Consumption Behavior, Savings Targets and Learning Dynamics

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Abstract

This paper introduces a novel model integrating a savings target mechanism with learning dynamics to overcome limitations in classical consumption-savings theories, such as the permanent income and life cycle hypotheses. By incorporating behavioral responses to economic changes, the model more accurately captures observed consumption patterns, especially in response to transitory income shocks and variations in the marginal propensity to consume (MPC). Governed by a quadratic hyperbolic decay fitness function, the learning mechanism ensures adaptability and long-term predictive reliability. Numerical simulations demonstrate the alignment of the model with empirical data, showcasing significant MPC responses to income shocks while maintaining key results such as consumption smoothing and long-term convergence to a steady state.

Keywords: Consumption, Income Shock, Marginal Propensity to Consume, Behavioral Economics

1 Introduction

Understanding the dynamics of consumption behavior is fundamental to economic analysis, particularly in the context of policy evaluation and forecasting. Traditional consumption-savings frameworks, such as Modigliani and Brumberg (1954) and Friedman (1957), provide foundational insights into how individuals plan their consumption and savings over their lifetime. However, these models often fail to explain observed consumption dynamics, such as the fact that transitory income shocks can affect agents' consumption patterns.

Empirical observations indicate that consumption exhibits significant responses to transitory income shocks, suggesting the existence of dynamics that traditional frameworks cannot capture. For example, Fagereng et al. (2021) highlight changes in agents' consumption behavior following lottery prizes, generating changes in consumption patterns that follow asymmetry and fat tails. Beyond that, various empirical works corroborate this view, such as Commault (2022), Fuster et al. (2021), Ganong et al. (2020), Kueng (2018), and Bunn et al. (2018). This discrepancy points to the necessity of a more nuanced approach that integrates the consolidated consumption-savings framework with a mechanism that accounts for sudden changes in consumption behavior given transitory income shocks.

This paper addresses the limitations of traditional frameworks by introducing a savings target function that affects the optimal program that agents have to solve. In this novel framework, individuals are punished for deviations from a savings target that reflects their needs and goals. To avoid such inconvenience, agents react to transitory income flows by temporarily changing their consumption patterns to avoid excessive savings, and vice versa. This strategy is based on the notion of targets as a way of dealing with uncertainty (Kahneman and Tversky, 1979; Kahneman and Tversky, 1984).

A second innovation of this paper is to integrate the savings target function with a learning mechanism governed by a quadratic hyperbolic decay fitness function. This mechanism allows agents to update their targets based on their perspective on the environment, reflecting their ability to adapt to economic changes (Ekerdt, 2010). In a stochastic income variation scenario, agents observe the difference between their effective and reference income to decide between updating or keeping the target available in the past period. This learning mechanism allows the study of a model from the perspective of heterogeneous agents dealing with uncertainty based on an adaptive process (Bischi and Tramontana, 2024; Cavalli et al., 2021).

Analytical and numerical results suggest that a savings target function can reproduce the consumption pattern similar to the empirical results found by Fagereng et al. (2021). It accounts for the intensity of the change with similar moments that characterize the behavior of jump followed by a smooth return to the steady state after a single period of income perturbation. By considering the savings target function with a learning mechanism, it is possible to verify the convergence of the system in the direction of a steady state compatible with a rational expectation equilibrium in line with the results found by Evans et al. (2022).

The results of this research can be useful for different research agendas whose impact of transitory income shocks on consumption can represent a challenge. Models interested in short-term fluctuations, such as Cantore and Freund (2021) and Gali (2018), can use the savings target mechanism as an effective shortcut to make small-scale DSGE models replicate the results of HANK models.[1] On the other hand, it brings a new perspective to researchers interested in understanding the convergence properties of models in which learning and uncertainty play a key role (Hommes et al., 2024; Evans et al., 2022).

[1] An introduction to HANK models and the role of changes in consumption patterns can be found in Kaplan and Violante (2022) and Kaplan et al. (2018).

2 The Classic Consumption-Savings Framework

The economic perspective on household savings, originally pioneered by Modigliani and Brumberg (1954) and further elaborated by Friedman (1957), suggests that individuals make deliberate choices about saving and consumption to achieve a balanced consumption pattern throughout their life cycle. This concept aligns with the use of concave utility functions that reflect a behavior of distributing consumption over time. A consolidated result of this approach is that temporary variations in the agents' income do not affect their consumption patterns, i.e., the MPC is indifferent to income fluctuations. For example, suppose that an individual maximizes their lifetime utility as in Eq. (1):

$$\max_{\{c_t, a_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where $u(c_t)$ denotes the utility of consumption in period t. This infinite horizon optimal program is subject to the following budget constraint:

$$a_t + c_t = a_{t-1}(1+r) + y_t. \tag{2}$$

To solve this problem, the agent must consider whether to consume c or save in a riskless asset a with a return r in each period. Beyond that, they have a given income flow y_t. By taking the First-Order Conditions (FOC) of this problem, we can find the Euler equation, linking present and future consumption decisions:

$$u'(c_t) = \beta(1+r)\mathbb{E}_t \left[u'(c_{t+1})\right]. \tag{3}$$

Here, the marginal utility of consumption in period t equals the marginal utility of consumption in period $t+1$ times an interest $(1+r)$ and a discount factor β. In steady state, for the case without uncertainty or borrowing constraints, the model predicts $\mathbb{E}_t \left[u'(c_{t+1})\right] = U'(c_{t+1})$, which implies $c_{t+1} = \beta(1+r)c_t$ under strict concavity.

As a result, agents' transitory income changes will not affect their consumption profile. To verify this statement, follow Proposition 1.

Proposition 1. *With no uncertainty or borrowing constraints, the marginal propensity to consume associated with Eq. (3) for $U'(c_t) = c_t^{-\gamma}$ is given by:*

$$\bar{\alpha} = 1 - \frac{(\beta(1+r))^{\frac{1}{\gamma}}}{(1+r)}.$$

For the case when $\gamma = 1$ such that $u(c_t) = \log(c_t)$, it becomes:

$$\bar{\alpha} = 1 - \beta.$$

Proof. See Appendix A. □

As can be seen, the MPC will depend only on parameters that are not affected by the income of the agents, determined exclusively by the relationship between the interest rate r, the discount factor β, and the relative risk aversion parameter γ. Note that, for the case in which the relative risk aversion coefficient is 1, we have the case of a log consumption function, in which changes in interest rates do not affect the marginal propensity to consume, being determined only by the parameter β. As shown by Cantore and Freund (2021), even for cases where $\gamma \neq 1$ and r_t are not constant, the necessary values of the relative risk aversion parameter to generate a response of MPC to match the empirical data is implausible from the empirical point of view. Let us mark this result as the implication of the classic case and call it the benchmark framework.

Despite being a consolidated way of approaching the problem of consumption and savings decisions, empirical studies consistently challenge this view. Works such as Parker et al. (2013), Stephens and Unayama (2011), and Johnson et al. (2006) reveal that people react by adjusting their consumption in response to both expected and unexpected transitory changes in income. By conducting experiments with lottery winners, Fagereng et al. (2021) found significant changes in the marginal propensities to consume after a transitory income shock. This result is corroborated by other methods such as quasi-experimental evidence (Kueng, 2018), survey instruments (Bunn et al., 2018; Fuster et al., 2021), and semi-structural methods (Ganong et al., 2020; Commault, 2022).

