, analogously to (Basso, Zhong'18, Basso et al.'19), and to establish the related TBA equations.
- Could one find a useful application of Yangian symmetry (Chicherin et al.'17, Kazakov, Levkovich-Maslyuk, Mishnyakov'23) for Checkerboard graphs?

Thanks for your attention!