The Network Effects of Fiscal Adjustments (Online Extended Version)

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Abstract

A large and increasing body of empirical evidence has established that fiscal adjustments based on government spending cuts are less costly in terms of losses in output growth than those based on tax increases. We show that the heterogeneous effects of tax-based and expenditure-based adjustments can be explained by the difference in their propagation channels in the network of industries. In theory, a tax-based adjustment plan is mainly a supply-side shock which propagates downstream (from supplier industries to customer industries) while an expenditure based plan is a demand-side shock which propagates upstream (from customer industries to supplier industries). In practice, the empirical investigation of these channels on US data based on Spatial Vector Autoregressions reveals that tax-based plans propagate through the network with an average output multiplier of close to -2, while the propagation of expenditure-based plans does not lead to any statistically significant contractionary effect on growth.

Keywords: industrial networks, fiscal adjustment plans, output. **JEL codes :** E60, E62.

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1 Introduction

Macroeconomic theory has traditionally attributed the large impact of fiscal adjustments on the real economy to the propagation mechanism that amplifies the initial impulse. Such a propagation mechanism has been firstly identified with the Keynesian Multiplier (see Diamond 1982 , and Christiano, Eichenbaum, and Rebelo 2011), which concentrates on the demand-side effects, but propagation to the real economy also depends on changes in the incentives of workers and firms, the supply side of the economy (see for example Christiano, Eichenbaum, and Rebelo 2011). While the propagation through the Keynesian Multiplier always implies stronger output effects of expenditure based adjustments than tax based adjustments, the results can be different in a model that includes supply-side effects. Alesina, Barbiero, et al. 2017 introduce the possibility of persistent adjustment plans in a standard New Keynesian framework to show that when fiscal adjustments are close to permanent, spending cuts are less recessionary than tax hikes.

The empirical literature on the macroeconomic effects of fiscal policies has notoriously found a wide range of estimates and is far from having reached a consensus for fiscal multipliers. A new fact, however, is consistently confirmed by a number of recent papers (e.g. Ramey 2018, Alesina, Favero, and Giavazzi 2015, Guajardo, Leigh, and Pescatori 2014): fiscal consolidations implemented by raising taxes imply larger output losses compared to consolidations relying on reductions in government spending.

In this paper we explore a new propagation mechanism of fiscal policy related to the work on the network effects of macroeconomic shocks (see Gabaix 2011, Acemoglu, Vasco M Carvalho, et al. 2012, Acemoglu, Akcigit, and Kerr 2016 and Ozdagli and Weber 2017). This mechanism has the potential of explaining the new fact in the empirical evidence and we investigate this possibility using US data over the period 1978-2014. A recent paper by Auerbach, Gorodnichenko, and Murphy 2019, using micro-level data on local defense spending finds the potential for large fiscal spillovers among entities that are strongly integrated economically. Differently from Auerbach, Gorodnichenko, and Murphy 2019, our focus is more on fiscal consolidation.

Network analysis of the transmission of macroeconomic shocks is based on the intuition that input-output linkages can neutralize the law of large numbers that makes local shocks irrelevant for the global economy because local shocks that hit sectors that are particularly important as suppliers to other sectors do translate into aggregate fluctuations. Studying the propagation of adjustments through input-output linkages produces some interesting theoretical implications. In fact, as shown by Acemoglu, Akcigit, and Kerr 2016, theory predicts that supply-side shocks propagate downstream more powerfully than upstream: downstream customers of directly hit sectors are affected more strongly than upstream suppliers. The converse is true for demand shocks that propagate more powerfully upstream. The reason for this asymmetric pattern lies in the fact that supply side shocks change the prices faced by customer industries while demand side shocks have much minor effects on prices and propagate upstream.

In the simplified benchmark model studied in much of the literature (Long Jr and Plosser 1983 and Acemoglu, Vasco M Carvalho, et al. 2012), both production functions and consumer preferences are Cobb-Douglas (so that income and substitution effects cancel out), and the asymmetry in the propagation of demand and supply shocks becomes extreme as there is no upstream effect from supply-side shocks and no downstream effect from demand-side shocks.

Fiscal adjustments based on changing taxation work mainly as supply-side adjustments while expenditure-based adjustments are one of the typical cases of demand-side adjustments. As their propagation is totally different, the size of their final effect on total output depends on different elements of the input-output matrix. The empirical analysis of a network-based propagation mechanism of fiscal adjustment can therefore be interesting to provide an assessment of the relevance of the theoretical mechanism and of its capability to explain the new fact in the empirical literature. This paper is organized as follows. We start by illustrating in Section 2 the theoretical mechanism of the network diffusion of a payroll-tax shock and a government expenditure shock. In Section 3 we describe how an empirical specification consistent with the theoretical mechanism can be identified and estimated, then we bring it to the data to illustrate our main results and some robustness check. Section 7 concludes.

2 A Theoretical Explanation

Our empirical identification strategy is essentially a linear regression whose structure has a particular theoretical justification; in fact, it could be interpreted as an estimation of a closed form solution of a macroeconomic model. In particular we adapt of the benchmark model designed to analyze the network transmission of demand and supply shocks (Long Jr and Plosser 1983 and Acemoglu, Vasco M Carvalho, et al. 2012), to capture the propagation of fiscal policy in the industrial network. In order to not divert the reader attention from our empirical approach and results, we prefer to be brief on the model technical details and we remand the interested reader to the next subsections where a full derivation and break down of the model is carried out. Consider a perfectly competitive economy with n sectors, where the market clearing condition for the generic industry i is:

$$y_i = c_i + \sum_{j=1}^{n} x_{ji} + G_i \tag{1}$$

c_i is household's consumption of good produced by industry i; x_{ij} is the quantity of goods produced in industry j used as inputs by industry i; G_i are government purchases which are funded by imposing either a lump sum or a distortionary payroll tax:

$$\sum_{j=1}^{n} p_i G_i = T + \tau w l \tag{2}$$

Each sector solves the following profit maximization problem:

$$\max_{l_i, \{x_{ij}\}_{j=1}^{n}} p_i \cdot y_i - w(1+\tau) l_i - \sum_{j=1}^{n} p_j x_{ij}$$

with:

$$y_i = e^{z_i} l_i^{\alpha_i^l} \prod_{j=1}^{n} x_{ij}^{\alpha_{ij}}$$

All alpha's are non negative, and we assume constant return to scale:

$$\alpha_i^l + \sum_{j=1}^{n} a_{ij} = 1$$

The FOC are:

$$a_{ij} = \frac{p_j x_{ij}}{p_i y_i}, \quad \alpha_i^l = \frac{w\,(1+\tau)\, l_i}{p_i y_i} \tag{3}$$

The economy is populated by measure one of agents, who maximize their utility[1] subject to a budget constraint:

$$\max_{l,\{c_i\}_{i=1}^n} (1-l)^\lambda \cdot \prod_{j=1}^n c_i^{\beta_i}$$
$$\text{s.t.} \sum_{i=1}^n p_i c_i \leq wl - T$$

with

$$\sum_{i=1}^n \beta_i = 1$$

The FOC are:

$$\frac{p_i c_i}{\beta_i} = k \quad \forall i \in \{1, ..., n\} \tag{4}$$

$$l = 1 - \frac{\lambda}{w} \cdot k \tag{5}$$

$$\sum_{i=1}^n p_i c_i = wl - T \tag{6}$$

Firms and households take all prices as given, and the market clearing conditions are satisfied in the goods market and the labour market. Government actions are taken as given and the wage is chosen as a numeraire $(w = 1)$.
Equations from 1 to 6 characterize the competitive equilibrium of the economy.
By log-differentiating the equations which characterize the equilibrium and after cumbersome algebra (see next subsections for the detailed derivation), we obtain the following closed form expression of a tax shock effect (we assume $dG = 0$: no change in government spending):

$$d\ln y_i = \alpha_i^l \cdot \big(d\ln(1-T) - d\ln(1+\tau) \big) + \sum_{j=1}^n a_{ij} \cdot d\ln y_j. \tag{7}$$

Introducing the input-output matrix A, allows us to rewrite equation (7) in matrix form:

$$\underset{n\times 1}{d\ln \mathbf{y}} = \underset{n\times 1}{\boldsymbol{\alpha}^l} \cdot \big(d\ln(1-T) - d\ln(1+\tau) \big) + \underset{n\times n}{A} \cdot d\ln \underset{n\times 1}{\mathbf{y}}. \tag{8}$$

[1]Following Acemoglu, Akcigit, and Kerr 2016 we consider the following simple functional form assumption for (dis-)utility of labour: $\gamma(l) = (1-l)^\lambda$

The sectorial propagation of a tax adjustment that determines its global impact is driven by the elements in the rows of the input-output matrix A, that describes industrial purchases from other sectors (input suppliers). Therefore, the network propagation mechanism of a tax shock is downstream ($d\ln \mathbf{y}^{down} = A \cdot d\ln \mathbf{y}$): downstream customers of directly hit sectors are affected while upstream suppliers are not. In practice a tax increase behaves as a supply-side shock which affect the price of the goods, this increase in the price is shifted upon other customer industries through the input linkages (downward propagation).

On the other hand, a government spending shock, generates the following change in the equilibrium level of y_i:

$$d\ln y_i = \sum_{j=1}^{n} \hat{a}_{ji} \cdot d\ln y_j + \frac{d\widetilde{G}_i}{p_i y_i} - \frac{\beta_i}{1+\lambda} \cdot \sum_{j=1}^{n} \frac{d\widetilde{G}_j}{p_i y_i}. \tag{9}$$

where $\widetilde{G}_i = p_i G_i$ and $\hat{a}_{ji} = \frac{x_{ji}}{y_i} = a_{ji}\frac{p_j y_j}{p_i y_i}$.
Introducing matrix $\hat{A}$, which is a transformation of the input-output matrix A, we can rewrite equation (9) in a compact matrix form:

$$\underset{n\times 1}{d\ln \mathbf{y}} = \underset{n\times n}{\hat{A}^T} \cdot \underset{n\times 1}{d\ln \mathbf{y}} + \underset{n\times n}{\mathbf{\Lambda}} \cdot \underset{n\times 1}{d\tilde{\mathbf{G}}}, \tag{10}$$

Equation (10) illustrates that, as the equilibrium price is not affected by the demand side shocks, directly hit sectors adjust the demand of their inputs in absence of price adjustments and shocks propagate upstream ($d\ln \mathbf{y}^{up} = \hat{A}' \cdot d\ln \mathbf{y}$). The sectorial propagation of an expenditure adjustment that determines its global impact is driven by the elements in the columns of a transformed input-output matrix $\hat{A}$, which describe a sector's sales to other industries. Basically, a spending cut, translates into a reduction in a sector's demand, whose reaction is to shrink the production of its output by purchasing less input.

We are aware of the fact that this theoretical model generates an asymmetric propagation of demand and supply shocks, thanks to the particular Cobb-Douglas specification (for instance in Vasco M. Carvalho et al. 2016, they adopt a CES production function which allows for both upstream and downstream propagation). Nevertheless, we prefer to use Cobb-Douglas for two reasons: 1) it is consistent with a dependent variable expressed in percent deviations; 2) it keeps the model easy. Since we interpret the theoretical model only as a tool to break down and getting an insight of a potential underlying economic mechanism, the plausibility of the model's assumptions (such as Cobb-Douglas production function) do not undermine the logic of our estimation strategy, which will be illustrated in section 3.

The next two subsections provide a detailed derivation of Equations 7 and 9. Subsections 2.3 and 2.4 illustrate an example of a tax hike and a spending cut respectively. Eventually, the last subsection (2.5) will go deeper on the choice of the Cobb-Douglas production function.

2.1 The Effect of a Tax Shock (Detailed Derivation)

Consider an increase in payroll tax which is implemented by keeping the government expenditure constant ($dG_i = 0$) and therefore by reallocating taxation between the non-distortionary and the distortionary component.
Take logs in the production function and totally differentiate by assuming no productivity shocks to obtain:

$$d \ln y_i = \alpha_i^l d \ln l_i + \sum_{j=1}^{n} a_{ij} d \ln x_{ij} \tag{11}$$

Totally differentiate the conditions for profit maximization:

$$\begin{aligned} d \ln y_i + d \ln p_i - d \ln p_j &= d \ln x_{ij} \\ d \ln y_i + d \ln p_i &= d \ln l_i + d \ln (1+\tau) \end{aligned}$$

as wages are chosen as numeraire.
Substitute these two equations into Equation 11:

$$d \ln y_i = \alpha_i^l \left(d \ln y_i + d \ln p_i - d \ln (1+\tau) \right) + \sum_{j=1}^{n} a_{ij} \left(d \ln y_i + d \ln p_i - d \ln p_j \right) \tag{12}$$

Using the household problem's optimality conditions (Equations 4, 5 and (6)), we have:

$$k = \frac{p_i \cdot c_i}{\beta_i} = wl - T$$

Plugging into the previous equation the optimal labor supply and considering that the wage is the numeraire, we obtain:

$$(1+\lambda) \cdot k = 1 - T$$

By log-differentiating the above equation, we obtain:

$$d \ln p_i + d \ln c_i = d \ln(1 - T) \tag{13}$$

and plugging (13) into (12) , yields:

$$d \ln y_i = \alpha_i^l \big(d \ln y_i - d \ln c_i + d \ln(1-T) - d \ln \left(1 + \tau\right) \big) + \sum_{j=1}^{n} a_{ij} \left(d \ln y_i - d \ln c_i + d \ln c_j \right).$$

Since $\alpha_i^l + \sum_{j=1}^{n} a_{ij} = 1$,, the $d \log y_i$ terms cancel out. Rearranging:

$$d \ln c_i = \alpha_i^l \big(d \ln(1-T) - d \ln(1+\tau) \big) + \sum_{j=1}^{n} a_{ij} d \ln c_j,$$

in matrix form

$$\begin{aligned}
\mathbf{d} \ln \mathbf{c} &= \alpha^l d \ln\left(1 - T\right) - \alpha^l d \ln\left(1 + \tau\right) + A \cdot \mathbf{d} \ln \mathbf{c} \\
\mathbf{d} \ln \mathbf{c} &= \left(I - A\right)^{-1} \left[\alpha^l d \ln\left(1 - T\right) - \alpha^l d \ln\left(1 + \tau\right) \right].
\end{aligned}$$

Next, combining the market clearing conditions with the first order conditions we obtain

$$y_j = c_j + \sum_{i=1}^{n} x_{ij}$$

$$\frac{y_j}{c_j} = 1 + \sum_{i=1}^{n} a_{ij} \frac{\beta_i y_i}{\beta_j c_i}$$

which, given that G is constant, implies

$$\mathbf{d} \ln \mathbf{c} = \mathbf{d} \ln \mathbf{y}$$

and finally

$$d \ln y_i = \alpha_i^l \cdot \big(d \ln\left(1 - T\right) - d \ln\left(1 + \tau\right) \big) + \sum_{j=1}^{n} a_{ij} \cdot d \ln y_j. \tag{14}$$

Using the input-output matrix A, allows us to rewrite equation (14) in matrix form:

$$d \ln \underset{n \times 1}{\mathbf{y}} = \underset{n \times 1}{\boldsymbol{\alpha}^l} \cdot \big(d \ln\left(1 - T\right) - d \ln\left(1 + \tau\right) \big) + \underset{n \times n}{A} \cdot d \ln \underset{n \times 1}{\mathbf{y}}. \tag{15}$$

We therefore derived Equation 15 which is Equation 8 in the theoretical model section.

2.2 The Effect of a Spending Shock (Detailed Derivation)

We consider now the effect of a government expenditure shock which is fully financed by lump-sum taxation, so for the sake of simplicity we set $\tau = 0$. By solving the following problem:

$$\min_{l_i, \{x_{ij}\}_{j=1}^n} w \cdot l_i + \sum_{j=1}^n p_j \cdot x_{ij}$$
$$\text{s.t.} \quad e^{z_i} l_i^{\alpha_i^l} \prod_{j=1}^n x_{ij}^{\alpha_{ij}} = 1$$

we derive the unit cost function of sector i:

$$C_i(p, w) = B_i \cdot w^{\alpha_i^l} \cdot \prod_{j=1}^n p_j^{a_{ij}}$$

where

$$B_i = \left(\frac{1}{\alpha_i^l}\right)^{\alpha_i^l} \cdot \prod_{j=1}^n \left(\frac{1}{a_{ij}}\right)^{a_{ij}}.$$

Zero profit condition for producers, implies that the cost of producing one extra unit of output ($C_i(p, w)$) coincides with the marginal revenue, the price p_i. By log differentiating the zero profit condition, we obtain:

$$\ln p_i = \ln B_i + \alpha_i^l \ln w + \sum_{j=1}^n a_{ij} \ln p_j. \tag{16}$$

Equation (16) illustrates that the vector of prices does not depend on government purchases G, suggesting that equilibrium price is not affected by the demand side shocks and instead, is fully determined by the supply side (notice that the equilibrium price is instead affected by changes in the payroll tax τ, which we assumed here to be zero for simplicity).