As specified in the empirical findings of Fagereng et al. (2021), the consumption pattern not only significantly responds to transitory income shifts, but also presents a jump in the immediate aftershock period and then smoothly returns to the steady state. The data presents a significantly sudden change in the consumption pattern followed by a return to a steady state over the next four years. At the same time, the classical model predicts a much lower change in consumption.

To fully understand how consumption patterns respond to a transitory income shock and why this effect does not appear in the benchmark analysis, it is important to understand what elements are missing in the classical case that could create such dynamics. While in the benchmark framework agents should only find the optimal consumption path for a given budget constraint and time horizon, we now start to pay attention to a second element: the reasons to save.

Given the conclusions derived from the classical case presented above, we can see that the reason why an agent is saving is not important since they are maximizing their utility only by considering the relationship between consumption today and consumption tomorrow expressed in Eq. (3). For now, we should consider savings to be more than a way to maximize consumption over time, but also an important instrument to deal with uncertainty and achieve specific goals.

Drawing from the work of Warneryd (1989), savings can be broadly defined as the act of regularly putting aside resources for specific goals. For example, reasons to save can include medical expenses in old age (De Nardi et al., 2010), retirement plans (Clark et al., 2017; Lee and Hanna, 2015), children's education (Sherraden et al., 2013), and the purchase of durable goods (Fernandez-Villaverde and Krueger, 2011).

Agents typically rely on behavioral strategies to determine their savings needs. For example, people often use reference points to guide their decision-making due to cognitive limitations (Kahneman, 2003). Although agents strive to make optimal decisions, human rationality limits the effectiveness of such processes, relying on heuristics to manage uncertainty. For example, studies by Kahneman and Tversky (1979, 1984) highlight that agents normally use reference points to navigate uncertainty.

Cognitive limitations and heuristics can result in deviations from the optimal outcomes the classical framework predicts. Enke et al. (2023) found that the presence of heuristics and bias affects the ability of individuals to make optimal decisions even with large financial incentives. In the same direction, the results of Griskevicius et al. (2013) and Ashby et al. (2011) indicate that past experiences significantly influence people's ability to set and achieve savings goals. In that sense, if agents' savings behavior is affected by heuristics and bias, what should be the consumption response in such a scenario?

3 Consumption Dynamics Under Savings Target

Even with interference from heuristics and bias, the classical framework remains a valuable tool for theoretical and empirical models. One potential approach to keep the fundamental structure of intertemporal optimal behavior using an Euler equation is to explore bias mechanisms that induce changes in MPC as described in the empirical literature.

Departing from the evidence described above regarding how behavioral elements can affect agents' savings decisions, this section aims to present a mechanism capable of connecting the results from a traditional consumption-savings framework with an innovative savings target function. As a result, we can describe how the behavioral element interacts with optimal consumption decisions and affects the consumption and savings dynamics.

3.1 The Savings Target Function

Consider a finite set of goals $\mathbb{G}$ that an agent aims to achieve, in which each element $g_i \in \mathbb{G}$ represents a specific objective or reason for saving:

$$\mathbb{G} = \{g_1, g_2, g_3, \ldots, g_n\}$$

Since $\mathbb{G}$ includes only goals achievable by financial means, we define that each goal $g_i \in \mathbb{G}$ has a corresponding savings amount $s_i \in \mathbb{S}$ necessary to achieve that goal. In this case, for a given generic function $f : \mathbb{G} \to \mathbb{S}$, we have an explicit relationship in $R = \{(g, f(g)) | g \in \mathbb{G}\}$. Assuming that all savings amounts $s_i \in \mathbb{S}$ are homogeneous and measurable in a common unit, we can assert the existence of an addition operation in $\sim$, ensuring that:

$$\sum_{i=1}^{n} f(g_i) = s \in \mathbb{R}$$

Here, s represents the target that captures the amount of savings required to achieve all goals g_i. Let us consider the case of fixed goals such that s is constant and is determined exogenously. Assuming that agents have a stock of savings represented by their assets a_t, it is possible to use a metric to check the deviations of the effective stock of savings from the exogenous target:

$$d(a_t, S) = |a_t - \bar{s}| \tag{4}$$

By using that distance, we can also construct a loss function to capture the cost of being away from the target:

$$f(a_t) = \phi \frac{|a_t - \bar{s}|^{1+\psi}}{1+\psi} \tag{5}$$

The output of $f(a_t)$ will be positive for any $a_t \neq \bar{s}$ and can be calculated as a cost or disutility flow with elasticity ψ and intensity ϕ. For simplicity, assume that $\psi = 1$, which leads to a quadratic function. A quadratic function keeps the problem more tractable, and given the fact that it is a well-known functional form for loss functions in economics, it makes the argument clearer to the reader.[2]

The increasing behavior of this function is justified by the fact that agents have a status quo bias, in which, for small changes in income, agents prefer to ignore the effort to recalculate their savings decisions (Godefroid et al., 2023). As changes become significant, agents feel more encouraged to change their behavior and move closer to standard optimizing behavior. This effect may also be related to an inattention bias in which agents cannot account for all available information (Gabaix, 2019).[3]

When an individual's stock of savings diverges from their reference level, there exists an incentive to return to the equilibrium level that meets the target by increasing or decreasing consumption. Saving less than the target means a loss as the agent moves away from their objective. On the other hand, excess savings mean an excessive loss of present utility since the savings stock compatible with the goal is already ensured.

The expected result of this mechanism is that a temporary increase in agents' income will lead to an increasing disincentive if the agent tries to save beyond the target, which will increase consumption, and vice versa. The symmetry in the punishment generated by being above or below the target characterizes an unrealistic hypothesis but, although unrealistic, provides greater clarity for the argument.

3.2 Rewriting the Optimal Consumption-Savings Problem

The savings target function can be applied to understand its effect on consumption decisions by approaching it via utility function or budget constraint. Let us start with the utility case by rewriting Eq. (1) as:

$$\max_{\{c_t, a_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \phi \frac{(a_t - \bar{s})^2}{2} \right) \tag{6}$$

[2] In Appendix B, the reader can find a solved version of the model considering the general case following Eq. (5).

[3] At the same time, the existence of transaction costs can also be related to this problem, as agents tend to keep their savings constant for small income changes since there is not always a new optimal allocation available given constant transaction costs as described by Garleanu and Pedersen (2016) and Baule (2010).

Once again, after taking the FOC of the problem by considering the budget constraint denoted in Eq. (2), we arrive at the Euler equation:

$$u'(c_t) + \phi(a_t - \bar{s}) = \beta(1+r)\,\mathbb{E}_t[u'(c_{t+1})]. \tag{7}$$

The result remains similar to that found in Eq. (3), with the addition of the second term on the left side of the equality that represents the cost in the form of disutility due to deviations of the current savings stock from the savings target. Considering the intertemporal problem and the optimal consumption path, we can see through Proposition 2 that the marginal propensity to consume now changes in time depending on the size of the gap between the effective savings stock a_t and the target $\bar{s}_t$.