Log-differentiate the firms' FOCs, and take into account the fact that in equilibrium prices do not change. Then we have:

$$d \ln y_i = d \ln x_{ij} \tag{17}$$

$$d \ln y_i = d \ln l_i. \tag{18}$$

In particular notice that from Equation 17 we obtain:

$$dx_{ij} = \frac{dy_i}{y_i} \cdot x_{ij} \tag{19}$$

Since we assumed $\tau = 0$, from the government budget constraint we have: $T = \sum_{i=1}^{n} p_i G_i$.
Using the fact that wage is the numeraire, from the household FOC we obtain:

$$p_i c_i = \frac{\beta_i}{1+\lambda} \cdot (1 - \sum_{j=1}^{n} p_j G_j),$$

by differentiating:

$$p_i \cdot dc_i = -\frac{\beta_i}{1+\lambda} \cdot \sum_{j=1}^{n} p_j \cdot dG_j. \tag{20}$$

By differentiating the resource constraint we have:

$$dy_i = dc_i + \sum_{j=1}^{n} dx_{ji} + dG_i \tag{21}$$

Now recall that:

$$a_{ij} = \frac{p_j x_{ij}}{p_i y_i} \implies x_{ij} = a_{ij} \frac{p_i y_i}{p_j}$$

Now rearrange Equation 21 and plug Equations: 17, 18, 19 and 20 into it:

$$\begin{aligned}
\frac{dy_i}{y_i} &= \frac{dc_i}{y_i} + \sum_{j=1}^{n} \frac{dx_{ji}}{y_i} + \frac{dG_i}{y_i} \\
\frac{dy_i}{y_i} &= -\frac{1}{y_i} \cdot \frac{\beta_i}{1+\lambda} \cdot \sum_{j=1}^{n} \frac{p_j \cdot dG_j}{p_i} + \sum_{j=1}^{n} x_{ji} \cdot \frac{dy_j}{y_j} \cdot \frac{1}{y_i} + \frac{dG_i}{y_i} \\
\frac{dy_i}{y_i} &= \sum_{j=1}^{n} a_{ji} \cdot \frac{p_j y_j}{p_i} \cdot \frac{dy_j}{y_j} \cdot \frac{1}{y_i} + \frac{dG_i}{y_i} - \frac{\beta_i}{1+\lambda} \cdot \sum_{j=1}^{n} \frac{p_j \cdot dG_j}{p_i y_i}.
\end{aligned}$$

Now set: $\widetilde{G}_i = p_i G_i$ and $\hat{a}_{ji} = \frac{x_{ji}}{y_i} = a_{ji} \frac{p_j y_j}{p_i y_i}$. We have:

$$d \ln y_i = \sum_{j=1}^{n} \hat{a}_{ji} \cdot d \ln y_j + \frac{d\widetilde{G}_i}{p_i y_i} - \frac{\beta_i}{1+\lambda} \cdot \sum_{j=1}^{n} \frac{d\widetilde{G}_j}{p_i y_i}, \tag{22}$$

where matrix $\hat{A}$, is the transformation of the input-output matrix A. Setting:

$$\mathbf{\Lambda} = \begin{bmatrix} (1 - \frac{\beta_1}{1+\lambda}) \cdot \frac{1}{p_1 \cdot y_1} & -\frac{\beta_1}{1+\lambda} \cdot \frac{1}{p_1 \cdot y_1} & \cdots & -\frac{\beta_1}{1+\lambda} \cdot \frac{1}{p_1 \cdot y_1} \\ -\frac{\beta_2}{1+\lambda} \cdot \frac{1}{p_2 \cdot y_2} & (1 - \frac{\beta_2}{1+\lambda}) \cdot \frac{1}{p_2 \cdot y_2} & \cdots & -\frac{\beta_2}{1+\lambda} \cdot \frac{1}{p_2 \cdot y_2} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\beta_n}{1+\lambda} \cdot \frac{1}{p_n \cdot y_n} & (1 - \frac{\beta_n}{1+\lambda}) \cdot \frac{1}{p_n \cdot y_n} & \cdots & -\frac{\beta_n}{1+\lambda} \cdot \frac{1}{p_n \cdot y_n} \end{bmatrix},$$

allows us to rewrite equation 22 in matrix notation:

$$\underset{n\times 1}{d\ln \mathbf{y}} = \underset{n\times n}{\hat{A}^T} \cdot \underset{n\times 1}{d\ln \mathbf{y}} + \underset{n\times n}{\mathbf{\Lambda}} \cdot \underset{n\times 1}{d\tilde{\mathbf{G}}}, \tag{23}$$

Equation 23 coincides with Equation 10 of the Theoretical Model Section.

2.3 The Effect of Tax Adjustment: a Simple Illustration

Consider the simple case in which we have three industries. We describe the network by triplets $\{i, j, k\}$ where j is a supplier of industry i and k is a customer of industry i. The network structure is the following simplified one: $\{2, 3, 1\}, \{1, 2, 3\}, \{3, 1, 2\}$.

Assume that $u(c_1, c_2, c_3, l) = \gamma_l \prod_{i=1}^{3} c_i^{1/3}$. Sector's i production function is $y_i = e^{z_i} l_i^{\alpha_i^l} x_{ij}^{\alpha_{ij}}$. Also set $z_i = 0$ for $\forall i$

Market clearing condition for sector i is $y_i = c_i + x_{ki} + G_i$. Combining

$$a_{ij} = \frac{p_j x_{ij}}{p_i y_i}, \quad \alpha_i^l = \frac{w\,(1+\tau)\, l_i}{p_i y_i}$$

and

$$\frac{p_i c_i}{\beta_i} = \frac{p_j c_j}{\beta_j}$$

to eliminate prices we get

$$a_{ij} = \frac{c_i x_{ij}}{c_j y_i}$$

$$\alpha_i^l = \frac{w\left(1+\tau\right)l_i}{p_i y_i}$$

using the fact that

$$p_i c_i = \beta_i\left(wl - T\right)$$

get

$$\alpha_i^l = \frac{w\left(1+\tau\right)l_i c_i}{\beta_i\left(wl-T\right)y_i}$$

$w = 1, \beta_i = \beta = 1/3, l = 1$

$$\alpha_i^l = \frac{3\left(1+\tau\right)l_i c_i}{\left(1-T\right)y_i}$$

Substituting these expressions into the production function, we obtain

$$y_i = l_i^{\alpha_i^l} x_{ij}^{\alpha_{ij}} = \left(\frac{\alpha_i^l\left(1-T\right)y_i}{3\left(1+\tau\right)c_i}\right)^{\alpha_i^l}\left(\frac{\alpha_{ij}c_j y_i}{c_i}\right)^{\alpha_{ij}}$$

Taking into account the fact that $\alpha_i^l + \alpha_{ij} = 1$ for our example, simplify expression above to

$$c_i = \left(\frac{\alpha_i^l\left(1-T\right)}{3\left(1+\tau\right)}\right)^{\alpha_i^l}\left(\alpha_{ij}c_j\right)^{\alpha_{ij}} = \left(\frac{\alpha_i^l}{3}\right)^{\alpha_i^l}\left(\alpha_{ij}\right)^{\alpha_{ij}}\left(\frac{\left(1-T\right)}{\left(1+\tau\right)}\right)^{\alpha_i^l}\left(c_j\right)^{\alpha_{ij}}$$

Let $\left(\frac{\alpha_i^l}{3}\right)^{\alpha_i^l}\left(\alpha_{ij}\right)^{\alpha_{ij}} = \Omega_{ij}$, then

$$c_i = \Omega_{ij}\left(\frac{1-T}{1+\tau}\right)^{\alpha_i^l} c_j^{\alpha_{ij}}, i = 1, 2, 3$$

Solving simultaneously the three equations, we obtain:

$$c_i = \widetilde{\Omega}_i\left(\frac{1-T}{1+\tau}\right)^{\eta_i}$$

where $\widetilde{\Omega}_i$ - some constant and

$$\eta_i = \frac{\alpha_i^l + \alpha_j^l a_{i,j} + \alpha_k^l a_{ij}a_{jk}}{1 - a_{ij}a_{jk}a_{ki}}$$

Taking the log differential of expression for c_i we get

$$d\ln c_i = \eta_i\left[d\ln(1-T) - d\ln(1+\tau)\right]$$

using the fact that

$$\mathbf{d}\ln\mathbf{c} = \mathbf{d}\ln\mathbf{y}$$

$$d\ln y_i = \eta_i\left[d\ln(1-T) - d\ln(1+\tau)\right]$$

so

$$\begin{aligned}
d\ln y_1 &= \frac{\alpha_1^l + \alpha_2^l a_{1,2} + \alpha_3^l a_{12}a_{23}}{1 - a_{12}a_{23}a_{31}}\left[d\ln(1-T) - d\ln(1+\tau)\right] \\
d\ln y_2 &= \frac{\alpha_2^l + \alpha_3^l a_{2,3} + \alpha_1^l a_{23}a_{31}}{1 - a_{23}a_{31}a_{12}}\left[d\ln(1-T) - d\ln(1+\tau)\right] \\
d\ln y_3 &= \frac{\alpha_3^l + \alpha_1^l a_{3,1} + \alpha_2^l a_{31}a_{12}}{1 - a_{31}a_{12}a_{23}}\left[d\ln(1-T) - d\ln(1+\tau)\right]
\end{aligned}$$

2.4 The Effect of an Expenditure Adjustment: a Simple Illustration

Consider now the case of an Expenditure adjustments expressed in nominal terms as $d\widetilde{G}_1, d\widetilde{G}_2, d\widetilde{G}_3$. As in the case of Tax adjustments we set $\beta_1 = \beta_2 = \beta_3 = 1/3$. For simplicity we assume that government expenditures are fully financed by lump-sum taxation and we set payroll tax to zero $\tau = 0$.

Utility function is $u(c_1, c_2, c_3, l) = \gamma_l \prod_{i=3}^{3} c_i^{1/3}$.Unit cost function can be written as:

$$C_i(p, w) = \mu_i w^{\alpha_i^l} p_j^{a_{ij}}$$

where $\mu_i = \left(\frac{\alpha_i^l}{a_{ij}}\right)^{a_{ij}} + \left(\frac{a_{ij}}{\alpha_i^l}\right)^{\alpha_i^l}$. In equilibrium we have

$$p_i = C_i(p, w) = \mu_i w^{\alpha_i^l} p_j^{a_{ij}}$$

Since $w = 1$we can solve the last equation for price

$$p_i = \gamma^{\frac{1}{1-a_{ij}a_{jk}a_{ki}}}$$

where $\gamma = \mu_i \mu_j^{a_{ij}} \mu_k^{a_{ij}a_{jk}}$. Taking into account the fact that prices do not respond to expenditure adjustments, we consider nominal values, denoted by $\tilde{}$.

$$d\widetilde{y}_i = d\widetilde{c}_i + a_{ki} d\widetilde{y}_k + d\widetilde{G_i}$$

From the household optimization problem we have

$$\widetilde{c}_i = \frac{1}{(1+\lambda)3} - \frac{\widetilde{G}_i + \widetilde{G}_j + \widetilde{G}_k}{(1+\lambda)3}$$

By differentiating and combining it with resource constraint will leads to:

$$d\widetilde{y}_i = -\frac{d\widetilde{G_i} + d\widetilde{G_j} + d\widetilde{G_k}}{(1+\lambda)3} + a_{ki} d\widetilde{y}_k + d\widetilde{G_i}, \forall i = 1, 2, 3$$

Solving this system of equations leads to

$$d\widetilde{y}_i = \frac{1}{1 - a_{ij}a_{jk}a_{ki}} \left\{ \begin{array}{c} d\widetilde{G}_i + a_{ki}a_{jk} d\widetilde{G}_j + a_{ki} d\widetilde{G}_k \\ -\frac{1+a_{ki}+a_{ki}a_{jk}}{(1+\lambda)3} [d\widetilde{G}_i + d\widetilde{G}_j + d\widetilde{G}_k] \end{array} \right\}$$

2.5 Spatial Variables and the Cobb-Douglas Production Function

Spatial variables are constructed consistently with the Cobb-Douglas production functions.
We defined the spatial variables in this way:

$$\Delta y_{i,t}^{up} = \sum_{j \neq i}^{n} \hat{a}_{ji} \cdot \Delta y_{j,t}$$

$$\Delta y_{it}^{down} = \sum_{j \neq i}^{n} a_{ij} \cdot \Delta y_{j,t}.$$

Where $\Delta y_{j,t}$ accounts for the percent growth rate of real value added of industry i recorded in year t.
For example, for industry 1 we have:

$$\Delta y_{1,t}^{up} = \frac{Sales_{1 \to 2} \cdot \Delta y_{2,t} + \cdots + Sales_{1 \to n} \cdot \Delta y_{n,t}}{Sales_1}$$

$$\Delta y_{1,t}^{down} = \frac{Sales_{2 \to 1} \cdot \Delta y_{2,t} + \ldots Sales_{n \to 1} \cdot \Delta y_{n,t}}{Sales_1}$$

They can be interpreted as a weighted average of the real value added of other industries, with weights given by the relative importance of every industry as a customer or a supplier to industry i.
Since the idea is to construct a variable which captures the network effect, we might wonder Why not expressing the spatial variables as the percent change of a linear combination rather than a linear combination of a percent change. That is, we could have expressed them in this alternative way:

$$\Delta y_{i,t}^{up} = \Delta \Big(\sum_{j \neq i}^{n} \hat{a}_{ji} \cdot Y_{j,t} \Big)$$

$$\Delta y_{i,t}^{down} = \Delta \Big(\sum_{j \neq i}^{n} a_{ij} \cdot Y_{j,t} \Big).$$

At this point we link a spatial variable to its corresponding dependent variable:

$$\Delta y_{i,t} = \beta^{up} \cdot \Delta y_{i,t}^{up} + \beta^{down} \cdot \Delta y_{i,t}^{down},$$

taking a logarithmic approximation yields:

$$\frac{\partial}{\partial t}\ln y_{i,t} = \beta^{up}\cdot\frac{\partial}{\partial t}\ln y_{i,t}^{up} + \beta^{down}\cdot\frac{\partial}{\partial t}\ln y_{i,t}^{down},$$

by integrating both terms we get:

$$\begin{aligned}\ln y_{i,t} &= \beta^{up}\cdot\ln\Big(\sum_{j\neq i}^{n}\hat{a}_{ji}\cdot Y_{jt}\Big) + \beta^{down}\cdot\ln\Big(\sum_{j\neq i}^{n}a_{ij}\cdot Y_{jt}\Big)\\ &= \ln\left(\Big(\sum_{j\neq i}^{n}\hat{a}_{ji}\cdot Y_{jt}\Big)^{\beta^{up}}\cdot\Big(\sum_{j\neq i}^{n}a_{ij}\cdot Y_{jt}\Big)^{\beta^{down}}\right),\end{aligned}$$

we finally obtain:

$$y_{i,t} = \Big(\sum_{j\neq i}^{n}\hat{a}_{ji}\cdot Y_{jt}\Big)^{\beta^{up}}\cdot\Big(\sum_{j\neq i}^{n}a_{ij}\cdot Y_{jt}\Big)^{\beta^{down}}.$$

The last expression is telling us that the industries are linked among themselves through a relationship which has nothing to do with a Cobb-Douglas production function.

Consider now the definition we employ in the paper:

$$\begin{aligned}\Delta y_{i,t} &= \beta^{up}\cdot\Delta y_{i,t}^{up} + \beta^{down}\cdot\Delta y_{i,t}^{down}\\ &= \beta^{up}\cdot\Big(\sum_{j\neq i}^{n}\hat{a}_{ji}\Delta y_{j,t}\Big) + \beta^{down}\cdot\Big(\sum_{j\neq i}^{n}a_{ij}\Delta y_{j,t}\Big),\end{aligned}$$

substituting again the percent changes with the logarithmic approximation, we have:

$$\frac{\partial}{\partial t}\ln y_{it} = \beta^{up}\cdot\Big(\sum_{j\neq i}^{n}\hat{a}_{ji}\cdot\frac{\partial}{\partial t}\ln y_{jt}\Big) + \beta^{down}\cdot\Big(\sum_{j\neq i}^{n}a_{ij}\cdot\frac{\partial}{\partial t}\ln y_{j,t}\Big),$$

by integrating both terms we have:

$$\begin{aligned}
\ln y_{it} &= \beta^{up} \cdot \Big(\sum_{j \neq i}^{n} \hat{a}_{ji} \cdot \ln y_{jt} \Big) + \beta^{down} \cdot \Big(\sum_{j \neq i}^{n} a_{ij} \cdot \ln y_{j,t} \Big) \\
&= \sum_{j \neq i}^{n} \underbrace{(\beta^{up} \cdot \hat{a}_{ji} + \beta^{down} \cdot a_{ij})}_{\phi_{ij}} \cdot \ln y_{j,t} \\
&= \ln \left(\prod_{j \neq i}^{n} y_{jt}^{\phi_{ij}} \right),
\end{aligned}$$

by removing the natural logarithm from both sides, we have:

$$y_{i,t} = \prod_{j \neq i}^{n} y_{j,t}^{\phi_{ij}}$$

When we assume a linear relationship between the spatial variables and the dependent variables (as done in a standard linear regression, also carried out in SAR frameworks), and the latter is expressed in percentage change, we automatically assume a multiplicative relationship between the absolute levels of the dependent variables. This is consistent with the choice of a Cobb-Douglas production function. In SAR frameworks, spatial variables are always expressed as a weighted average of the dependent variables, and a linear regression of dependent variables over spatial variables is carried out. Therefore, if we underpin a SAR model with a theoretical model and data are expressed in percentage change (as we do in this paper), it seems appropriate to adopt a Cobb-Douglas production function.