Proposition 2. *With no uncertainty or borrowing constraints, the lifetime consumption path associated with Eq. (7) for* $u'(c_t) = c_t^{-\gamma}$ *is given by:*

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \left(\frac{c_0^{-\gamma}}{(\beta(1+r))^t} + \sum_{k=0}^{t-1}\frac{\phi(a_k - \bar{s})}{(\beta(1+r))^{t-k}}\right)^{\frac{1}{-\gamma}} = (1+r)a_0 + \sum_{t=0}^{\infty}\left(\frac{1}{(1+r)}\right)^t y_t.$$

For the case when $u(c_t) = \log(c_t)$*, the marginal propensity to consume can be found as:*

$$\alpha_t = 1 - \frac{\beta}{(1+\phi c_{t-1}(a_{t-1} - \bar{s}))}.$$

Proof. See Appendix A. □

On the other hand, if we set the problem departing from a budget constraint perspective, we can see that the agent optimizes their lifetime consumption by using a Perceived Budget Constraint (PBC),[4] in which the expected value of their assets relies on a behavioral cost (seen by the agent as a monetary cost), which leads to:

$$a_t^p = a_{t-1}(1+r) + y_t - c_t - \phi\,\frac{(a_t - \bar{s})^2}{2}. \tag{8}$$

As the associated cost of the savings target function is a behavioral effect viewed in a budget constraint perspective, it has no counterpart in any agent budget, i.e., it is a behavioral wedge accounted as a constraint every period that $a_t \neq \bar{s}$. The Actual Budget Constraint (ABC) captures the real movements of the assets and is computed like in Eq. (2). In that case, the PBC affects the ABC indirectly via changes in consumption decisions.

If we solve the problem by considering the budget constraint perspective, we have the following Euler equation:

$$u'(c_t)(1 + \phi(a_t - \bar{s})) = \beta(1+r)\,\mathbb{E}_t[u'(c_{t+1})]. \tag{9}$$

[4]The argument here is based on the adaptive learning literature in which agents can have biased perceptions of the law of motion of a given price or quantity. For a survey, see Hommes (2021) and Evans (2021).

This equation essentially delivers a multiplicative effect, whereas in the utility case, we see an addition as in Eq. (7). Having been computed into the budget constraint, the savings target now affects consumption decisions as a perceived monetary cost associated with consumption.

Proposition 3. *With no uncertainty or borrowing constraints, the lifetime consumption path associated with Eq. (9) for* $u'(c_t) = c_t^{-\gamma}$ *is given by:*

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t c_0\left(\frac{(\beta(1+r))^t}{\prod_{k=0}^{t-1}(1+\phi(a_k-s_k))}\right)^{\frac{1}{\gamma}} = (1+r)a_0 + \sum_{t=0}^{\infty}\left(\frac{1}{(1+r)}\right)^t y_t.$$

For the case when $u(c_t) = \log(c_t)$*, the marginal propensity to consume is:*

$$\alpha_t = 1 - \frac{\beta}{(1+\phi(a_{t-1}-\bar{s}))}.$$

Proof. See Appendix A. □

For both cases, we can see that the MPC, previously constant depending only on parameters, now varies over time depending on consumption and the savings stock. For each period t, the consumption profile tends to change, except in the case where $a_{t-1} = \bar{s}$. Assuming that the goal $\bar{s}$ is always satisfied in equilibrium, we will have $\bar{s} = a^*$. For this particular case, the effect of the mechanism is nullified, and we return to the classic case. As the mechanism comes into action only when a temporary shock hits the agent's income, we will have an increase in levels as in the standard model for dynamics that affect permanent income over time. This includes admitting that the goal also fits this new permanent income level.

To better observe how temporary changes in income impact the consumption profile of agents under the savings target, we can explore the behavior of consumption and assets over time. For the case of a log-consumption utility and the savings target function on the budget constraint, the dynamics of consumption and assets can be represented on the map **M1**:

$$\mathbf{M1}: \begin{cases} c_t = (1-\beta(1+\phi(a_{t-1}-S)))^{-1}(a_{t-1}(1+r)+y_t) \\ a_t = a_{t-1}(1+r)+y_t-c_t. \end{cases}$$

The equation relating to consumption dynamics allows us to observe the effect of the savings target on the model over time. As discussed previously, we can start from the idea that in steady state, agents align with their savings goal, that is, $s = a^*$. Therefore, we need to define the value at which this equality is satisfied. For this, we have Proposition 4.

Proposition 4. *Assuming that $s = a^*$, the map **M1** has a unique fixed point $\{c^*, a^*\}$ that satisfies:*

$$c^* = \frac{(1-\beta)\bar{Y}}{1-\beta(1+r)},$$
$$a^* = \frac{\beta\bar{Y}}{1-\beta(1+r)}.$$

Proof. See Appendix A. □

The single fixed point of the M1 map is defined only by the permanent income $\bar{y}$ and the parameters β and r. It is important to note that the analysis developed so far focuses on transitory income shifts. For the case of permanent income shifts, the model will behave exactly as in the classical case. As pointed out in Corollary 1, changes in permanent income do not affect the MPC if agents guarantee a savings target compatible with their new long-term income.

Corollary 1. *Any change in permanent income $\bar{y}$ does not affect the marginal propensity to consume as long as $s = a^*$.*

Proof. See Appendix A. □

3.3 Learning Dynamics

If we consider, in a second moment, the evolution of consumption over a longer period, it becomes pertinent also to the fact that agents can update their target. To do this, we can now study a situation in which agents can change their target to be perfectly compatible with their current savings stock or maintain their previous target. In that sense, the target will evolve according to the following rule:

$$s_t = s_{t-1}\,\omega_t + a_t(1-\omega_t), \tag{10}$$

which means that agents keep their last target available with a probability ω_t or update it to match the last available stock of savings with a probability $(1-\omega_t)$. As stated in Ekerdt (2010), goals change over time and strongly depend on individual income. To account for this fact, agents rely on a mechanism in the spirit of Bischi and Tramontana (2024) and Cavalli et al. (2021), in which the probability of a target update by agents will be determined by their fitness capacity, which reflects their ability to adapt their target given income changes. In this case, ω_t follows a quadratic hyperbolic decay fitness function:

$$\omega_t = \frac{1}{1+\varphi\,(y_t-\bar{y})^2}, \tag{11}$$

with the parameter $\varphi \in (0,\infty)$ representing the intensity in which agents can adapt their behavior in consequence of changes in their perception of the situation of the economy. For deviations of the effective income y_t from a permanent income measure $\bar{y}$, agents feel more comfortable updating their target as the status quo bias (or even a transaction cost) becomes less relevant. Substituting Eq. (11) into (10) and reorganizing, we arrive at:

$$s_t = \frac{s_{t-1} + a_t\,\varphi(y_t - \bar{y})^2}{1 + \varphi(y_t - \bar{y})^2}. \tag{12}$$

In extreme cases, when $\varphi \to 0$, the dynamics is the same as in the case presented in the previous section, where the savings target will always be equal to the initial target (a^*, for example). In this situation, agents do not have adaptability capacity and continue to carry their initial target for all periods. On the other hand, in the case where $\varphi \to +\infty$, we return to the classic case since $a_t = s_t$ for all t. In this situation, agents perfectly adapt their target in each period and suffer no penalty effect.