3 From Theory to Empirics

3.1 Empirical Strategy

Suppose it would be possible to identify tax and expenditure fiscal corrections, τ_t, and g_t, exogenous for the estimation of their effect on output growth and orthogonal to each other. In this case equations (7) and (9) from the previous section can naturally be nested in the following panel specification to model the effect of fiscal adjustments on the value added growth in each industry i:

$$\Delta y_{i,t} = c_i + \left(\delta \cdot \tau_t + \beta^{down} \cdot \underbrace{\sum_{j \neq i}^{n} a_{ij} \cdot \Delta y_{j,t}}_{\Delta y_{i,t}^{down}} \right) \cdot TB_t + \\ + \left(\gamma \cdot g_t + \beta^{up} \cdot \underbrace{\sum_{j \neq i}^{n} \hat{a}_{ji} \cdot \Delta y_{j,t}}_{\Delta y_{i,t}^{up}} \right) \cdot EB_t + \varepsilon_{it} \tag{24}$$

where TB_t is a dummy that takes a value of 1 when a tax-adjustment takes place and zero otherwise while EB_t is a dummy that takes a value of 1 when an expenditure-based adjustment takes place and zero otherwise. In practice, to take the above model to the data, two problems need to be solved. The first one is to identify fiscal adjustments exogenous with respect to output fluctuations, the second one is to deal with the fact that in practice τ_t, and g_t are correlated as typically government decides first the total amount of the needed adjustment and successively their allocation in the two components. We solve the first problem by adopting a narrative identification approach to fiscal adjustment plans (see the next section for details), while we map correlated τ_t, and g_t adjustments into mutually exclusive components by adopting the following identification strategy.

$$\tau_t = \delta_0^{TB} \cdot e_t \cdot TB_t + \delta_0^{EB} \cdot e_t \cdot EB_t + \epsilon_t \tag{25}$$

$$g_t = \vartheta_0^{TB} \cdot e_t \cdot TB_t + \vartheta_0^{EB} \cdot e_t \cdot EB_t + \upsilon_t \tag{26}$$

$$e_t = g_t + \tau_t \tag{27}$$

where the TB_t and EB_t labels are attributed by considering the dominant component of each adjustment plans. The estimated parameters in this system allow to track the relative contribution of tax and spending measures to EB and TB plans and to map correlated adjustments into mutually exclusive ones. As the TB_t and EB_t variables are mutually exclusive, the following model can be estimated and simulated to derive the impact of expenditure-based and tax-based fiscal adjustments

$$\begin{aligned}\Delta y_{i,t} = c_i + &\left(\delta \cdot e_t + \beta^{down} \cdot \underbrace{\sum_{j \neq i}^{n} a_{ij} \cdot \Delta y_{j,t}}_{\Delta y_{i,t}^{down}}\right) \cdot TB_t + \\ &+ \left(\gamma \cdot e_t + \beta^{up} \cdot \underbrace{\sum_{j \neq i}^{n} \hat{a}_{ji} \cdot \Delta y_{j,t}}_{\Delta y_{i,t}^{up}}\right) \cdot EB_t + \varepsilon_{it}\end{aligned} \tag{28}$$

The TB adjustments, being mainly supply shocks, have a network effect that goes through the connection of industry i with its supplier industries (changes in suppliers' output flows down to customer industry i: downstream propagation). This is captured by including in the specification a spatial variable which is a weighted average of value added growth in all other sectors ($\Delta y_{i,t}^{down}$); weights are given by the elements of the rows of the input-output matrix A.

Symmetrically, the EB adjustments, being mainly demand shocks, have a network effect that goes through the connection of industry i with its customer industries (changes in customers' output flows up to supplier industry i: upstream propagation). This is captured by including in the specification another spatial variable, which is a weighted average of value added growth in all other sectors. Weights are now given by the elements of the columns of the transformed input-output matrix $\hat{A}$.

The net distinction between demand and supply shocks, provide a theoretical justification for the interaction of the shocks and the networks: downstream channel is activated when a TB shock occurs and upstream channel is activated when an EB shock occurs. This assumption will be tested in the robustness section (Section 6)

when we switch the interactions: downstream channel is activated when EB occurs and upstream channel is activated when TB occurs, thus going against what theory predicts.
Furthermore, since the two network variables, $\Delta y_{i,t}^{down}$ and $\Delta y_{i,t}^{up}$ exhibit a non-negligible correlation level, keeping them separated by means of the interaction with the shocks, turns out useful for ruling out multicollinearity problems.

This empirical framework combines the Spatial Autoregressions (SARs) (Ord 1975 , J. LeSage and Pace 2009) or more precisely spatial autoregressive panel data models with fixed effects and the GVAR or global vector autoregression (Chudik and Pesaran 2016) approach to identify direct and indirect effects of fiscal adjustments. Difference between two approaches is that historically the spatial approach is more about cross-section and GVAR is more about time dimension, even though now they are very close to each other. Note that we use industry specific spatial variables and we define them by excluding the value added growth of the sector modelled in each equation. In practice this is carried out by removing the main diagonal from the input-output matrix A and its transformation $\hat{A}$; these new matrices are denoted by adding a 0 subscript to the original ones: A_0 and $\hat{A}_0$.

3.2 Interpretation of output effects

To interpret the output effects of fiscal stabilization in each industry and in the economy described by equation (28), we have to take into account the role of the effects in related industries. Following J. P. LeSage and Parent 2007 we define three scalars to measure the average total, direct and indirect effect. Using vector notation we rewrite equation (28) as follows:

$$\begin{aligned}\Delta \mathbf{y}_t = \mathbf{c} + \left(\delta \cdot \mathbf{1}_n \cdot e_t + \beta^{down} \cdot A_0 \cdot \Delta \mathbf{y}_t\right) \cdot TB_t + \\ + \left(\gamma \cdot \mathbf{1}_n \cdot e_t + \beta^{up} \cdot \hat{A}_0^T \cdot \Delta \mathbf{y}_t\right) \cdot EB_t + \varepsilon_t\end{aligned} \tag{29}$$

The sectorial effects of EB and TB adjustments can now be computed as:

$$\left(\frac{\partial \Delta \mathbf{y}}{\partial e_t} \mid TB_t = 1\right) = (I_n - \beta^{down} \cdot A_0)^{-1} \cdot \mathbf{1}_n \cdot \delta = \mathbf{S}^{TB}\left(\beta^{down}, A_0\right) \cdot \mathbf{1}_n \cdot \delta,$$
$$\left(\frac{\partial \Delta \mathbf{y}}{\partial e_t} \mid EB_t = 1\right) = (I_n - \beta^{up} \cdot \hat{A}_0^T)^{-1} \cdot \mathbf{1}_n \cdot \gamma = \mathbf{S}^{EB}\left(\beta^{up}, \hat{A}_0^T\right) \cdot \mathbf{1}_n \cdot \gamma.$$

Given estimates for $\gamma, \beta^{down}, \delta, \beta^{up}$ the matrices $\mathbf{S}^{EB}\left(\beta^{down}, A_0\right)$ and $\mathbf{S}^{TB}\left(\beta^{up}, \hat{A}_0'\right)$ become observable, and therefore the average total, direct and indirect effect of fiscal stabilization can be computed as follows:

- *Average direct effect:* the weighted average of the diagonal elements of $\mathbf{S}^{EB}$ and $\mathbf{S}^{TB}$ with weights given by the sectorial contribution to total value added.
- *Average total effect:* the sum across the i-th row of $\mathbf{S}^{EB}$ and $\mathbf{S}^{TB}$ represents the total impact of expenditure-based and tax-based adjustments on value added of industry i. We obtain the average total effect on total value added by taking the weighted average of these effects with weights defined as in the computation of the average direct effect.
- *Average indirect effect*: the difference between the average total effect and the average direct effect.

3.3 Example: 3 Industries Economy

To illustrate the procedure consider a simple example with three industries for which the relevant adjustment is exclusively a tax based one.

$$\underbrace{\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta y_{3t} \end{bmatrix}}_{\Delta\mathbf{y}} = \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}_{\mathbf{c}} + \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{1}_3} \cdot \delta \cdot e_t + \underbrace{\begin{bmatrix} 0 & a_{12} \cdot \beta & a_{13} \cdot \beta \\ a_{21} \cdot \beta & 0 & a_{23} \cdot \beta \\ a_{31} \cdot \beta & a_{32} \cdot \beta & 0 \end{bmatrix}}_{A_0} \cdot \underbrace{\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta y_{3t} \end{bmatrix}}_{\Delta\mathbf{y}} + \underbrace{\begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \end{bmatrix}}_{\epsilon}$$

$$\begin{aligned} \frac{\partial \Delta\mathbf{y}}{\partial e_t} &= \begin{bmatrix} 1 & -a_{12} \cdot \beta & -a_{13} \cdot \beta \\ -a_{21} \cdot \beta & 1 & -a_{23} \cdot \beta \\ -a_{31} \cdot \beta & -a_{32} \cdot \beta & 1 \end{bmatrix}^{-1} \cdot \mathbf{1}_3 \cdot \delta \\ &= \begin{bmatrix} 1 - \beta^2 \cdot a_{23}a_{32} & \beta \cdot a_{12} + \beta^2 \cdot a_{32}a_{13} & \beta \cdot a_{13} + \beta^2 \cdot a_{12}a_{23} \\ \beta \cdot a_{21} + \beta^2 \cdot a_{23}a_{31} & 1 - \beta^2 \cdot a_{31}a_{13} & \beta \cdot a_{23} + \beta^2 \cdot a_{21}a_{13} \\ \beta \cdot a_{31} + \beta^2 \cdot a_{21}a_{32} & \beta \cdot a_{32} + \beta^2 \cdot a_{31}a_{12} & 1 - \beta^2 \cdot a_{21}a_{12} \end{bmatrix} \cdot \mathbf{1}_3 \cdot \frac{\delta}{d}, \end{aligned}$$

where, d is the determinant of matrix $(I_3 - \beta \cdot A_0)$:

$$d = 1 - \beta^2 \cdot (a_{12}a_{21} + a_{13}a_{31} + a_{32}a_{23}) - \beta^3 \cdot (a_{12}a_{23}a_{31} + a_{21}a_{13}a_{32}).$$

Given the above result, we provide the analytic form for the total, direct and indirect effect of a tax shock of sector 1:

- $\frac{1-\beta^2 \cdot a_{23}a_{32}}{d} \cdot \delta = \left[1 + \frac{\beta^2 \cdot (a_{13}a_{31} + a_{12}a_{21}) + \beta^3 \cdot (a_{12}a_{23}a_{31} + a_{21}a_{13}a_{32})}{d}\right] \cdot \delta$ gives the response of value added in industry 1 to the TB adjustment if this industry were the only

one affected by it, and it represents the tax shock "*direct effect*" on sector 1. Notice that it can be interpreted as the summation of the tax coefficient δ plus the network effect triggered by a tax shock which hits only sector 1 itself, called *feedback loop* by J. LeSage and Pace 2009. We call the former "*instantaneous effect*", while the latter, "*network direct effect*".
Furthermore, the average of the direct effects on all sectors of a tax shock, provides the Average Direct Effect of a tax shock.

- $\frac{\beta \cdot a_{12} + \beta^2 \cdot a_{32} a_{13}}{d} \cdot \delta$ gives the response of value added in industry 1 to the TB adjustment if industry 2 were the only one one affected by it.

- $\frac{\beta \cdot a_{13} + \beta^2 \cdot a_{12} a_{23}}{d} \cdot \delta$ gives the response of value added in industry 1 to the TB adjustment if industry 3 were the only one affected by it.

- The summation of the previous two effects represent the "*(network) indirect effect*" of industry 1; that is, the effect on industry 1 of a shock which hits all the industries except for industry 1 itself.
The average of the (network) indirect effects of every industry accounts for the Average Indirect Effect.

- The summation of all the direct and indirect effects is the "*total effect*" of a tax shock for sector 1. The average of all the total shocks, represent the Average Total Effect.

3.4 Total, Direct and Indirect Effect in Details

3.4.1 Determinant Decomposition

In this section we explain in details what the determinant d of matrix $(1 - \beta \cdot A_0)$ represents from an economic point of view.[2]
First of all, notice that the reciprocal of the determinant can be interpreted as the convergence point of a geometric summation:

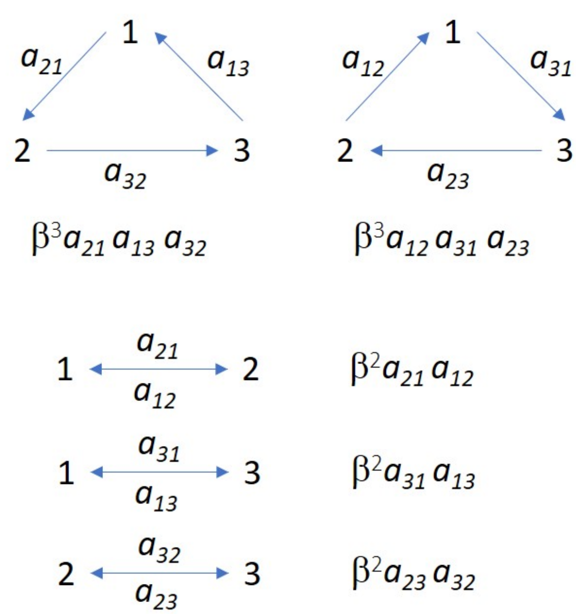
$$\frac{1}{d} = \sum_{i=1}^{\infty} \Big(\beta^2 \cdot \big(a_{12} a_{21} + a_{13} a_{31} + a_{32} a_{23} \big) + \beta^3 \cdot \big(a_{12} a_{23} a_{31} + a_{21} a_{13} a_{32} \big) \Big)^i = \sum_{i=1}^{\infty} K^i$$

where $K = \beta^2 \cdot (a_{12} a_{21} + a_{13} a_{31} + a_{32} a_{23}\big) + \beta^3 \cdot \big(a_{12} a_{23} a_{31} + a_{21} a_{13} a_{32})$.

[2] To ease notation we write simply β instead of β^{down}.

Basically, suppose that a little change in a sector's output occurs, then, such a change triggers a cascade effect of other changes in other sectors and then come back to it via the input-output network. If we imagined a kind of temporal-sequence in transferring such a shock, we would see that at every step the change that occurs is exactly K. Suppose that a unit change occurs, then this change is transferred to other sectors and come back, by generating a total change of K, but then this change is transferred again and comes back to its initial point thus generating a further change of K^2 and so on and so forth. The limit point of such a series is exactly the reciprocal of the determinant of matrix $I_3 - \mathbf{T}$.

Notice that every element of K represents a particular sectors' relationship, as it is represented in the figure below:

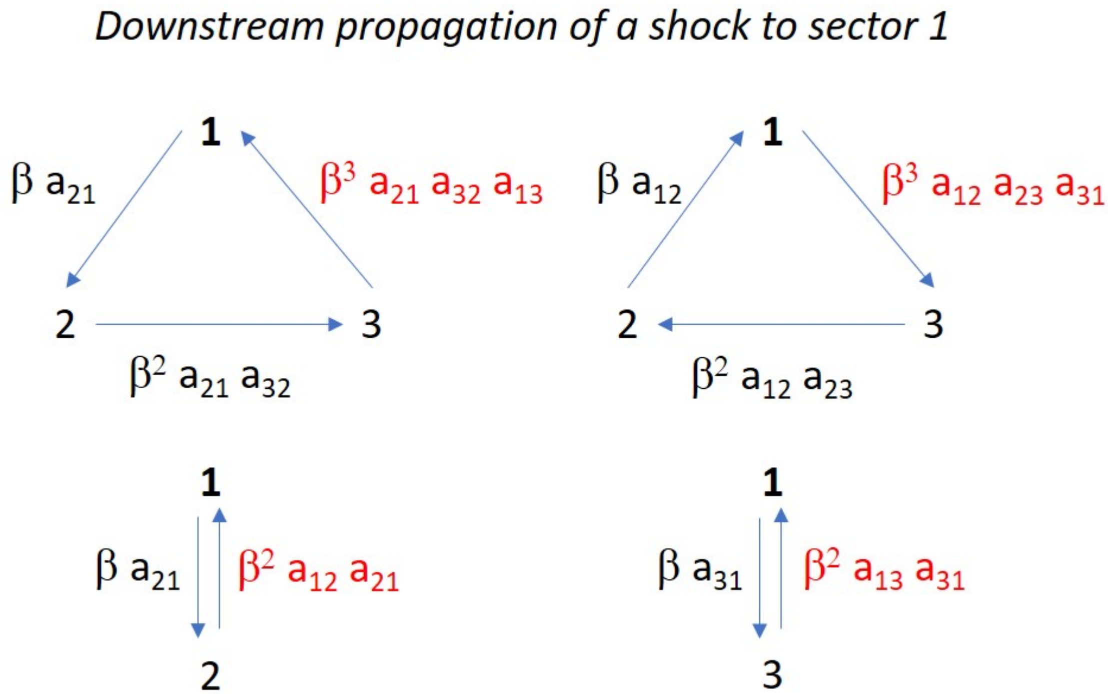


Now, all these changes actually occurs simultaneously, therefore, every variation is subject to an amplification due to the sectors' interconnections. Such an amplification is exactly expressed by the reciprocal of d, which can be seen as a "simultaneity multiplier".

3.4.2 Shock Effect Decomposition

At this point, we can examine more carefully all the elements of $\mathbf{S} = (I - \beta \cdot A_0)^{-1}$. In particular we can make the following distinction:

1. **Direct effect**: the direct effect to sector i of a macro shock δ ($DE_{\delta,i}$), is the change in sector i growth rate as if it would be the only sector in the economy subject to that shock:
$$DE_{\delta,i} = \frac{1}{d} \cdot \mathbf{S}_{ii} \cdot \delta.$$
Basically, the direct effects are collected on the main diagonal of the previous matrix. For instance, the direct effect of a shock to sector 1 is:
$$DE_{\delta,1} = \delta + \frac{\beta^2 \cdot (a_{31}a_{13} + a_{21}a_{12}) + \beta^3 \cdot (a_{12}a_{23}a_{31} + a_{21}a_{13}a_{32})}{d}\delta.$$
Notice that the direct effect could be decomposed into two parts: the shock itself (δ) plus the network effect that such a shock triggers, amplified by the simultaneity multiplier. Notice in fact that the instantaneous effect of a direct shock to only sector 1, is transferred on sector 2 and then goes back to sector 1, for a total change of $\beta^2 \cdot a_{21}a_{12}$. The same is true for sector 3 for a total change of $\beta^2 \cdot a_{31}a_{13}$. Moreover, the shock, once transferred upon sectors 2 and 3, it can get back to sector 1 indirectly via the connections between sector 2 and sector 3, for a total change of $\beta^3 a_{21}a_{32}a_{13}$ when the shock flows down to sector 2 and $\beta^3 a_{31}a_{23}a_{12}$ when the shock flows down to sector 3. This scheme is represented also in the figure below:



2. **Indirect effect**: The indirect effect to sector i of a macro shock δ, ($IE_{\delta,i}$), is the summation of all the effects of shocks which hit other sectors and then are transferred to sector i:

$$IE_{\delta,i} = \frac{1}{d} \cdot \left(\sum_{j \neq i}^{n} \mathbf{S}_{ij} \right) \cdot \delta.$$

For instance, the indirect effect of a macro shock on sector 1 is:

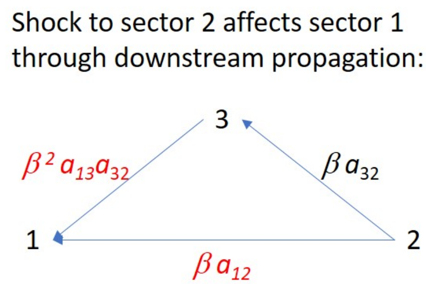
$$IE_{\delta,1} = \frac{1}{d} \cdot \Big[(\beta \cdot a_{12} + \beta^2 \cdot a_{32} a_{13}) + (\beta \cdot a_{13} + \beta^2 \cdot a_{12} a_{23}) \Big] \cdot \delta$$

$$= \frac{\beta \cdot (a_{12} + a_{13}) + \beta^2 \cdot (a_{32} a_{13} + a_{12} a_{23})}{d} \cdot \delta$$

Consider now the generic element of matrix $\mathbf{S}$, which we call $\mathbf{S}_{ij}$. Such an element provides the specific impact of a shock which hits directly sector j and then is transferred to sector i. Such an effect could be interpreted as an instantaneous effect which is then amplified via the simultaneity multiplier. To better understand this point, consider for instance the following element:

$$\mathbf{S}_{12} = \beta \cdot a_{12} + \beta^2 \cdot a_{32}a_{13}.$$

This element provides the "instantaneous" effect of a shock which hits sector 2 but then is transferred upon sector 1 (for this reason we call it "indirect"). Such a transfer occurs through the direct linkage between sector 2 and 1 ($\beta \cdot a_{12}$, sales of sector 2 to sector 1) and through the indirect connections via sector 3 ($\beta^2 \cdot a_{32}a_{13}$,that is, the sales of sector 2 to sector 3 and then the sales of sector 3 to sector 1). This propagation is shown in the figure below:



At this point we can show what the total, direct and indirect effects are defined:

1. The direct effect of a shock is not equal for every sector, since it depends on the network interactions specific for that sector. For this reason, we computed the effect on the economy, which is a weighted average of the single industries' effects, where weights are collected into the $n \times 1$ vector W (every industry weight is the relative size of the sector with respect to GDP):

$$DE_\delta = \frac{\delta}{d} \cdot diag(\mathbf{S}) \cdot W.$$

2. The total effect of a macro shock is by consequence the summation of the direct and indirect effect. Formally:

$$TE_{\delta,i} = \frac{\delta}{d} \cdot \Big(\sum_{j=1}^{n} \mathbf{S}_{ij} \Big).$$

Again, we report the formula for the total effect on the economy:

$$TE_\delta = \frac{\delta}{d} \cdot \mathbf{S} \cdot W.$$

3. The average indirect effect will be computed indirectly by taking the difference between the total and the direct effect:

$$IE_\delta = TE_\delta - DE_\delta.$$

4. The same things will be replicated identically for the expenditure shock ($EB_t = 1$):

$$ADE_\gamma = \frac{\gamma}{|I_n - \Gamma|} \cdot \frac{1}{n} \cdot Tr(S(\mathbf{\Gamma})).$$

$$ATE_\gamma = \frac{\gamma}{|I_n - \Gamma|} \cdot \frac{1}{n} \cdot \mathbf{1'} \cdot S(\mathbf{\Gamma}) \cdot \mathbf{1}.$$

$$AIE_\gamma = ATE_\gamma - ADE_\gamma$$

4 Data

4.1 Fiscal Shocks

4.1.1 Database of Exogenous Fiscal Adjustment Plans for US

We identify exogenous fiscal adjustment by adopting the narrative method (C. D. Romer and D. H. Romer 2009, C. D. Romer and D. H. Romer 2010). This method refers to presidential speeches, congressional debates, budget documents, congressional reports, to identify the size, timing, and principal motivation for all major postwar tax policy actions. Legislated changes are then classified into endogenous (those induced by short-run counter-cyclical concerns and those taken because of change in government spending) and exogenous (those that are responses to the state of government debt or to concerns about long-run economic growth).

Following Alesina, Favero, and Giavazzi 2019 we also acknowledge that fiscal consolidation policy is actually implemented through multi-year plans that involve an intertemporal and an intratemporal dimension. The intertemporal dimension is relevant in that plans involve both measures that are implemented upon announcement (the unanticipated component of the plan) and measures that are announced for the future n years (the anticipated component of the plan); the intratemporal dimension depends on the fact that adjustment plans are implemented with a mix of measures on the expenditure side and on the revenue side.

We adopt the annual database on fiscal adjustment plans constructed by Alesina, Favero, and Giavazzi 2015 and concentrate on US data only, by carrying out only slight modifications to the original database. A detailed description of the data is provided in the next subsection.

When fiscal policy is conducted through multi-year plans narrative exogenous fiscal adjustments in each year are made of three components: the unexpected adjustments (announced upon implementation at time t), the past announced adjustments (implemented at time t but announced in the previous years) and the future announced corrections.

We identify plans as sequences of fiscal corrections announced at time t to be implemented between time t and time $t+k$; we call k the anticipation horizon. We define the unanticipated fiscal shocks at time t as the surprise change in the primary surplus at time t:

$$e_t^u = \tau_t^u + g_t^u$$

where τ_t^u is the surprise increase in taxes announced at time t and implemented in the same year, and g_t^u is the surprise reduction in government expenditure also announced at time t and implemented in the same year. We denote instead as $\tau^a_{t,j}$ and $g^a_{t,j}$ the tax and expenditure changes announced by the fiscal authorities at date t with an anticipation horizon of j years (*i.e.* to be implemented in year $t+j$). In Pescatori et al. 2011's database, fiscal plans almost never extend beyond a 3-years horizon: thus we take $j = 3$ as the maximum anticipation horizon[3]. We therefore define the observed anticipated shocks in period t as follows

$$\begin{aligned}
&\tau^a_{t,0} = \tau^a_{t-1,1} \\
&\tau^a_{t,j} = \tau^a_{t-1,j+1} + \left(\tau^a_{t,j} - \tau^a_{t-1,j+1}\right) \;\; j \geqslant 1 \\
&g^a_{t,0} = g^a_{t-1,1} \\
&g^a_{t,j} = g^a_{t-1,j+1} + \left(g^a_{t,j} - g^a_{t-1,j+1}\right) \;\; j \geqslant 1 \\
&e^a_{t,j} = \tau^a_{t,j} + g^a_{t,.j}
\end{aligned}$$

Fiscal corrections in each year can be written as follows

$$e_t = e^u_t + e^a_{t,0} + \sum_{j=1}^{horz} e^a_{t,j}$$

Plans are labeled as tax-based or expenditure-based by adopting the following rule:

$$\begin{gathered}
if \quad \left(\tau^u_t + \tau^a_{t,0} + \sum_{j=1}^{horz} \tau^a_{t,j}\right) > \left(g^u_t + g^a_{t,0} + \sum_{j=1}^{horz} g^a_{t,j}\right) \qquad (30) \\
then\ TB_t = 1\ and\ EB_t = 0, \\
else\ TB_t = 0\ and\ EB_t = 1, \forall t
\end{gathered}$$

By construction tax-based (TB) and expenditure-based (EB) plans are mutually exclusive and the labelling is almost never marginal.

Insert Table I here

Table I represents the correlation matrix of fiscal adjustments. Taking into account rather high correlation between tax and expenditure adjustments (0.5962 for

[3]In the sample there are a few occurrences of policy shifts anticipated four and five years ahead. Their number is too small to allow us to include them in our estimation.

unanticipated component) as well as between anticipated and unanticipated components (0.5702 for the tax component) it is important to consider fiscal adjustment plans and not separate shocks.

The orthogonalization that we create using classification TB/EB is not the only way, although it is the most reasonable in our view. Another option would be to orthogonalize τ and g running the following regression $\tau = \theta * g + \epsilon^{\tau}$ and then use g and $\hat{\epsilon^{\tau}}$.[4] The advantage of using TB/EB classification relative to the alternative is twofold. First, coefficient θ will capture only average connection between τ and g, while our classification TB and EB allows for changes across time. Taking into account correlation matrix, θ is positive and is around 0.6. So consideration of the orthogonal tax adjustment $\tau - \theta * g$ will bias results for TB years, since our TB plans are indeed pure tax adjustments. In general, any alternative, that allows include separately τ and g will complicate interpretation of results and will make impossible to disentangle the effect of two components (see for details Giavazzi, Paradisi, 2013).

In Figure 1 we plotted the fiscal adjustments as a percentage of GDP. In particular we plot aggregate tax component of fiscal plan

$$\left(\tau_t^u + \tau_{t,0}^a + \sum_{j=1}^{horz} \tau_{t,j}^a \right) \tag{31}$$

and aggregate expenditure component of fiscal plan

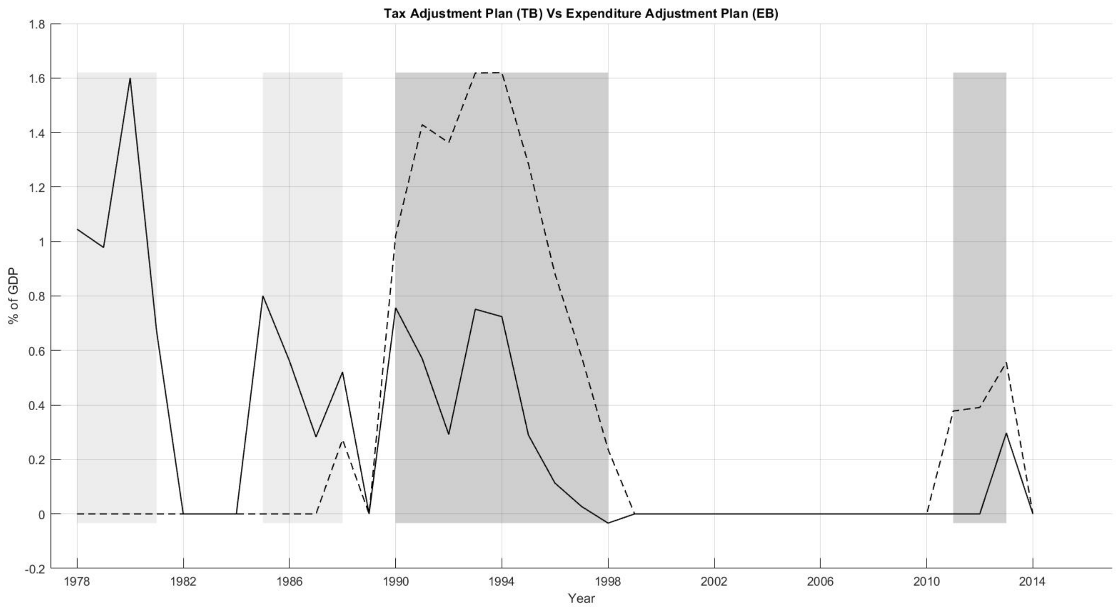
$$\left(g_t^u + g_{t,0}^a + \sum_{j=1}^{horz} g_{t,j}^a \right) \tag{32}$$

As in Alesina, Favero, and Giavazzi 2015, we scale all the measures by GDP on the year prior to the consolidation in order to avoid potential endogeneity issues.

Figure 1 represents our fiscal shocks database. The solid line represents the left hand side of inequality 30, that is, the total tax adjustment fiscal plans; the dashed line instead, represents the left hand side of inequality 30, that is, the total expenditure adjustment fiscal plans. The light gray areas represent the years when a TB fiscal plan occurs ($TB_t = 1$); we identify two TB periods: 1978-1981 and 1985-1988. The darker areas account for the years when an EB fiscal plan occurs ($EB_t = 1$); we identify two EB periods: 1990-1998 and 2011-2013.

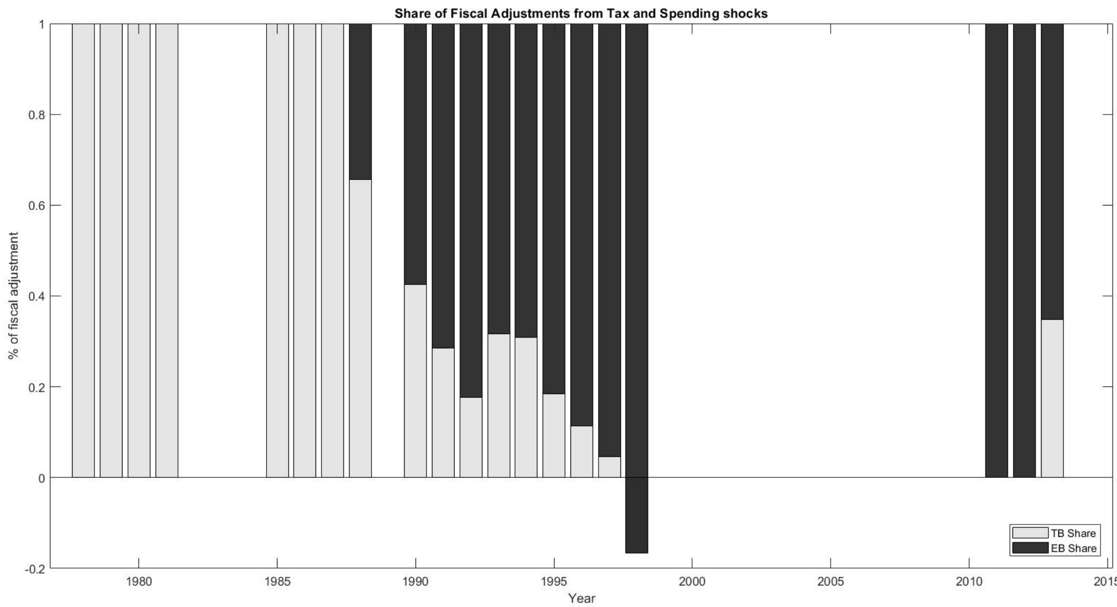
[4] This example ignores intertemporal correlation

Figure 1: Fiscal Adjustments Database



In Figure 2 we provide a decomposition of the fiscal adjustments to illustrate the mixed nature of the plans. Notice that the TB plans are all pure tax hikes but year 1988, which is the result of a mixed fiscal plan (around 30% of the fiscal adjustment plan comes from a spending cut). At the same time, EB plans are mainly made of spending cuts rather than tax hikes, even though they exhibit a more heterogeneous structure: on average, 18% of an EB fiscal adjustment comes from a tax increase).

Figure 2: Tax and Expenditure Share



4.1.2 Details on Database of Exogenous Fiscal Adjustment Plans

For the purpose of this project we follow Alesina et al.(2015) and use the annual fiscal adjustment plans data. Our focus is US. We adopt the same data employed by Alesina et Al.(2015), however we do several modifications. The reason for such a discrepancy can be explained by the fact that we deal in a different way with long-run adjustments. In fact, Romer and Romer(2010) distinguish between deficit driven and long-run growth driven adjustments. Long-run driven adjustments can be both positive and negative. In order to take them into account we follow the rule: sum up positive and negative components of long-run growth driven adjustments together with deficit driven adjustments and include the sum into the database if and only if it is non negative [5].