After developing the target updating mechanism that determines agents' learning dynamics, we can reanalyze the consumption-savings problem by rewriting Eq. (8) considering Eq. (12). For this new case, the intertemporal optimization problem now becomes:

$$\max_{\{c_t, a_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t\, u(c_t), \tag{13}$$

subject to:

$$a_t^p = a_{t-1}(1+r) + y_t - c_t - \frac{\phi}{2}\left(a_t - \frac{s_{t-1} + a_t\,\varphi(y_t - \bar{y})^2}{1 + \varphi(y_t - \bar{y})^2}\right). \tag{14}$$

At this point, income variations are now given by a white noise process such that $y_t = \bar{y} + \varepsilon_t$ with $\varepsilon_t \sim N(0, \sigma)$. Once again, after taking the FOC of the problem, we arrive at the following Euler equation:

$$u'(c_t)\left(1 + \frac{\phi(a_t - s_{t-1})}{1 + \varphi\,(y_t - \bar{y})^2}\right) = \beta(1+r)\,\mathbb{E}_t[u'(c_{t+1})]. \tag{15}$$

Increases in the difference between agents' effective income and their measure of permanent income lead to a reduction in the second term on the left side of the Euler equation, gradually approaching the benchmark-classical case described in Eq. (3).

We can organize the results obtained so far to generate a new map that expresses the dynamics of the model with an endogenous target. In this case, we can express the map **M2** as:

$$\mathbf{M2}: \begin{cases} c_t = (1 - \beta(1 + \phi(a_{t-1} - s_{t-1})))^{-1}\,(a_{t-1}(1+r) + \bar{y} + \varepsilon_t) \\ a_t = a_{t-1}(1+r) - c_t + \bar{y} + \varepsilon_t \\ s_t = \left(s_{t-1} + a_t\,\varphi\,(\varepsilon_t)^2\right)\left(1 + \varphi\,(\varepsilon_t)^2\right)^{-1} \end{cases}$$

Map M2 allows us to observe the dynamics of the savings target and its impact on consumption and savings. As we no longer have $s = a^*$ for all t anymore, it becomes convenient to understand under what conditions this result may occur. For this, we set Proposition 5.

Proposition 5. *For any $\varphi > 0$ and $\varepsilon_t \sim N(0, \sigma^2)$ with $\sigma^2 > 0$, s_t converges to a_t as $t \to \infty$.*

Proof. See Appendix A. □

This result aligns with the empirical results of Hommes et al. (2024) and Evans et al. (2022), which point to the convergence of agent behavior into optimal intertemporal behavior in the long run. In this case, s_t will fluctuate toward the equilibrium point $s^* = a^*$ given the stochastic income flow. Beyond this point, s_t will present only symmetric fluctuations around the steady state as $\varepsilon_t \sim N(0, \sigma)$.

With the target convergence conditions known, the study of the existence and uniqueness of the fixed point of the system becomes simpler, since we can assume, once again, that $s = a^*$ as $t \to \infty$. Under that specific condition, the fixed point for the M2 map is similar to the M1 map, as shown in Proposition 6.

Proposition 6. *The map **M2** has a unique fixed point $\{c^*, a^*, s^*\}$ that satisfies:*

$$c^* = \frac{(1-\beta)\bar{y}}{1-\beta(1+r)},$$

$$a^* = \frac{\beta\bar{y}}{1-\beta(1+r)},$$

$$s^* = \frac{\beta\bar{y}}{1-\beta(1+r)}.$$

Proof. See Appendix A. □

In this way, the system presents a steady state similar to that found for map M1. This result maintains the results of map M1, showing that the analysis, from an equilibrium point of view, is similar to map M1 when $\{c_0, a_0, s_0\} = \{c^*, a^*, s^*\}$. On the other hand, the M2 map has a learning mechanism capable of guaranteeing the convergence of the system in cases where the initial conditions differ from the steady state when $\varphi > 0$ and $\sigma^2 > 0$.

4 Numerical Simulations

Finally, we can study the dynamics of the system and understand how the savings target function and the learning mechanism can be used to approximate the behavior of real consumption series both in the short run with jumps in the MPC and in the long run considering a convergence compatible with the optimal steady-state.

MPC Jumps and Consumption Dynamics

Initially, we explore the system's behavior departing from the steady state given a single deterministic income perturbation. The idea is to understand how well the

Table 1 Parameter values for numerical simulation 1

Symbol	Parameter	Value
β	Discount Rate	0.902
r	Interest Rate	0.0406
ϕ	Savings Target Intensity	3.3
$\bar{y}$	Permanent Income Level	0
σ	Income Volatility	0
μ	Learning Intensity	1

savings target function can be used to address the dynamics of MPC jumps. We explore the case of a deterministic income shock in a single period and try to match the evidence provided by Fagereng et al. (2021). Furthermore, the model was calibrated according to the specifications presented in Table 1. The values of β and r follow Fagereng et al. (2021) with data from Norway for the period 1967 to 2014. ϕ was calibrated to match the data. As will be better explored in the next exercise, the learning intensity was assumed to be unit such that $\mu = 1$. For this first exercise, $\bar{y}$ is set at 0 as a way to better compare the relationship between the MPC and the consumption dynamics considering only the income from the single-period shock. Also, for this first simulation, $\sigma = 0$ rules out any stochastic effect, since we want to observe only the impact of a single-period deterministic income shock.

Initially, we must observe the crucial aspect of the savings target function, which is the capacity to affect consumption dynamics to match data results for transitory income shocks. Figure 4 presents the consumption dynamics for the benchmark model, a version with the savings target function, and also data showing the response of consumption to a single period of an additional income increase at one point for a period of 5 years. While the benchmark model presents only a subtle change in consumption, the version with the savings target presents the same jump behavior in the data with an immediate peak of approximately 0.52, 5 times greater than the result provided by the benchmark model. Also, the period after the jump follows a smooth convergence back to the steady state in the savings target version following the data.

Table 2 shows the statistical moments of the data compared to the case of benchmark and savings target simulations. The use of the savings target function can effectively approximate all moments of the simulated model. It is also important to note that, in addition to the savings target mechanism, the model is parsimonious and still manages to deliver quite reasonable approximations in relation to the benchmark model.