Moreover, it is worth to notice two years of Reagan presidency. The rule described above leads to drop from the sample the deficit-driven adjustment implemented in the US in years 1983-84 because it was smaller than the contemporaneous negative long-run growth-driven adjustment.

[5] Modifications are light because positive long-run driven adjustments, that is tax increase due to long-run growth reasons, are very uncommon.

Slight modifications in years 1980 and 1981 are due to the same logic. In 1980 we include the positive long-run growth tax increases[6]. In 1981 following the rule we consider the sum of the deficit - driven tax hike and long-run growth driven tax decrease.

Other slight modifications, consistent with the previous reasoning, are in years 1985, 1986, 1990. We record initial announcement of the Social Security Amendment 1983 as in Alesina et al.(2015) in the announced part of the plan, however additionally we record revisions to already announced adjustments for the years 1985, 1986, 1990 as a surprise component[7]. Revisions result in further austerity.

Overall, the differences between Alesina et al.(2015) database and our database are minimal. Importantly, following Alesina et al.(2015), we scale all the measures by GDP on the year prior to the consolidation in order to avoid potential endogeneity issues[8].

To illustrate the procedure of fiscal plan construction consider the case of 1990 OBRA (Omnibus Budget Reconciliation Act) - 1990, which is considered as exclusively motivated by a deficit reduction motive and therefore exogenous for the estimation of the output effect of fiscal corrections [9].

Insert Table XIV here

Table XIV illustrates how the plan is reclassified by DeVries et Al. and R&R using different sources. OBRA - 1990 plans fiscal adjustment both on revenue and expenditure side over the period 1991-1995. R&R concentrate only on the revenue adjustment and lump in the first quarter of 1991 all the relevant adjustment (that therefore adds up adjustment to be implemented in 1991 and 1992), the post 1992 adjustment are not included because of their small size. "...almost all the revenue provisions were effective January 1, 1991. Thus the first full fiscal year the changes were scheduled to be in effect was fiscal 1992. We therefore use the estimated revenue effect from the Budget for that year as our revenue estimate. That is, we estimate

[6]In 1980 the Crude Oil Windfall Profit Tax Act was signed. It is scheduled as a series of tax increases. However, such reforms were not due to deficit driven reason but for long-run growth reasons.

[7]Budgets 1985, 1987, 1989, 1991 provide revision estimates.

[8]Romer and Romer 2010 scale their fiscal shocks by the nominal GDP in the year at the time of the change

[9]Difference in the table relative to Devries et al. 2013 is due to two facts. First the scaling is done using the GDP of the year prior to consolidation. Second, to be consistent with Alesina et al. only for the revenue part we use the CBO 1998 document Projecting Federal Tax Revenues and the Effects of Changes in the tax Law, p.31 (the difference is very small and does not influence main results of the paper)

that there was a tax increase of \$35.2 billion in 1991Q1..." Devries et al. 2013 after the reclassification from fiscal to calendar year, use the implementation rather than the announcement as a criterion to attribute shocks to each period[10].

Table XV illustrates reclassification of shocks in to the fiscal adjustment plans that identifies separately the announced and implemented shocks.

Insert Table XV here

4.2 Industrial Networks

4.2.1 I-O matrices

The generic element of the input-output matrix A is constructed as follows:

$$a_{ij} = \frac{p_j \cdot x_{ij}}{p_i \cdot y_i} = \frac{\text{SALES}_{j \to i}}{\text{SALES}_i}$$

where x_{ij} is the quantity of good employed by sector i and supplied by industry j. We used the industry-by-industry total requirement table of year 1997[11], provided by the Bureau of Economic Analysis (BEA), to construct the empirical counterpart of matrix A. In sub-section 4.2.4, we explain in depth all the steps required to construct matrix A starting from the raw data.

For the sake of clarity, let's assume the number of sectors, n, to be equal to 3; thus, we have:

$$A = \begin{bmatrix} a_{11} = \dfrac{\text{SALES}_{1\to 1}}{\text{SALES}_1} & a_{12} = \dfrac{\text{SALES}_{2\to 1}}{\text{SALES}_1} & a_{13} = \dfrac{\text{SALES}_{3\to 1}}{\text{SALES}_1} \\[2ex] a_{21} = \dfrac{\text{SALES}_{1\to 2}}{\text{SALES}_2} & a_{22} = \dfrac{\text{SALES}_{2\to 2}}{\text{SALES}_2} & a_{23} = \dfrac{\text{SALES}_{3\to 2}}{\text{SALES}_2} \\[2ex] a_{31} = \dfrac{\text{SALES}_{1\to 3}}{\text{SALES}_3} & a_{32} = \dfrac{\text{SALES}_{2\to 3}}{\text{SALES}_3} & a_{33} = \dfrac{\text{SALES}_{3\to 3}}{\text{SALES}_3} \end{bmatrix},$$

The elements of a generic row, i, represent the inputs that industry i employs in its

[10] R&R propose several measures of the tax adjustments, generated respectively by including or not the retroactive components of the measures. There are no cases of retroactive components in deficit driven adjustments, and the retroactive components of a long run do not affect our measure of revenue adjustments.

[11] We assume the elements a_{ij} to be constant over time. The assumption is backed by empirical evidence, witnessing stable industrial linkages. We address this issue thoroughly in sub-section 4.2.4.

production. Basically, they reflect the transfers from suppliers of inputs to industry i. For this reason it can be described as the downstream matrix: any change which affects the supplying industries, should propagate downward to hit the customer industry i (from suppliers to customers: downstream propagation).
On the contrary, if we take the Hadamard product of matrix A and a scaling matrix Σ, we obtain the upstream matrix $\hat{A}$:

$$\hat{A} = A \circ \Sigma = \begin{bmatrix} \frac{\text{SALES}_{1\to1}}{\text{SALES}_1} & \frac{\text{SALES}_{2\to1}}{\text{SALES}_1} & \frac{\text{SALES}_{3\to1}}{\text{SALES}_1} \\ \frac{\text{SALES}_{1\to2}}{\text{SALES}_2} & \frac{\text{SALES}_{2\to2}}{\text{SALES}_2} & \frac{\text{SALES}_{3\to2}}{\text{SALES}_2} \\ \frac{\text{SALES}_{1\to3}}{\text{SALES}_3} & \frac{\text{SALES}_{2\to3}}{\text{SALES}_3} & \frac{\text{SALES}_{3\to3}}{\text{SALES}_3} \end{bmatrix} \circ \begin{bmatrix} 1 & \frac{\text{SALES}_1}{\text{SALES}_2} & \frac{\text{SALES}_1}{\text{SALES}_3} \\ \frac{\text{SALES}_2}{\text{SALES}_1} & 1 & \frac{\text{SALES}_2}{\text{SALES}_3} \\ \frac{\text{SALES}_3}{\text{SALES}_1} & \frac{\text{SALES}_3}{\text{SALES}_2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\text{SALES}_{1\to1}}{\text{SALES}_1} & \frac{\text{SALES}_{2\to1}}{\text{SALES}_2} & \frac{\text{SALES}_{3\to1}}{\text{SALES}_3} \\ \frac{\text{SALES}_{1\to2}}{\text{SALES}_1} & \frac{\text{SALES}_{2\to2}}{\text{SALES}_2} & \frac{\text{SALES}_{3\to2}}{\text{SALES}_3} \\ \frac{\text{SALES}_{1\to3}}{\text{SALES}_1} & \frac{\text{SALES}_{2\to3}}{\text{SALES}_2} & \frac{\text{SALES}_{3\to3}}{\text{SALES}_3} \end{bmatrix}.$$

The generic column j of matrix $\hat{A}$ contains the sales of sector j to sector i. Basically, every column represents the transfers from sector j to its customers i. For this reason matrix $\hat{A}$ is said to be the upstream matrix: sector j is now the supplier while the other industries are the customers.

4.2.2 Value Added

In our baseline specification the dependent variable Δy_{it}, is industry value added. Value added is the difference between an industry's or an establishment's total output and the cost of its intermediate inputs. It equals gross output (sales or receipts and other operating income, plus inventory change) minus intermediate inputs (consumption of goods and services purchased from other industries or imported). Value added consists of compensation of employees, taxes on production and imports less subsidies (formerly indirect business taxes and non tax payments), and gross operating surplus (formerly "other value added").

We employ annual data from the BEA database; in particular, we choose industry value added at the disaggregation level of 15 sectors. Notice that 15 sectors is somehow an optimal disaggregation level for our purpose: it allows us to break down the US economy in its main industrial sectors, but at the same time it is not a too "thin" decomposition, letting us capture fluctuations due to macroeconomic shocks like fiscal adjustment plans. Not surprisingly, our results do not hold anymore as we employ the BEA database with 65 industries. In fact, our results do hold for EB plans in this case, but they do not hold for TB plans. This might be explained by the fact that TB plans are on aggregate level, while EB plans are sector specific.

4.2.3 Spatial variables

The combination of value added and the I-O matrices allows us to construct two spatial variables: the ΔY^{up} - the upstream spatial variable, which captures the propagation of shocks from customers up to their suppliers - and the ΔY^{down} - the downstream spatial variable, which captures the propagation of shocks from suppliers down to their customers. Analytically we have:

$$
\begin{aligned}
\Delta y_{i,t}^{down} &= \sum_{j \neq i}^{n} a_{ij} \cdot \Delta y_{j,t} \\
\underset{n\times 1}{\Delta \mathbf{y}_t^{down}} &= \underset{n\times n}{\begin{bmatrix} 0 & a_{12} & \dots & a_{1n} \\ a_{21} & 0 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & 0 \end{bmatrix}} \cdot \underset{n\times 1}{\begin{bmatrix} \Delta y_{1,t} \\ \Delta y_{2,t} \\ \vdots \\ \Delta y_{n,t} \end{bmatrix}} \\
\Delta \mathbf{y}_t^{down} &= A_0 \cdot \Delta \mathbf{y}_t,
\end{aligned}
$$

where $\Delta y_{j,t}$ is the value added percent change of sector j in year t and A_0 corresponds to matrix A with zeros on its main diagonal. As anticipated in the Section 3, we exclude the main diagonal when constructing spatial variables: this strategy is typically implemented to avoid endogeneity issues.

On the other hand, the spatial upstream variable is constructed as follows:

$$\Delta y_{i,t}^{up} = \sum_{j \neq i}^{n} \hat{a}_{ji} \cdot \Delta y_{j,t}$$

$$\underset{n\times 1}{\Delta \mathbf{y}_t^{up}} = \underset{n\times n}{\begin{bmatrix} 0 & \hat{a}_{21} & \dots & \hat{a}_{n1} \\ \hat{a}_{12} & 0 & \dots & \hat{a}_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{a}_{1n} & \hat{a}_{2n} & \dots & 0 \end{bmatrix}} \cdot \underset{n\times 1}{\begin{bmatrix} \Delta y_{1,t} \\ \Delta y_{2,t} \\ \vdots \\ \Delta y_{n,t} \end{bmatrix}}$$

$$\Delta \mathbf{y}_t^{up} = \hat{A}_0^T \cdot \Delta \mathbf{y}_t,$$

where $\hat{A}_0^T$ corresponds to the transposed upstream matrix $\hat{A}$ with zeros on its main diagonal.
We also follow the SAR literature and we row-normalize both the downstream and upstream weight-matrices. [12]

4.2.4 Detailed derivations of the I-O matrix A

The Bureau of Economic Analysis (BEA) provides 4 requirement tables. In particular, we are interested in an industry by industry total requirement table.
The construction of the total requirement table is detailed over page 12-8 (page 8 of chapter 12) of the "Concepts and Methods of the U.S. Input-Output accounts" - a guide released by the BEA, which provides a full explanation of the industrial network data.
Consider a generic industry, say Z, whose total output is denoted with y. Since supply and demand must coincide, y is equal to F - final uses - plus x - demand from other industries which use the output of industry Z as input:

$$y = F + x.$$

Now we define the coefficient matrix A as:

$$A = \frac{x}{y},$$

that is, the share of industry Z output used as production input by the other industries.
Therefore, we have $x = A \cdot y$, and plugging it into the previous equation we have:

$$y = F + A \cdot y,$$

[12]Results with non-row-normalized data are robust and available in sub-section 6.3.

whose close-form solution is:

$$y = (I - A)^{-1} F = f(F).$$

In the I-O terminology used by the BEA, function f, which links final uses with the industry output, is called total requirement table. In economic theory we usually refer to such a transformation as the Leontief Inverse matrix.
In order to construct such a table, the BEA starts from storing raw data into two tables: the Make Table and the Use Table. The empirical counterpart of $(I - A)^{-1}$ is constructed in several steps, illustrated by the BEA guide.

The first step consists of reshaping the Use table, which is a non-symmetric commodity-by-industry table. The Use table shows the uses of commodities by intermediate and final users. Differently from the Make table, the rows in the Use table present the commodities or products, and the columns display the industries and final users that utilize them. The sum of the entries in a row is the output of that commodity. The columns show the products consumed by each industry and the three components of value added, compensation of employees, taxes on production and imports less subsidies, and gross operating surplus. Value added is the difference between an industry's output and the cost of its intermediate inputs. Total value added is equal to GDP. The sum of the entries in a column is that industry's output. We can derive the analytic form of the Use table, by introducing a specific terminology:

- INP_j^i = Commodity j used as input by industry i. This is the generic element of the Use table.
- SALES_j = Total output of industry j,

we rewrite the generic element of the Use table - assuming for simplicity that the number of commodities and industries is three ($n = 3$) - in this way:

$$\mathrm{USE} = \begin{bmatrix} \mathrm{INP}_1^1 & \mathrm{INP}_1^2 & \mathrm{INP}_1^3 \\ \\ \mathrm{INP}_2^1 & \mathrm{INP}_2^2 & \mathrm{INP}_2^3 \\ \\ \mathrm{INP}_3^1 & \mathrm{INP}_3^2 & \mathrm{INP}_3^3 \end{bmatrix}.$$

At this point we can derive a commodity by industry direct requirement table by dividing each industry's input by its corresponding total industry output. We denote such a matrix with letter B and we can express its generic element using the previous notation in this way:

$$B_{ij} = \frac{\text{INP}_i^j}{\text{SALES}_j},$$

where i denotes the row and j the column of matrix B. Therefore, the analytic form of matrix B is :

$$\text{B} = \begin{bmatrix} \dfrac{\text{INP}_1^1}{\text{SALES}_1} & \dfrac{\text{INPUT}_1^2}{\text{SALES}_{\ 2}} & \dfrac{\text{INP}_1^3}{\text{SALES}_3} \\ \\ \dfrac{\text{INP}_2^1}{\text{SALES}_1} & \dfrac{\text{INPUT}_2^2}{\text{SALES}_{\ 2}} & \dfrac{\text{INP}_2^3}{\text{SALES}_3} \\ \\ \dfrac{\text{INP}_3^1}{\text{SALES}_1} & \dfrac{\text{INPUT}_3^2}{\text{SALES}_{\ 2}} & \dfrac{\text{INP}_3^3}{\text{SALES}_3} \end{bmatrix}.$$

The BEA guide provides also a numerical example - with 3 industries ($n = 3$) - which we report here for the sake of clarity:

	1	2	3	Final demand	Total Commodity Output
1	50	120	120	40	330
2	180	30	60	130	400
3	50	150	50	20	270
Scrap	1	3	1	0	5
VA	47	109	34	/	190
Total Industry Output	328	412	265	190	/

Consider the first row: 50 units of commodity 1 are used by industry 1, 120 are used by industry 2 and 120 are used by industry 3; 40 units of commodity 1 are demanded as final product, therefore, the overall production of commodity 1 amounts to 50 plus 120 plus 120 plus 40: 330 units.
At the same time, we can derive the direct requirement table by following the instructions explained above:

	1	2	3
1	0.152	0.291	0.453
2	0.549	0.073	0.226
3	0.152	0.364	0.189
Scrap	0.003	0.007	0.004
VA	0.143	0.265	0.128
Total	1	1	1

The first element of the first row is obtained by dividing 50 by 328, for instance. The second element of the first row is obtained dividing 120 by 412 and so on and so forth.
By removing scrap and value added from the above table, we obtain a symmetric commodity-by-industry matrix, denoted with B, whose generic elements are described above:

$$B = \begin{bmatrix} 0.152 & 0.291 & 0.453 \\ 0.549 & 0.073 & 0.226 \\ 0.152 & 0.364 & 0.189 \end{bmatrix}.$$

At this point put aside for a while the direct requirement matrix just derived, and focus on the Make table, which shows the production of commodities by industries. The rows present the industries, and the columns display the commodities that the industries produce. Looking across a row, all the commodities produced by that industry are identified, and the sum of the entries is that industry's output. Looking down a column, all the industries producing that commodity are identified, and the sum of the entries is the output of that commodity.
As we did previously, we now introduce a useful notation, which allows to better interpret what we are computing:

- Y_j = Total production of commodity j.
- OUT_j^i = Commodity j produced by industry i
- NSR_i^{-1} = The inverse of the non-scrap ratio of industry i,

The analytical form of the Make table is the following:

$$\text{MAKE} = \begin{bmatrix} \text{OUT}_1^1 & \text{OUT}_2^1 & \text{OUT}_3^1 \\ \\ \text{OUT}_1^2 & \text{OUT}_2^2 & \text{OUT}_3^2 \\ \\ \text{OUT}_1^3 & \text{OUT}_2^3 & \text{OUT}_3^3 \end{bmatrix}$$

At this point, we divide each row for the total commodity output to obtain the market share matrix, which shows the proportion of commodity output produced by each industry, whose analytic form is the following:

$$\text{MS} = \begin{bmatrix} \frac{\text{OUT}_1^1}{Y_1} & \frac{\text{OUT}_2^1}{Y_2} & \frac{\text{OUT}_1^3}{Y_3} \\ \\ \frac{\text{OUT}_1^2}{Y_1} & \frac{\text{OUT}_2^2}{Y_2} & \frac{\text{OUT}_3^2}{Y_3} \\ \\ \frac{\text{OUT}_1^3}{Y_1} & \frac{\text{OUT}_2^3}{Y_2} & \frac{\text{OUT}_3^3}{Y_3} \end{bmatrix}$$

Again, we show a sample Make table, with $n = 3$:

	1	2	3	Scrap	Total Industry Output
1	300	25	0	3	328
2	30	360	20	2	4412
3	0	15	250	0	265
Total Commodity Output	330	400	270	5	/

Consider the first column which corresponds to industry 1 output: industry 1 makes 300 of commodity 1, 30 of commodity 2 and it does not produce commodity 3. Overall, industry 1 makes 300 plus 30 plus 0, 330 of total commodity output. Following the instructions described above, we derive the market share table:

	1	2	3
1	0.909	0.063	0
2	0.091	0.900	0.074
3	0	0.038	0.926
Total Commodity Output	1	1	1

The third step is to make adjustments for scrap. The I-O accounts include a commodity for scrap, which is a byproduct of industry production. No industry produces scrap on demand; rather, it is the result of production to meet other demands. In order to make the I-O model work correctly - that is, not requiring industry output because of a demand for scrap inputs- we have to eliminate scrap as a secondary product. At the same time, we must also keep industry output at the same level. This adjustment is accomplished by calculating the ratio of non-scrap output to industry output for each industry and then applying these ratios to the market shares matrix in order to account for total industry output. More precisely, the non-scrap ratio is defined as follows:

$$(\text{Non-scrap ratio})_i = \frac{\text{Industry i output}}{\text{Industry i output - scrap i}} = \text{NSR}_i$$

Therefore, using the numbers from the previous example, we have:

	Tot.Ind.Out.	Scrap	Δ	Non-Scrap Ratio
1	328	3	325	0.991
2	412	2	410	0.995
3	265	0	265	1

The market shares matrix is adjusted for scrap by dividing each row coefficient by the non-scrap ratio for that industry. In the resulting transformation matrix, called W, the implicit commodity output of each industry has been increased.
We might write the generic element of the adjusted market share matrix W in this way:

$$(\text{Market share adjusted})_{ij} = W_{ij} = \frac{\text{OUT}_j^i \cdot NSR_i^{-1}}{Y_j},$$

whose analytical form is:

$$\text{W} = \begin{bmatrix} \dfrac{\text{OUT}_1^1 \cdot NSR_1^{-1}}{Y_1} & \dfrac{\text{OUT}_2^1 \cdot NSR_1^{-1}}{Y_2} & \dfrac{\text{OUT}_3^1 \cdot NSR_1^{-1}}{Y_3} \\[2ex] \dfrac{\text{OUT}_1^2 \cdot NSR_2^{-1}}{Y_1} & \dfrac{\text{OUT}_2^2 \cdot NSR_2^{-1}}{Y_2} & \dfrac{\text{OUT}_3^2 \cdot NSR_2^{-1}}{Y_3} \\[2ex] \dfrac{\text{OUT}_1^3 \cdot NSR_3^{-1}}{Y_1} & \dfrac{\text{OUT}_2^3 \cdot NSR_3^{-1}}{Y_2} & \dfrac{\text{OUT}_3^3 \cdot NSR_3^{-1}}{Y_3} \end{bmatrix}$$

The resulting transformation matrix W of our example is:

$$W = \begin{bmatrix} 0.917 & 0.063 & 0 \\ 0.091 & 0.904 & 0.074 \\ 0 & 0.038 & 0.926 \end{bmatrix}.$$

We now have all the elements to compute a symmetric direct requirement table. Recall now that the transformation matrix is an industry by commodity table, while the direct requirement table B is a commodity by industry table. Therefore, by multiplying them, we can construct a symmetric industry-by-industry direct requirement table, denoted with WB.

$$WB = \begin{bmatrix} \frac{\mathrm{OUT}_1^1 \cdot NSR_1^{-1}}{Y_1} & \frac{\mathrm{OUT}_2^1 \cdot NSR_1^{-1}}{Y_2} & \frac{\mathrm{OUT}_3^1 \cdot NSR_1^{-1}}{Y_3} \\ \frac{\mathrm{OUT}_1^2 \cdot NSR_2^{-1}}{Y_1} & \frac{\mathrm{OUT}_2^2 \cdot NSR_2^{-1}}{Y_2} & \frac{\mathrm{OUT}_3^2 \cdot NSR_2^{-1}}{Y_3} \\ \frac{\mathrm{OUT}_1^3 \cdot NSR_3^{-1}}{Y_1} & \frac{\mathrm{OUT}_2^3 \cdot NSR_3^{-1}}{Y_2} & \frac{\mathrm{OUT}_3^3 \cdot NSR_3^{-1}}{Y_3} \end{bmatrix} \cdot \begin{bmatrix} \frac{\mathrm{INP}_1^1}{\mathrm{SALES}_1} & \frac{\mathrm{INPUT}_1^2}{\mathrm{SALES}_2} & \frac{\mathrm{INP}_1^3}{\mathrm{SALES}_3} \\ \frac{\mathrm{INP}_2^1}{\mathrm{SALES}_1} & \frac{\mathrm{INPUT}_2^2}{\mathrm{SALES}_2} & \frac{\mathrm{INP}_2^3}{\mathrm{SALES}_3} \\ \frac{\mathrm{INP}_3^1}{\mathrm{SALES}_1} & \frac{\mathrm{INPUT}_3^2}{\mathrm{SALES}_2} & \frac{\mathrm{INP}_3^3}{\mathrm{SALES}_3} \end{bmatrix}$$

The generic element of matrix WB is the following:

$$WB_{ij} = \sum_{s=1}^{3} W_{is} \cdot B_{sj}$$

For instance let's derive the analytic form of the second element of the first row:

$$WB_{12} = \frac{\frac{\mathrm{OUT}_1^1 \cdot NSR_1^{-1}}{Y_1} \cdot \mathrm{INP}_1^2 + \frac{\mathrm{OUT}_2^1 \cdot NSR_1^{-1}}{Y_2} \cdot \mathrm{INP}_2^2 + \frac{\mathrm{OUT}_3^1 \cdot NSR_1^{-1}}{Y_3} \cdot \mathrm{INP}_3^2}{SALES_2}$$

$$= \frac{\mathrm{SALES}_{1 \to 2}}{\mathrm{SALES}_2}.$$

Again, let's derive another element: WB_{21}:

$$WB_{21} = \frac{\frac{\text{OUT}_1^2 \cdot NSR_2^{-1}}{Y_1} \cdot \ \text{INP}_1^1 + \frac{\text{OUT}_2^2 \cdot NSR_2^{-1}}{Y_2} \cdot \text{INP}_2^1 + \frac{\text{OUT}_3^2 \cdot NSR_2^{-1}}{Y_3} \cdot \text{INP}_3^1}{SALES_1}$$

$$= \frac{\text{SALES}_{2\to 1}}{\text{SALES}_1}.$$

Therefore, the analytic form of matrix WB is:

$$WB = \begin{bmatrix} \frac{\text{SALES}_{1\to 1}}{\text{SALES}_1} & \frac{\text{SALES}_{1\to 2}}{\text{SALES}_2} & \frac{\text{SALES}_{1\to 3}}{\text{SALES}_3} \\ \frac{\text{SALES}_{2\to 1}}{\text{SALES}_1} & \frac{\text{SALES}_{2\to 2}}{\text{SALES}_2} & \frac{\text{SALES}_{2\to 3}}{\text{SALES}_3} \\ \frac{\text{SALES}_{3\to 1}}{\text{SALES}_1} & \frac{\text{SALES}_{3\to 2}}{\text{SALES}_2} & \frac{\text{SALES}_{3\to 3}}{\text{SALES}_3} \end{bmatrix}.$$

Notice that this matrix coincide with our theoretical matrix A transposed. Recall in fact that from the profit maximization problem we obtained:

$$a_{ij} = \frac{p_j \cdot x_{ij}}{p_i \cdot y_i} = \frac{\text{SALES}_{j\to i}}{\text{SALES}_i}$$

where x_{ij} is the quantity of good employed by sector i and supplied by industry j (as usual, i is the number of the row while j the number of the column):

$$A = \begin{bmatrix} a_{11} = \frac{\text{SALES}_{1\to 1}}{\text{SALES}_1} & a_{12} = \frac{\text{SALES}_{2\to 1}}{\text{SALES}_1} & a_{13} = \frac{\text{SALES}_{3\to 1}}{\text{SALES}_1} \\ a_{21} = \frac{\text{SALES}_{1\to 2}}{\text{SALES}_2} & a_{22} = \frac{\text{SALES}_{2\to 2}}{\text{SALES}_2} & a_{23} = \frac{\text{SALES}_{3\to 2}}{\text{SALES}_2} \\ a_{31} = \frac{\text{SALES}_{1\to 3}}{\text{SALES}_3} & a_{32} = \frac{\text{SALES}_{2\to 3}}{\text{SALES}_3} & a_{33} = \frac{\text{SALES}_{3\to 3}}{\text{SALES}_3} \end{bmatrix}.$$

Therefore, the following identity is true:

$$A = (WB)^T.$$

The above identity is crucial, since it shows that there is a discrepancy between the theoretical Input-Output matrix and the empirical one. We have to pay lot of attention when working with these data in order not to forget to take the transpose of the empirical matrix, in order to replicate what theory suggests.

At this point, we can finally derive the ultimate table, which is the one available for downloading on the BEA website: the total requirement table industry-by-industry. In the BEA guide they provide computations for obtaining the commodity-by-commodity total requirement table (they do B times W rather than W times B), however at page 24 of chapter 12 they show the formulas they employ to derive the industry-by-industry total requirement table, which we indicate with TR:

$$TR = (I - WB)^{-1},$$

which shows the industry output required per dollar of each industry product delivered to final users.
At this point a little of matrix algebra turns out useful. Consider a $n \times n$ matrix $S = I_n - A$. Since the transpose of an invertible matrix is also invertible, and its inverse is the transpose of the inverse of the original matrix, we can write the following:

$$(S^T)^{-1} = (S^{-1})^T.$$

Morevoer, the following identity holds:

$$S^T = (I - A)^T = I - A^T,$$

then:

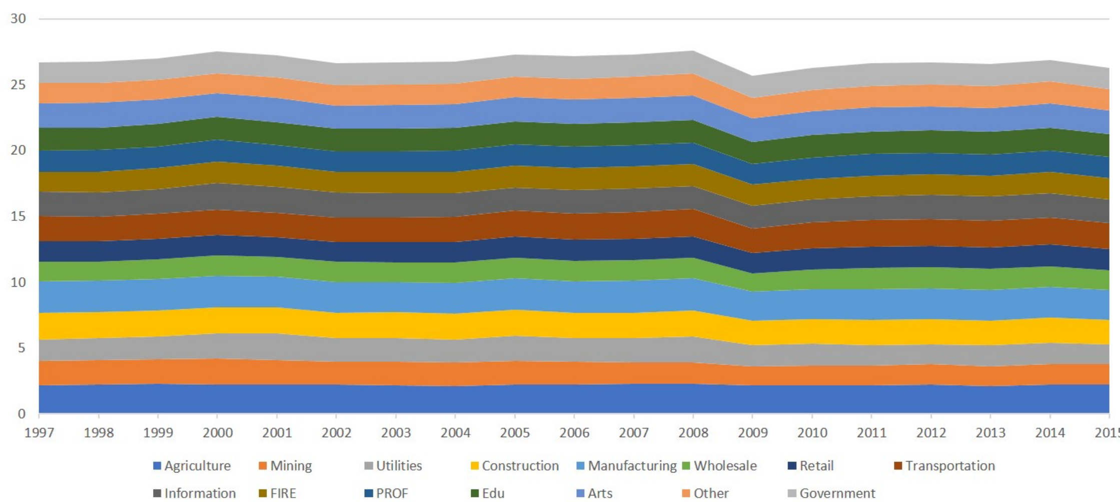
$$\begin{aligned} TR &= (I - WB)^{-1} \\ &= (I - A^T)^{-1} \\ &= (S^T)^{-1} \\ &= (S^{-1})^T \\ &= \left((I - A)^{-1}\right)^T \\ &= H^T \end{aligned}$$

Matrix $H = (I - A)^{-1}$ is exactly our theoretical Leontief inverse matrix: the industry-by-industry total requirement table available for free-downloading on the BEA website, coincide with the **transposed** Leontief inverse matrix. At this point, we are able to define a transformation which allows us to pass from the row data to the empirical counterpart of the theoretical I-O matrix A:

$$A = f(TR_{BEA}) = I_n - \left[(TR_{BEA})^T\right]^{-1}, \tag{33}$$

where, A is therefore a function of the row data TR_{BEA}, the 15 industry-by-industry total requirement table. By taking the transpose of the original table the empirical counterpart of matrix A is obtained.

The last issue to discuss is the following: the industry-by-industry total requirement table's spreadsheet, contains 19 tables, one per each year since 1997 to 2015 (estimates are yearly updated). Which one to use?
We choose the mid sample table of table of 1997.
to check for the potential relevance of this choice we computed total requirement for every industry and year, by simply summing up the columns of the total requirement tables, and then we looked at the evolution of these values over time. Results are shown in the figure below:



The values are substantially stable over time pointing to stable connections among industries over time.

4.2.5 Industrial Network Breakdown

At this point the reader might wonder what is the effect of a fiscal adjustment on every sector and what is its specific industrial propagation effect. For instance, think about the possibility for a policy maker to devise a sector specific fiscal adjustment.

Is it possible for Her to minimize the recessionary effect of it by choosing a sector rather than another? Or at least to target sectors with common features which can minimize the propagation effect? What are the sectors more vulnerable?

To answer these questions we need to analyze our network. We employ a slightly modified version of the Bonacich Centrality measure, which involves somehow the Leontief inverse matrix. This is not surprising, given what seen before: every element of the Leontief inverse corresponds to the infinite sum of the propagation (feedback effect). If a sector is weakly connected with all the others, also its feedback effect will be small, by consequence we could consider the sector not central to the network. The spatial coefficient represents the strength of the transmission channel: if smaller than one in absolute value, convergence of the geometric sum is preserved:

$$\text{Centrality} = I_n + \beta \cdot A + \beta^2 \cdot A^2 + ... = (I_n - \beta \cdot A)^{-1}.$$

We also remove "the one" from the main diagonal of the Leontief inverse, in order to remove the direct effect: recall from Section 3, that the elements on the main diagonal could be rewritten as "one" plus another term representing the feedback. Therefore, our centrality measures have the form of:

$$\text{Centrality} = (I_n - \beta \cdot A)^{-1} - I_n$$

Summing up the elements on row i, we obtain the effects triggered by all other sectors on sector i. On the contrary, summing the elements on column i gives the propagation effect triggered by sector i on all other sectors.

In particular, we construct four centrality measures:

$$\begin{aligned}
\underset{n\times 1}{\text{Passive Customerness}} &= (I_n - \hat{\beta}^{down} \cdot A)^{-1} - I_n) \cdot \mathbf{1}_n \\
\underset{n\times 1}{\text{Passive Supplierness}} &= (I_n - \hat{\beta}^{up} \cdot \hat{A}^T)^{-1} - I_n) \cdot \mathbf{1}_n \\
\underset{n\times 1}{\text{Active Supplierness}} &= (I_n - \hat{\beta}^{down} \cdot A)^{-1} - I_n)^T \cdot \mathbf{1}_n \\
\underset{n\times 1}{\text{Active Customerness}} &= (I_n - \hat{\beta}^{up} \cdot \hat{A}^T)^{-1} - I_n)^T \cdot \mathbf{1}_n
\end{aligned}$$

where matrices A and $\hat{A}$ account for the non-row-stochastic I-O matrices, and β^{down} and β^{up} represent the ML estimates of the spatial coefficients.