By now, we should focus on the main mechanism explaining what characterizes the system's response to transitory variations in income. The idea is to study the response of the MPC to an increase in income. As can be seen in Figure 4, unlike the benchmark model which presents a constant MPC, the model with savings target presents a behavior similar to empirical evidence. Immediately after the shock, the MPC presents a jump compatible with the empirical results, followed by a smooth transition back to the steady state. An important detail is that, on average, about 2/3

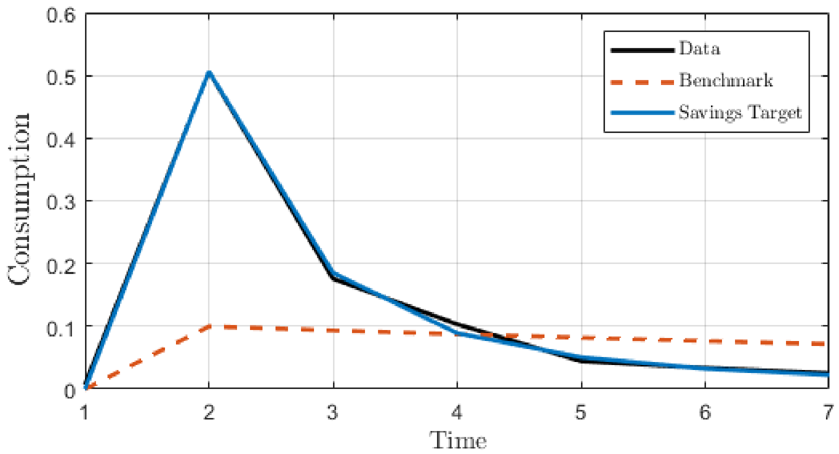

Fig. 1 Yearly consumption dynamics for data and model outputs for the benchmark and savings target cases. Data source: Fagereng et al. (2021).

Table 2 Moments for data and benchmark and savings target simulations for a single-period deterministic income shock.

Moment	Data	Savings Target	Benchmark
Mean	0.1482	0.1363	0.0841
Variance	0.0339	0.0334	0.0001
Skewness	1.4461	1.7105	0.1185
Kurtosis	3.5224	4.0509	1.7444

of the effect disappears immediately after the first year, and only 1/3 of the effect is characterized by a smooth return to the steady state.

Learning Dynamics and Long-run Convergence

Given that the behavior of the MPC follows what would be expected from empirical evidence, the next step is to understand to what extent the savings target function combined with the learning mechanism is capable of providing a long-term convergence behavior that follows the results of Hommes et al. (2024) and Evans et al. (2022). As the model now presents a stochastic element that determines income variation, all the following results are the average of 1000 Monte Carlo simulations. Table 3 summarizes the model calibration for this next exercise. The values of β, r, and ϕ follow the same values as in the first simulation. For now, $\bar{y}$ is set to 0.1 to allow a non-zero steady state. For now, the model has stochastic income variation, and the learning mechanism is tested for different values of μ.

If we consider a time horizon of 60 years, we can observe how different intensities in the agents' learning process affect the evolution of the targets. In Figure 4, we can

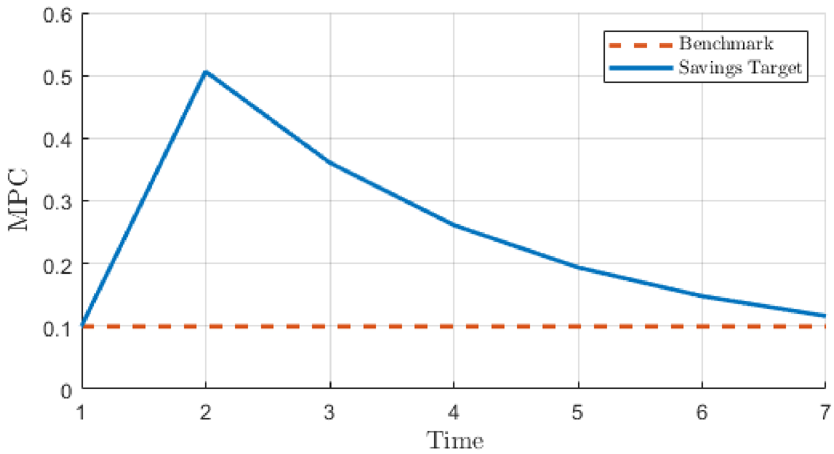

Fig. 2 MPC Dynamics with a Jump caused by an income increase in period 5.

Table 3 Parameter values for numerical simulation 2

Symbol	Parameter	Value
β	Discount Rate	0.902
r	Interest Rate	0.0406
ϕ	Savings Target Intensity	3.3
$\bar{y}$	Permanent Income Level	0.1
σ	Income Volatility	0.2
μ	Learning Intensity	1, 10, 100

see that the target approaches the effective savings stock as described by Proposition 4. For any positive value of μ and σ, we can observe convergence that will depend on both income variation and learning intensity. In the case of Figure 4, we have different learning intensities while keeping the income variation constant. We can associate the process of convergence of the savings stock with the process of convergence in relation to a steady state studied by the research agenda focused on economic growth. After all, the availability of savings, in this model, is directly affected by the movement of targets.

If we observe the long-term consumption dynamics shown in Figure 4, we will see that consumption tends to be higher initially for the case of low-intensity learning, but consumption tends to approach the optimal long-term level more quickly for cases where μ is higher. From period 20 onward, all simulations with savings targets start to have a lower consumption than the benchmark model. This is because the benchmark model captures a consumption dynamic that guarantees the maximization of utility; therefore, any deviation that generates an increase in consumption in advance implies a loss in future consumption. The increase in consumption in the initial periods for

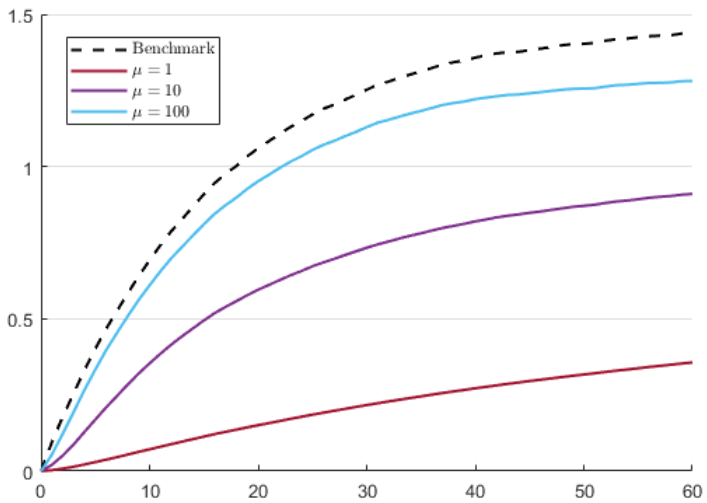

Fig. 3 Evolution of Targets on time in comparison with the benchmark savings.

models with savings targets implies a loss of return on a possible savings flow that will guarantee a higher return later.

It is important to note that the convergence process studied in this exercise happens in a scenario of constant income volatility and no other process affecting the MPC. This exercise focuses on clearly showing the effect of the learning intensity on the convergence time and its compatibility with long-term scenarios. However, overconsumption with roots in behavioral wedges could be included as one of the reasons for the slow convergence of some economies to an optimal investment level.

5 Conclusion

The savings target model with learning dynamics presented in this paper provides a comprehensive framework to analyze consumption behavior. By effectively integrating short-term responses to economic changes with long-term principles of consumption smoothing and asset accumulation, the model is shown to be adaptable and theoretically robust, offering a valuable tool for understanding the complex interplay between consumption, savings, and income dynamics.