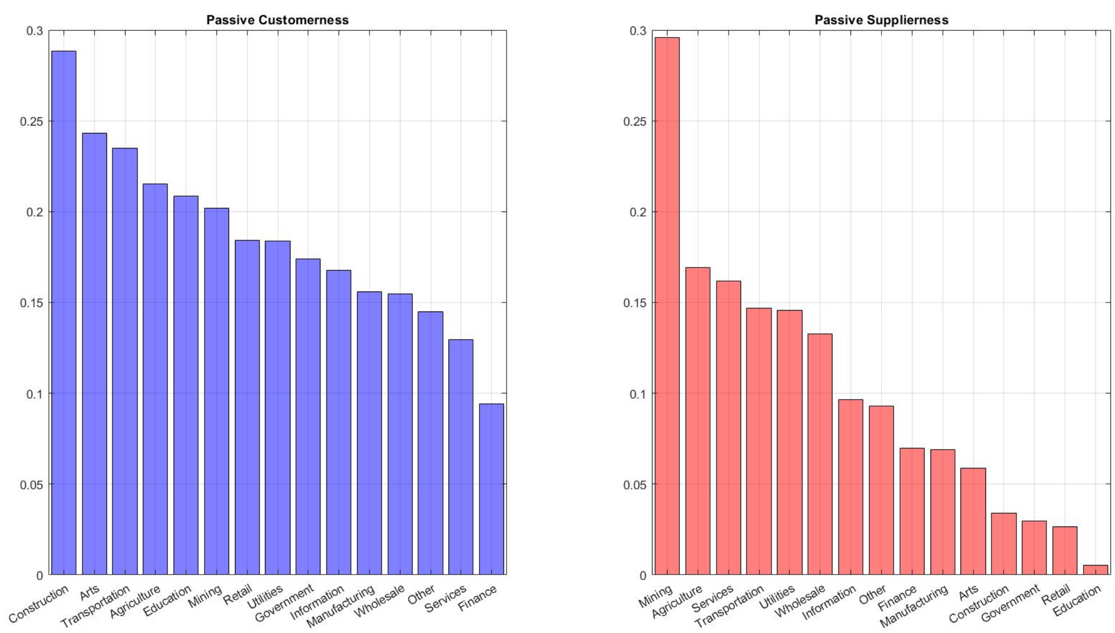
The four centrality measures represent respectively:

- *Passive Customerness*: how much industry i is harmed by its suppliers (downstream propagation).
- *Passive Supplierness*: how much industry i is harmed by its customers (upstream propagation).
- *Active Supplierness*: how much industry i harms its customers (downstream propagation).
- *Active Customerness*: how much industry i harms its suppliers (upstream propagation).

The first two centrality measures are considered "passive" indicators, since they represent the effect of a change in customers/suppliers output on an industry. On the contrary, the last two indicators account for an "active" measure, since they represent the effect of a change in an industry's output on all the other customers/suppliers.

Figure 3 shows in the left panel the Passive Customerness, while in the right one the Passive Supplierness.

Figure 3: Passive Customerness and Supplierness

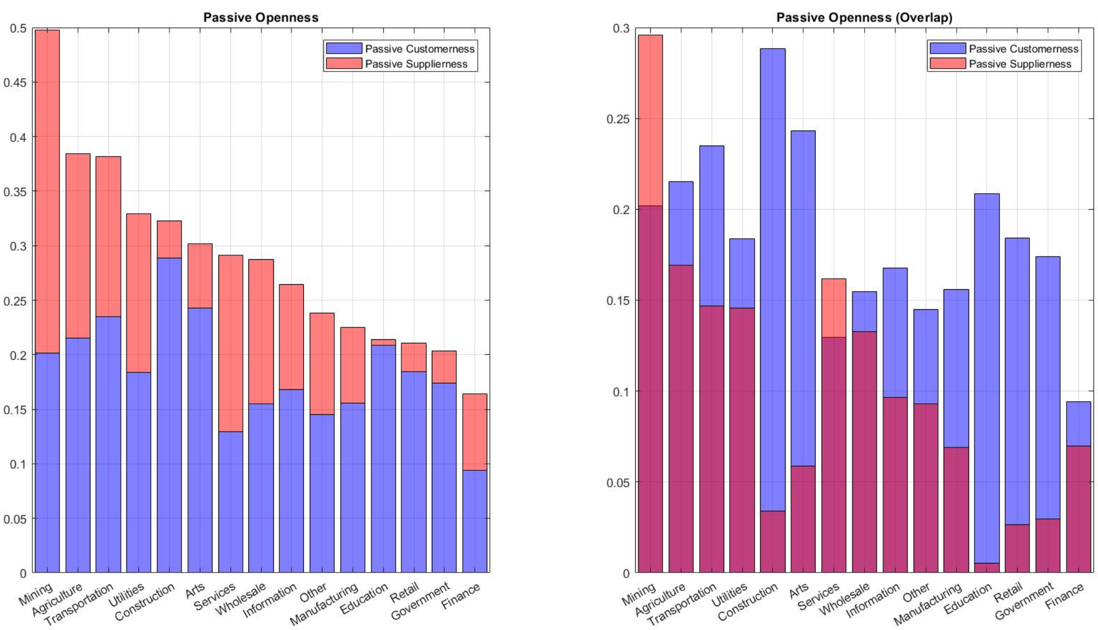


To measure the overall centrality of the sector we also constructed a new measure, which should describe the degree of interconnection (for this reason we named it "Openness") with the other sectors.

$$\underset{n\times 1}{\text{Passive Openness}} = \text{Passive Customerness} + \text{Passive Supplierness}.$$

Figure 4 shows on the left panel the Passive Openness, while on the right one it shows the overlap of the Passive Customerness and Passive Supplierness (both in absolute values).

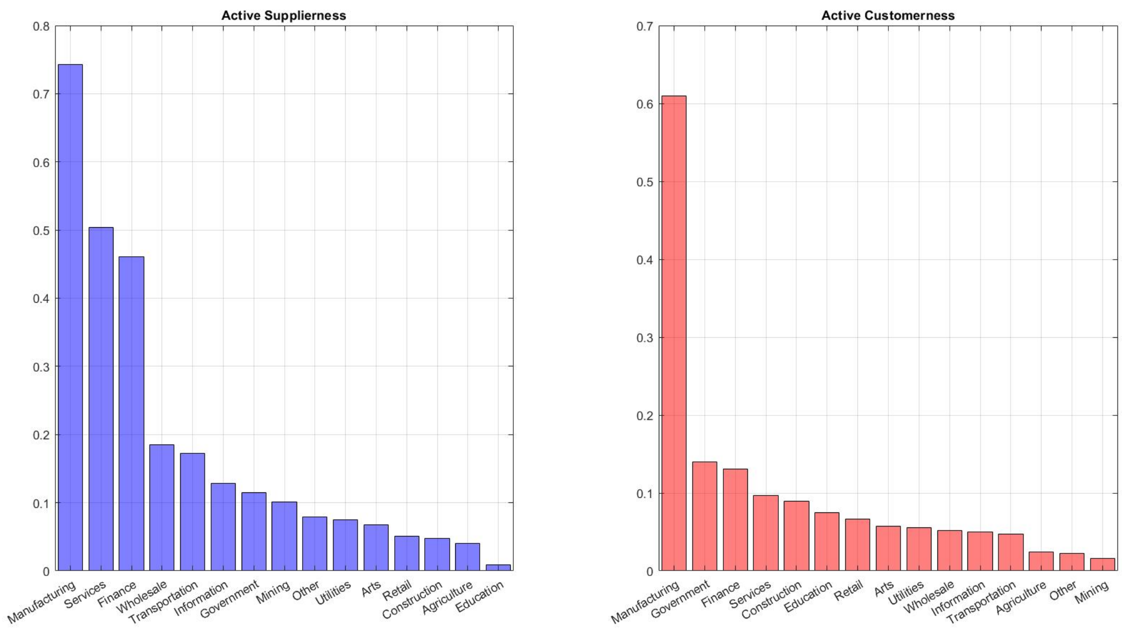
Figure 4: Passive Openness



Looking at the right panel, it is possible to notice that except for Mining and Services, Customerness overcomes by far Supplierness. For this reason we have a stronger downstream propagation of fiscal shocks than upstream, which can help explain why tax based adjustments, have a stronger recessive effect.
Overall, Mining, Agriculture and Transportation seem to be the most damaged sectors from a fiscal adjustment plan, making them the more vulnerable sectors; while Retail, Government and Finance are the least affected ones.

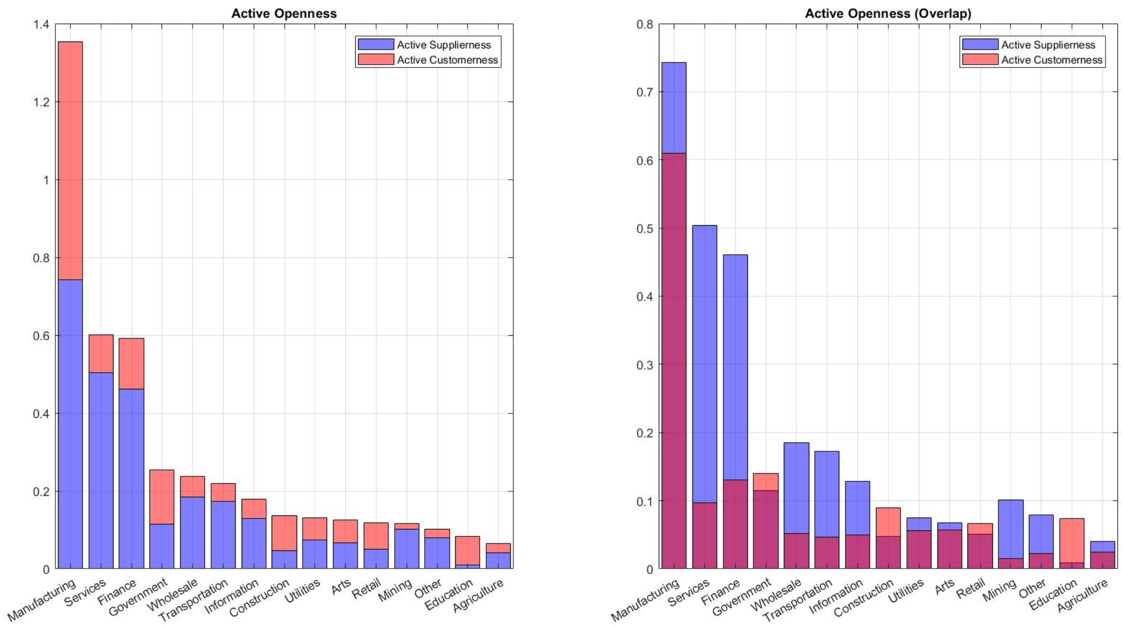
Furthermore, it might seem reasonable that targeting more isolated sectors should prevent the negative shock to spread around the economy. Figure 5 shows the Active Supplierness and Customerness in the left and right panel respectively. It is interesting to notice that Manufacturing, Services and Finance account for the most of the downstream propagation. The situation is even exacerbated for the Active Customerness, where Manufacturing is responsible for the most of the transmission through the upstream channel.

Figure 5: Active Customerness and Supplierness



Symmetrically, we construct the 6th indicator: the "Active Openness", which is reported in Figure 6:

Figure 6: Active Customerness and Supplierness



We see that Manufacturing is absolutely an explosives' fuse, which might trigger a very strong propagation effect; also Services and Finance are sectors capable of strongly propagating shocks towards the rest of the economy. On the other hand, Agriculture and Education are the sectors less dangerous in terms of shock propagation, by consequence it would be convenient for the policy maker to implement sector specific shocks towards them, rather than hitting Manufacturing (even if tailored fiscal policies might not be politically implementable for equality reasons).
Such a conclusion might be correct so as it might not. In fact, our estimates are based on generic fiscal adjustments and not on specific sectors.
Vice-versa, since we constructed industry specific spending shocks, we might argue that a federal government spending cut towards Agriculture, Other Services and Mining could have less harmful effects in terms of upstream propagation, while we would discourage the policy maker to cut Manufacturing spending (like the military expenditure) to avoid very negative and strong spillovers.
To conclude this descriptive statistics section, we report two scatter-plots, one for downstream and the other one for upstream, which plot the sectors into a plane: on the horizontal axis it is reported the passive measure of propagation (passive customerness and passive supplierness for downstream and upstream propagation

mechanism respectively) indicated with the term "**vulnerability**", while on the vertical axis we report the active measure of propagation (active supplierness and active customerness for downstream and upstream propagation mechanism respectively) indicated with the term "**harmfulness**" (for this reason we named them V-H graph). It is interesting to notice from both Figure 7 and 8 the existence of a negative relationship between sector's harmfulness and vulnerability.

Figure 7: **V-H graph** - Downstream Propagation

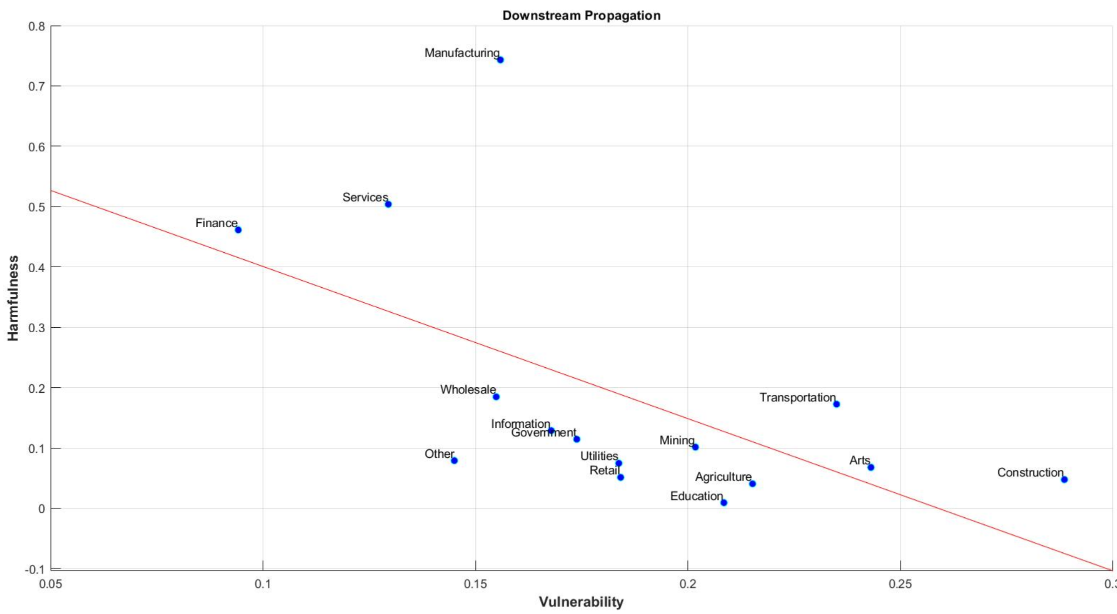
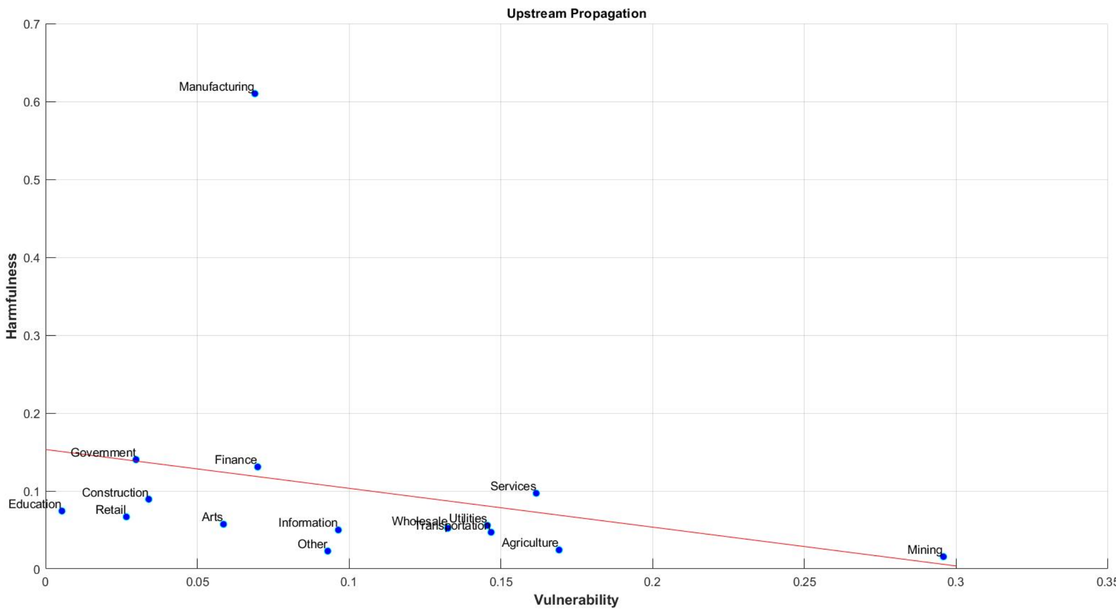


Figure 8: **V-H graph** - Upstream Propagation



5 Empirical Results

We bring the following empirical model to the data:

$$\Delta y_{i,t} = c_i + \left(\beta^{down} \cdot \Delta y_{i,t}^{down} + \delta^u \cdot e_t^u + \delta^a \cdot e_{t,0}^a + \delta^f \cdot e_t^f\right) \cdot TB_t + \\ + \left(\beta^{up} \cdot \Delta y_{i,t}^{up} + \gamma^u \cdot e_{i,t}^u + \gamma^a \cdot e_{i,t,0}^a + \gamma^f \cdot e_{i,t}^f\right) \cdot EB_t. \tag{34}$$

equation (37) is a panel specification that allows to track the effect on output growth of EB and TB based fiscal plans. We model the multi-period structure of fiscal plans by allowing separate coefficients on the unexpected, announced and future components of the fiscal adjustments. Total adjustments are separated into their three components and each component is allowed to a have a different impact on output growth. To keep the specification parsimonious the future announced component is identified as the sum of future announcements at all horizons:

$$e_t^f = \sum_{j=1}^{horz} e_{t,j}^a.$$

TB adjustments do not have a heterogeneous effect sector by sector: a tax shock impact on every industry is the same. However, since the purchases of government goods and services differ across sectors, we assume EB adjustments to impact each industry in an idiosyncratic way.