One of the key contributions of this work is the introduction of the savings target function, which reflects the agents' goals and needs, and its impact on their consumption patterns. This function allows for a more realistic depiction of how individuals adjust their savings in response to transitory income shocks, deviating from the assumptions of traditional models that often overlook such behavioral nuances. By incorporating a disutility flow for deviations from the savings target, the model captures the asymmetrical and fat-tailed nature of consumption responses observed in empirical data.

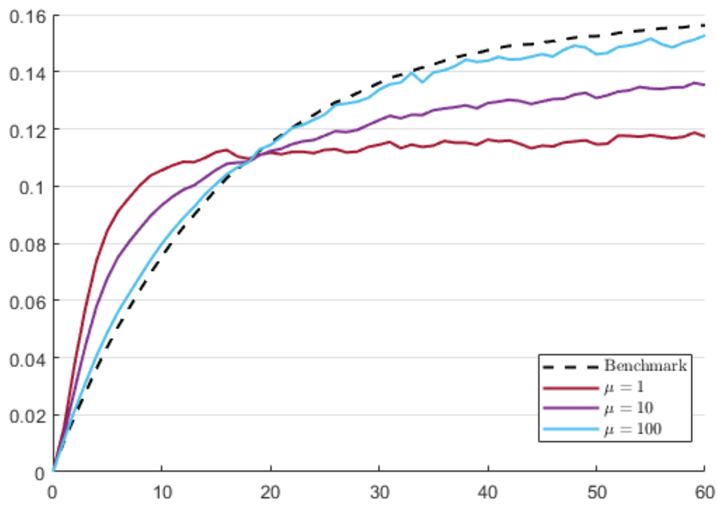

Fig. 4 Evolution of consumption on time in comparison with the benchmark model.

Additionally, the integration of a learning mechanism governed by a quadratic hyperbolic decay fitness function is a significant innovation. This mechanism enables agents to adapt their savings targets based on their perceptions of the economic environment, reflecting their ability to adjust to stochastic income variations. This adaptive process provides a more dynamic and flexible approach to understanding consumption behavior under uncertainty, contrasting with the static assumptions of the classical frameworks.

The results of this research have important implications both for theoretical and applied economics. From a theoretical perspective, the model offers a refined understanding of consumption dynamics, highlighting the importance of behavioral factors and adaptive processes in shaping economic outcomes. It challenges the traditional view that consumption is solely driven by intertemporal optimization and provides a more nuanced perspective that accounts for the complexity of real-world decision-making.

From an applied standpoint, the model's ability to replicate observed consumption patterns makes it a valuable tool for policy analysis. Policymakers can use the insights gained from this model to design interventions that take into account the behavioral tendencies of individuals, leading to more effective strategies to manage economic fluctuations and growth strategies. For example, understanding how consumers adjust their savings target in response to income changes can inform policies aimed at smoothing consumption and mitigating the adverse effects of economic shocks.

Furthermore, the model opens up several avenues for future research. Expanding the framework to explore more complex income processes, such as those involving multiple sources of uncertainty or non-linear income dynamics, could provide deeper insights into the factors influencing consumption and savings decisions. Additionally, investigating the impact of different policy interventions in general equilibrium frameworks can be useful, as results like the Ricardian equivalence may not work under savings target.

Declarations

Conflict of interest. The author has no conflict of interest to declare that are relevant to the content of this article.

Appendix A Mathematical Proofs

Proof of Proposition 1

Agents solve the following optimal program:

$$\max_{\{c_t,c_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\gamma}}{1-\gamma} \right)$$

subject to the budget constraint:

$$a_t = a_{t-1}(1+r) + \bar{y} - c_t$$

with no uncertainty or borrowing constraints, the optimal problem delivers the following Euler equation:

$$c_{t+1} = \beta(1+r)c_t$$

by iterating forward the budget constraint, we obtain:

$$c_0 + \frac{1}{(1+r)} c_1 + \frac{1}{(1+r)^2} c_2 + ... = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)} \right)^t y_t$$

after applying the relationship between c_t and c_{t+1} defined in the Euler equation, we have that:

$$c_0 + \frac{1}{(1+r)} c_0[\beta(1+r)]^{\frac{1}{\gamma}} + \frac{1}{(1+r)^2} c_0[\beta(1+r)]^{\frac{2}{\gamma}} + ... = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)} \right)^t y_t$$

by collecting the terms on the left that depend on t, we arrive at a geometrical series and the intertemporal budget constraint can be expressed as:

$$c_0 \sum_{t=0}^{\infty} \left(\frac{(\beta(1+r))^{\frac{1}{\gamma}}}{(1+r)} \right)^t = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)} \right)^t y_t$$

by using the convergent series property such that $\sum_{t=0}^{\infty} x^t = \frac{1}{1-x}$ if $|x| < 1$, we arrive at:

$$\sum_{t=0}^{\infty} \left(\frac{(\beta(1+r))^{\frac{1}{\gamma}}}{(1+r)} \right)^t = \frac{1}{1 - (\beta(1+r))^{\frac{1}{\gamma}} (1+r)^{-1}}$$

finally, we can rewrite the optimal consumption path as:

$$c_0 \left(\frac{1}{\alpha} \right) = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)} \right)^t y_t$$

such that the marginal propensity to consume (MPC) is:

$$\alpha = 1 - (1+r)^{-1} (\beta(1+r))^{\frac{1}{\gamma}}$$

for the case when $\gamma = 1$ such that $u(c_t) = log(c_t)$, the MPC becomes:

$$\alpha = 1 - \beta$$

Proof of Proposition 2

Agents solve the following optimal program:

$$\max_{\{c_t, c_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \phi \frac{(a_t - S)^2}{2} \right)$$

subject to the budget constraint:

$$a_t = a_{t-1}(1+r) + \bar{y} - c_t$$

The First-Order Conditions (FOC) of the problem are given by:

$$c_t^{-\gamma} = \lambda_t$$
$$\lambda_t + \phi(a_t - \bar{s}) = \mathbb{E}_t \lambda_{t+1} \beta(1+r)$$

By combining both FOCs, we have:

$$c_t^{-\gamma} + \phi(a_t - \bar{s}) = \mathbb{E}_t c_{t+1}^{-\gamma} \beta(1+r)$$

with no uncertainty or borrowing constraints, the optimal problem delivers the following Euler equation:

$$c_{t+1}^{-\gamma} = c_t^{-\gamma} \frac{1}{\beta(1+r)} + \frac{\phi(a_t - \bar{s})}{\beta(1+r)}$$

by iterating forward the budget constraint and considering there is no Ponzi schemes, we obtain:

$$c_0 + \frac{1}{(1+r)} c_1 + \frac{1}{(1+r)^2} c_2 + ... = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)}\right)^t y_t$$

after applying the relationship between c_t and c_{t+1} defined in the Euler equation and collecting the terms, the intertemporal budget constraint can be expressed as:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left(\frac{c_0^{-\gamma}}{(\beta(1+r))^t} + \sum_{k=0}^{t-1} \frac{\phi(a_k - \bar{s})}{(\beta(1+r))^{t-k}}\right)^{\frac{1}{-\gamma}} = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)}\right)^t y_t$$

In this case, the consumption path can be calculated numerically and the MPC can be obtained using a finite diference or similar method. On the other hand, if we consider the case of $u(c_t) = ln(c_t)$, with no uncertainty or borrowing constraints, the optimal problem delivers the following Euler equation:

$$c_{t+1} = c_t \left(\frac{\beta(1+r)}{1 + c_{t-1}\phi(a_{t-1} - \bar{s})}\right)$$

in this case, we have a multiplicative c_t to make the marginal disutility of the savings target compatible with the relative price for (c_t, c_{t+1}). Once again, after applying the relationship between c_t and c_{t+1} defined in the Euler equation and collecting the terms, the intertemporal budget constraint can be expressed as:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t c_0 \left(\frac{(\beta(1+r))^t}{\prod_{k=0}^{t-1}(1 + c_k\phi(a_k - s_k))}\right) = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)}\right)^t y_t$$

Assuming that $\beta\ (|c_t\phi(a_k - \bar{s})|)^{-1} \leq 1$ for all t, the marginal propensity to consume can be explicitly found:

$$\alpha_t = 1 - \frac{\beta}{(1 + c_{t-1}\phi(a_{t-1} - \bar{s}))}$$

Proof of Proposition 3

Agents solve the following optimal program:

$$\max_{\{c_t, c_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

subject to the budget constraint:

$$a_t = a_{t-1}(1+r) + \bar{y} - c_t - \phi\frac{(a_t - \bar{s})^2}{2}$$

FOC are given by:

$$\frac{1}{c_t} = \lambda_t$$
$$\lambda_t + \lambda_t \phi(a_t - \bar{s}) = \mathbb{E}_t \lambda_{t+1} \beta(1+r)$$

with no uncertainty or borrowing constraints, the optimal problem delivers the following Euler equation:

$$c_{t+1} = c_t \left(\frac{\beta(1+r)}{1+\phi(a_t - \bar{s})} \right)^{\frac{1}{\gamma}}$$

by iterating forward the budget constraint and considering there is no Ponzi schemes, we obtain:

$$c_0 + \frac{1}{(1+r)} c_1 + \frac{1}{(1+r)^2} c_2 + ... = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)} \right)^t y_t$$

after applying the relationship between c_t and c_{t+1} defined in the Euler equation and collecting the terms, the intertemporal budget constraint can be expressed as:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t c_0 \left(\frac{(\beta(1+r))^t}{\prod_{k=0}^{t-1}(1+\phi(a_k - s_k))} \right)^{\frac{1}{\gamma}} = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)} \right)^t y_t$$

For the case when $u(c_t) = log(c_t)$, and assuming that $\beta(|\phi(a_k - \bar{s})|)^{-1} \leq 1$ for all t, the marginal propensity to consume can be explicitly found:

$$\alpha_t = 1 - \frac{\beta}{(1+\phi(a_{t-1} - \bar{s}))}$$

Proof of Proposition 4

By considering that $c_t = c$, $a_t = a$ and $y_t = \bar{y}$ for all t, such that $\bar{y}$ is a constant permanent income level, map M1 becomes:

$$\mathbf{M1} : \begin{cases} c = \left(1 - \beta(1+\phi(a-s)c)^{-1}\right)(a(1+r) + \bar{y}) \\ a = a(1+r) + \bar{y} - c \end{cases}$$

By assuming that $s = a$, we have that:

$$\mathbf{M1} : \begin{cases} c = (1-\beta)(a(1+r) + \bar{y}) \\ a = a(1+r) + \bar{y} - c \end{cases}$$

By manipulating the second equation, we can find that:

$$c = ar + \bar{y}$$

After substituting in the first equation and manipulating, we will find that:

$$a^* = \frac{\beta \bar{y}}{1 - \beta(1 + r)}$$

Finally, by substituting a^* in c, we arrive at:

$$c^* = \frac{(1 - \beta)\bar{y}}{1 - \beta(1 + r)}$$

Proof of Corollary 1

By recalling Eq. (5) and rewrite it by using Eq. (2), we have that:

$$\chi_t = a_{t-1}(1 + r) + y_t - c_t - s$$

As in the steady-state $\{c_t, a_t\} = \{c^*, a^*\}$, we can rewrite previous equation in the steady-state as:

$$\chi^* = (1 + r)a^* - c^* + \bar{y} - s$$

Using the results found in Proposition 4 and also the fact that $s = a^*$, we can find that:

$$\chi^* = (1 + r)\left(\frac{\beta \bar{y}}{1 - \beta(1 + r)}\right) - \left(\frac{(1 - \beta)\bar{y}}{1 - \beta(1 + r)}\right) + \bar{y} - \left(\frac{\beta \bar{y}}{1 - \beta(1 + r)}\right)$$

$$\chi^* = \bar{y}\left(1 + \frac{\beta(1 + r) - 1}{1 - \beta(1 + r)}\right) = 0$$

so $\chi^* = 0$ for any permanent income level $\bar{y}$. As proved in proposition 2, the marginal propensity to consume reduces to the classical case and become constant when the effective stock of savings and the target are equal, i.e., for $\chi_t = 0$

Proof of Proposition 5

We can rewrite equation Eq. (12) to explicitly visualize S_{t-1} and a_t being multiplied by weights that depends on y_t:

$$s_t = s_{t-1}\left(\frac{1}{1 + \varphi(y_t - \bar{y})^2}\right) + a_t\left(\frac{\varphi(y_t - \bar{y})^2}{1 + \varphi(y_t - \bar{y})^2}\right)$$

Since $y_t = \bar{y} + \varepsilon_t$ with $\varepsilon_t \sim N(0, \sigma^2)$, we have that:

$$s_t = s_{t-1}\left(\frac{1}{1 + \varphi\varepsilon_t^2}\right) + a_t\left(\frac{\varphi\varepsilon_t^2}{1 + \varphi\varepsilon_t^2}\right)$$

The average of the squared errors can be written as:

$$\mathbb{E}\left[\sum_{t=0}^{T}\varepsilon_t^2\right] = \mathbb{E}\left[\sum_{t=0}^{T}(\sigma, Z)^2\right] = \sigma^2\mathbb{E}\left[\sum_{t=0}^{T}Z^2\right]$$

with $Z \sim N(0,1)$ being a standardized normal distribution and σ^2 the variance of the stochastic element. Being this true, we can see that:

$$\sigma^2\mathbb{E}\left[\sum_{t=0}^{T}Z^2\right] = \sigma^2 T$$

For the case in which $T = \infty$, by the law of large numbers, we know that the average value of ε_t will be:

$$\frac{1}{T}(\sigma^2 T) = \sigma^2$$

In this case, we can see that:

$$\mathbb{E}\left[s_t\right] = \mathbb{E}\left[s_{t-1}\left(\frac{1}{1+\varphi\varepsilon_t^2}\right) + a_t\left(\frac{\varphi\varepsilon_t^2}{1+\varphi\varepsilon_t^2}\right)\right]$$

$$s = s\left(\frac{1}{1+\varphi\sigma^2}\right) + a\left(\frac{\varphi\sigma^2}{1+\varphi\sigma^2}\right)$$

$$s\left(\frac{\varphi\sigma^2}{1+\varphi\sigma^2}\right) = a\left(\frac{\varphi\sigma^2}{1+\varphi\sigma^2}\right)$$

which implies that $s = a$ for any $\sigma^2 > 0$ and $\varphi > 0$ when $t \to \infty$.

Proof of Proposition 6

Considering that $c_t = c$, $a_t = a$ and $s_t = s$ for all t, and the fact that $\varepsilon = 0$ since $\varepsilon_t \sim N(0, \sigma)$, we have that:

$$\mathbf{M2}: \begin{cases} c = \left(1 - \beta(1 + \phi(a-s)c)^{-1}\right)\left(a(1+r) + \bar{y}\right) \\ a = a(1+r) - c + \bar{y} \\ s = s \end{cases}$$

The target s makes the system present infinite feasible solutions, but for the case in which φ and σ are strictly positive (assumed by definition), by means of proposition 4, we can assume $s = a$ as the unique steady state solution. Given this equality, we can rewrite the system as:

$$\mathbf{M2}: \begin{cases} c = (1-\beta)\left(a(1+r) + \bar{y}\right) \\ a = a(1+r) + \bar{y} - c \\ s = a \end{cases}$$

By manipulating second equation, we can find that:

$$c = ar + \bar{y}$$

After substituting in the first equation and manipulating, we will find that:

$$a^* = \frac{\beta \bar{y}}{1 - \beta(1+r)}$$

Substituting a^* in c, we arrive at:

$$c^* = \frac{(1-\beta)\bar{y}}{1 - \beta(1+r)}$$

Finally, as $s = a$, s^* is given by:

$$s^* = \frac{\beta \bar{y}}{1 - \beta(1+r)}$$

Appendix B Consumption with Metric Target

Problem Setup and Euler Equation

Agents solve the following optimal program:

$$\max_{\{c_t, c_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

subject to the perceived law of motion of a:

$$a_t^p = a_{t-1}(1+r) + \bar{y} - c_t - \frac{|a_t - s_t|^{1+\psi}}{1+\psi}$$

The First-Order Conditions (FOC) of the problem are given by:

$$\frac{1}{c_t} = \lambda_t$$
$$\lambda_t(1 + |a_t - s_t|^{\psi} \operatorname{sng}(a_t - s_t)) = \mathbb{E}_t \lambda_{t+1} \beta(1+r)$$

with no uncertainty or borrowing constraints, the optimal problem delivers the following Euler equation:

$$\frac{c_{t+1}}{c_t} = \frac{\beta(1+r)}{1 + |a_t - s_t|^{\psi} \operatorname{sng}(a_t - s_t)}$$

Isolating c_{t+1} and iterating it one period forward, we have that:

$$c_{t+1} = c_t \beta(1+r)(1 + |a_t - s_t|^{\psi} \operatorname{sng}(a_t - s_t))^{-1}$$

$$c_{t+2} = (c_t\beta(1+r)(1+|a_t-s_t|^{\psi}\,\mathrm{sng}\,(a_t-s_t))^{-1})\beta(1+r)(1+|a_{t+1}-s_{t+1}|^{\psi}\,\mathrm{sng}\,(a_{t+1}-s_{t+1}))^{-1}$$

Proceeding with the induction and rescaling for c_0 we have that:

$$c_t = c_0\left(\frac{(\beta(1+r))^t}{\prod_{k=0}^{t-1}(1+|a_k-s_k|^{\psi}\,\mathrm{sng}\,(a_k-s_k))}\right)$$

Target Dynamics

Remembering the target evolves according to:

$$s_t = \omega_t\, s_{t-1} + (1-\omega_t)a_{t-1}$$

such that the probability ω for each t is given by:

$$\omega_t = \frac{1}{1+\varphi\varepsilon_t^2}$$

where $\varepsilon_t \sim N(0,\sigma^2)$ is a normally distributed stochastic learning input and φ is the intensity of choice. Given the quadratic characteristic of the fitness function ω, the distribution of the process can be seen as a chi square distribution with t degrees of freedom such that:

$$\mathbb{E}[\varepsilon_t^2] = \frac{1}{t}\sum_{i=1}^{t}(X_i-\mu)^2$$

if consider that $t\to\infty$, we have:

$$\lim_{t\to\infty}\frac{1}{t}\sum_{i=1}^{t}(X_i-\mu)^2 = \sigma^2$$

By considering that $s_t = s_{t-1} = s$, $a_t = a_{t-1} = a$, and that $\mathbb{E}[\varepsilon_t^2] = \sigma$, we have the equilibrium target given by:

$$s = \left(\frac{1}{1+\varphi\sigma}\right)s + \left(1-\left(\frac{1}{1+\varphi\sigma}\right)\right)a$$

$$s\left(1-\left(\frac{1}{1+\varphi\sigma}\right)\right) = \left(1-\left(\frac{1}{1+\varphi\sigma}\right)\right)a$$

such that $s^* = a^*$ if σ and φ are greater than zero. Otherwise, s has infinite possible solutions.

Consumption Dynamics

The intertemporal budget constraint is given by:

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t c_t + \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t c_t = (1+r)a_0 + \sum_{t=0}^{\infty}\left(\frac{1}{(1+r)}\right)^t y_t - \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \frac{|c_t-s_t|^{1+\psi}}{1+\psi}$$

By using the fact that $a_t = s_t$ when $t \to \infty$ and the usual transversality condition, it becomes:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t c_t = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)}\right)^t y_t$$

After substituting for the c_t found previously after iterating the Euler equation, we have:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t c_0 \left(\frac{(\beta(1+r))^t}{\prod_{k=0}^{t-1}(1+|a_k - s_k|^{\psi}\,\mathrm{sng}\,(a_k - s_k))}\right) = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)}\right)^t y_t$$

For the case in which $a_t = s_t$, we have the lifetime consumption path given by:

$$c_0 \sum_{t=0}^{\infty} \beta^t = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)}\right)^t y_t$$

Using the convergent series property such that $\sum_{t=0}^{\infty} x^t = \frac{1}{1-x}$ if $|x| < 1$, we arrive at:

$$\sum_{t=0}^{\infty} \beta^t = \frac{1}{1-\beta}$$

Finally, we can rewrite the optimal consumption path as:

$$c_0 \left(\frac{1}{\alpha}\right) = (1+r)a_0 + \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)}\right)^t y_t$$

Such that the marginal propensity to consume (MPC) when $a_t = s_t$ is:

$$\alpha^* = 1 - \beta$$

On the other hand, assuming that $|\beta(1+|a_t - s_t|^{\psi}\,\mathrm{sng}\,(a_t - s_t))^{-1}| \leq 1$ for all cases in which $a_t \neq s_t$, the marginal propensity to consume will be given by:

$$\alpha_t = 1 - \beta(1+|a_t - s_t|^{\psi}\,\mathrm{sng}\,(a_t - s_t))^{-1}$$

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