Such a heterogeneity is modelled following Acemoglu, Akcigit, and Kerr 2016 who weigh the spending adjustments using the input-output matrix to construct industry specific spending shocks. In particular, we pre-multiply each spending shock by the elements of the last row of matrix $\hat{A}_0$, as the n^{th} row of the transformed input-output matrix corresponds to government sector.

The generic element of the n^{th} row of matrix $\hat{A}_0$, indicated with ω_j, is:

$$\omega_j = \frac{Sales_{j \to G}}{Sales_j}, \quad j \neq n$$

which is used to scale the EB adjustments to find their sectorial counterpart:

$$e_{j,t}^u = \omega_j \cdot e_t^u$$

$$e_{j,t,0}^a = \omega_j \cdot e_{t,0}^a$$

$$e_{j,t}^f = \omega_j \cdot e_t^f$$

We estimate the coefficients of the model following the procedure outlined by J. LeSage and Pace 2009. In particular, we develop a Maximum Likelihood Estimator[13] for our panel specification. In order to keep into account the different volatility of the sectors, we make our MLE heteroscedasticity robust. For instance, the mining and agriculture sector exhibit a huge and different volatility, relative to all the remaining sectors.

Maximum Likelihood estimates[14] are shown in Table II:

Insert Table II here

J. LeSage and Pace 2009 also recommends to estimate the coefficients of the model via Bayesian MCMC when addressing heteroscedasticity. Table III reports the estimates of the coefficients when adopting this alternative approach.[15] Since the posterior distribution of the spatial coefficients is unknown and non-normal, we report the mean, the standard deviation, the t-statistic (even if these should be treated with care, because of slight non-normal shape[16] and quantiles (the most reliable descriptive statistic):

Insert Table III here

Notice that the two estimation methods deliver almost identical results like in J. P. LeSage 1997 in the standard homoscedastic SAR case.

After estimation we compute the output effect of fiscal policy by simulating a combination of the unexpected and future component of a fiscal adjustment that replicates the average in-sample TB and EB fiscal plans in US data of the size of 1% of GDP. For TB plans, the unexpected component is 11.5% and the future component is 88.5%. For EB plans the unexpected component is 19.8% and the future component is 80.2%.

5.1 The Effect of Tax Based Adjustments

We report in Table IV the results of a Monte Carlo simulation of the output effects of TB adjustments. The left part of Table IV, reports results obtained using MLE, while the right side of Table IV shows the results obtained by drawing parameters from the posterior distributions obtained via Bayesian MCMC:

[13] In sub-section 5.3.1, we provide a full analytic derivation of the estimator.

[14] In Table II we do not report fixed effects and variances. All the estimates are illustrated in the additional tables sub-section (sub-section 8.1, Table XVI and Table XVII).

[15] See sub-section 5.3.1 for a detailed description of the estimation and simulation procedures.

[16] Histograms of the posteriors are reported in the Additional Figures Section (Section 9)

Insert Table IV here

Note that MLE-MC simulation delivers again similar results to Bayesian MCMC simulation. The average total effect of TB adjustment, is estimated at -2%. Furthermore, we carry out a shock-effect decomposition, as we outlined in Section 3: the average total effect is decomposed into the instantaneous, the network-direct and network-indirect effects. The instantaneous effect is a combination of the estimated coefficients on unexpected and future components of a TB plan, weighed by .115 and .885 respectively:

$$\phi_\delta = .115 \cdot \hat{\delta}^u + .885 \cdot \hat{\delta}^f.$$

The Average Total Effect (ATE) is the sum of the instantaneous effect, the average network direct effect (NDE) and the average network indirect effect (NIE). While the total network effect (TNE) is the summation of the NDE and NIE. Table V reports the breakdown of the TB adjustment effects, both in absolute and relative (to the ATE) terms:

Insert Table V here

The relevant finding here is that the TNE contributes for more than 50 per cent of the total output effect, suggesting the relevance of the industrial network in the propagation of the TB fiscal adjustments.

5.2 The Effect of Expenditure Based Adjustments

This subsection is the analogue of the previous one for EB adjustments. Table VI is the analogue of IV, and reports the results of the simulated effect of EB adjustments:

Insert Table VI here

Again, the two alternative methodologies deliver almost identical results.
As in the case of TB adjustments, the total effect is also decomposed into instantaneous (now with weights .198 and .802 respectively), network-direct and network indirect effects. Table VII, which is the analogue of Table V, reports the breakdown of the EB total effect:

Insert Table VII here

The magnitude of the effect is definitely smaller than the one of TB adjustments: -0.71% against -2%. The effect is also slightly less statistically significant (0.08 per cent versus 0.02 per cent). Overall, we find not only a smaller total effect of EB plans on the economy, but also a significantly lesser importance of the network: network propagation is accountable for 27% of the total effect when an EB plan occurs, against 52% of a TB plan.

5.3 Estimation and Simulation Procedure in Details

5.3.1 MLE and MC simulation

In this sub-section we explain in details the procedure adopted to estimate via Maximum Likelihood the model

$$\Delta y_{i,t} = c_i + \left(\beta^{down} \cdot \Delta y^d_{i,t} + \delta^u \cdot e^u_t + \delta^a \cdot e^a_{t,t} + \delta^f \cdot e^f_t\right) \cdot TB_t +$$
$$+ \left(\beta^{up} \cdot \Delta y^u_{i,t} + \gamma^u \cdot e^u_{i,t} + \gamma^a \cdot e^a_{i,t,t} + \gamma^f \cdot e^f_{i,t}\right) \cdot EB_T.$$

First of all, by subsuming the fixed effects and the fiscal shocks in a matrix called X and doing the same for their coefficients (we group them into a vector named β), we can use the following compact representation of our model:

$$\begin{aligned}
&(H_t)^{-1} \cdot \underset{n\times 1}{\Delta y_t} = \underset{n\times(n+6)}{X_t} \cdot \beta + \varepsilon_t \\
&(H_t)^{-1} = I_n - (\beta^{down} \cdot A_0 \cdot TB_t + \beta^{up} \cdot \hat{A}'_0 \cdot EB_t) \\
&\varepsilon_t \sim \mathcal{N}(0, \Omega), \forall t \in \{1, ..., T\} \\
&\Omega = diag(\sigma^2_1, ..., \sigma^2_n) \\
&\varepsilon_t \perp \varepsilon_{t+i}, \quad \forall t \in \{1, ..., T\}, \forall i \in \mathcal{Z}
\end{aligned}$$

LeSage&Pace (2009) shows how to implement the calculation of the Maximum Likelihood Estimator for such a model. However, our model specification differs slightly from theirs (the standard SAR framework). In particular, we have a panel dataset and the network is activated in different years according with some dummy variables: TB_t and EB_t.

In order to derive the log-likelihood of our model at time t let's start off by setting $(H_t)^{-1} \cdot \Delta y_t = Z_t$, we have that:

$$Z_t = (H_t)^{-1} \cdot \Delta y_t \sim \mathcal{N}(X_t\beta, \Omega),$$

Therefore we have

$$\Delta y_t \sim \mathcal{N}(H_t X_t \beta, H_t \Omega H'_t)$$

The density function of the random vector Δy_t is:

$$f(\underset{n\times 1}{\Delta y_t} | X_t, \rho, \beta, \Omega) = \frac{1}{\sqrt{(2\pi)^n \cdot |H_t \Omega H'_t|}} \exp\left\{ -\frac{1}{2} \cdot (\Delta y_t - H_t X_t \beta)' \cdot (H_t \Omega H'_t)^{-1} \cdot (\Delta y_t - H_t X_t \beta) \right\},$$

with $\rho = (\beta^{down} \quad \beta^{up})$.
Given that:

$$(H_t \Omega H_t')^{-1} = (H_t')^{-1} \cdot \Omega^{-1} \cdot H_t^{-1}$$

and

$$|H_t \Omega H_t'| = |H_t|^2 \cdot |\Omega|$$

we have:

$$\begin{aligned} f(\Delta y_t|\cdot) &= (2\pi)^{-n/2} \cdot |H_t|^{-1} \cdot |\Omega|^{-1/2} \cdot \exp\Big\{ -\frac{1}{2}(Z_t - X_t\beta)' \cdot H_t' \cdot (H_t')^{-1} \cdot \Omega^{-1} \cdot H_t^{-1} \cdot H_t \cdot (Z_t - X_t\beta)\Big\} \\ &= (2\pi)^{-n/2} \cdot |(I_n - \beta^{down} A_0 TB_t - \beta^{up} \hat{A}_0' EB_t)^{-1}|^{-1} \cdot |\Omega|^{-1/2} \exp\Big\{ -\frac{1}{2}\varepsilon_t' \Omega^{-1} \varepsilon_t \Big\} \\ &= (2\pi)^{-n/2} \cdot |I_n - \rho_1 \cdot W_1 \cdot TB_t - \rho_2 \cdot W_2 \cdot EB_t| \cdot |\Omega|^{-1/2} \exp\Big\{ -\frac{1}{2}\varepsilon_t' \Omega^{-1} \varepsilon_t \Big\}, \end{aligned}$$

with $\rho_1 = \beta^{down}$, $\rho_2 = \beta^{up}$, $A_0 = W_1$ and $\hat{A}_0' = W_2$ (to ease notation).
At this point we need to find the likelihood of the random vector $\boldsymbol{\Delta y_t}$:

$$\boldsymbol{\Delta y_t} = \begin{bmatrix} \Delta y_1 & \dots & \Delta y_T \end{bmatrix}'.$$

Since our model is static and we have assumed

$$cov(\varepsilon_t, \varepsilon_{t-k}) = \underset{n\times n}{\mathbf{0}},$$

we consider our variables Δy_t, to be *iid*. By consequence, the following holds:

$$\begin{aligned} f(\underset{nT\times 1}{\boldsymbol{\Delta y_t}}|X_1, \dots, X_T, \rho, \beta, \Omega) = \prod_{t=1}^{T} f(\underset{n\times 1}{\Delta y_t}|X_t, \rho, \beta, \Omega) = \big((2\pi)^n |\Omega|\big)^{-T/2} \cdot \\ \cdot \prod_{t=1}^{T} |I_n - \rho_1 \cdot W_1 \cdot TB_t - \rho_2 \cdot W_2 \cdot EB_t| \exp\big\{ -\frac{1}{2} \cdot \sum_{t=1}^{T} \varepsilon_t' \Omega^{-1} \varepsilon_t \big\}. \end{aligned}$$

Now we divide the time series of length T in three different subperiods. In doing so, consider the following new parameters:

- t_1: set of years when a tax based fiscal adjustment occurs. Formally:

  $$t_1 := \{1, ..., t, ..., T_1 | t \text{ such that } TB_t = 1\}$$

  We set:

  $$H_t | t \in t_1 = (I_n - \rho_1 \cdot W_1)^{-1} = H_\tau$$

- t_2: set of years when an expenditure tax based fiscal adjustment occurs. Formally:
$$t_2 := \{1, ..., t, ..., T_2 | t \text{ such that } EB_t = 1\}$$
We set:
$$H_t | t \in t_2 = (I_n - \rho_2 \cdot W_2)^{-1} = H_\gamma$$
- t_3: set of years when neither a tax based fiscal adjustment nor an expediture based fiscal adjustment occurs. Formally:
$$t_3 := \{1, ..., t, ..., T_3 | t \text{ such that } TB_t = 0 \wedge EB_t = 0\}$$
We set:
$$H_t | t \in t_3 = (I_n)^{-1} = I_n$$

Therefore, we have that t_1, t_2 and t_3 account for a partition of the whole time series and $T = T_1 + T_2 + T_3$. By consequence we have:

$$\begin{aligned}
\prod_{t=1}^{T} |I_n - \rho_1 W_1 TB_t - \rho_2 W_2 EB_t| &= \prod_{t=1}^{T} |H_t^{-1}| \\
&= \prod_{t=1}^{T} \frac{1}{|H_t|} \\
&= \prod_{t \in t_1}^{T_1} \frac{1}{|H_t|} \cdot \prod_{t \in t_2}^{T_2} \frac{1}{|H_t|} \cdot \prod_{t \in t_3}^{T_3} \frac{1}{|H_t|} \\
&= |H_\tau|^{-T_1} \cdot |H_\gamma|^{-T_2} \cdot |I_n|^{-T_3} \\
&= |I_n - \rho_1 \cdot W_1|^{T_1} \cdot |I_n - \rho_2 W_2|^{T_2}
\end{aligned}$$

At this point, we rewrite the probability density function of our dependent variable as:

$$\begin{aligned}
f(\boldsymbol{\Delta y_t} | X_1, \ldots, X_T, \rho, \beta, \Omega) = (2\pi)^{-nT/2} \cdot |\Omega|^{-T/2} \cdot \\
\cdot |I_n - \rho_1 \cdot W_1|^{T_1} \cdot |I_n - \rho_2 W_2|^{T_2} \cdot \exp\left\{ -\frac{1}{2} \cdot \sum_{t=1}^{T} \varepsilon_t' \cdot \Omega^{-1} \cdot \varepsilon_t \right\}.
\end{aligned}$$

Eventually, we express the log-likelihood of our dataset:

$$\begin{aligned}
\log \mathcal{L}(\rho, \beta, \Omega | \Delta y_1, \ldots, \Delta y_T, X_1, \ldots, X_T) = -\frac{nT}{2} \ln(2\pi) - \frac{T}{2} \cdot \ln(|\Omega|) + \\
+ T_1 \cdot \ln(|I_n - \rho_1 \cdot W_1|) + T_2 \cdot \ln(|I_n - \rho_2 W_2|) - \frac{1}{2} \cdot \sum_{t=1}^{T} \varepsilon_t' \cdot \Omega^{-1} \cdot \varepsilon_t.
\end{aligned}$$

with:

$$\varepsilon_t = Z_t - X_t \cdot \beta = H_t^{-1} \cdot \Delta y_t - X_t\beta = (I_n - \rho_1 W_1 TB_t - \rho_2 W_2 EB_t) \cdot \Delta y_t - X_t \cdot \beta.$$

Furthermore, we impose the condition $\lambda_{\text{min}}^{-1} < \hat{\rho}_1 < \lambda_{\text{max}}^{-1}$ and $\mu_{\text{min}}^{-1} < \hat{\rho}_2 < \mu_{\text{max}}^{-1}$, where λ and μ are the eigenvalues of the spatial matrices W_1 and W_2 respectively.[17] Such a condition guarantees that the Variance-Covariance Matrix of the ML estimator is positive definite.

At this point we concentrate the log-likelihood by computing the partial derivatives of it. Let's start with deriving the concentrated estimator of β. In our model β contains the $n = 15$ fixed effects plus the 6 coefficients in front of the fiscal shocks: unexpected, announced and future for both taxes and expenditures.

$$\frac{\partial \log \mathcal{L}(\rho, \beta, \Omega|\cdot)}{\partial \beta} = -\frac{1}{2} \cdot \frac{\partial(\sum_{t=1}^{T} \varepsilon_t' \cdot \Omega^{-1}\varepsilon_t)}{\partial \beta}.$$

Note that:

$$\sum_{t=1}^{T} \varepsilon_t' \cdot \Omega^{-1}\varepsilon_t = \begin{bmatrix} \varepsilon_1' & \cdots & \varepsilon_T' \end{bmatrix} \cdot \Sigma^{-1} \cdot \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_T \end{bmatrix} = \varepsilon' \cdot \Sigma^{-1} \cdot \varepsilon,$$

where:

$$\underset{nt \times nT}{\Sigma} = \begin{bmatrix} \Omega & \underset{n\times n}{\mathbf{0}} & \cdots & \underset{n\times n}{\mathbf{0}} \\ \underset{n\times n}{\mathbf{0}} & \Omega & \cdots & \underset{n\times n}{\mathbf{0}} \\ \vdots & \vdots & \ddots & \vdots \\ \underset{n\times n}{\mathbf{0}} & \underset{n\times n}{\mathbf{0}} & \cdots & \Omega \end{bmatrix}$$

Also:

$$\Sigma^{-1} = \begin{bmatrix} \Omega & \underset{n\times n}{\mathbf{0}} & \cdots & \underset{n\times n}{\mathbf{0}} \\ \underset{n\times n}{\mathbf{0}} & \Omega & \cdots & \underset{n\times n}{\mathbf{0}} \\ \vdots & \vdots & \ddots & \vdots \\ \underset{n\times n}{\mathbf{0}} & \underset{n\times n}{\mathbf{0}} & \cdots & \Omega \end{bmatrix}^{-1} = \begin{bmatrix} \Omega^{-1} & \underset{n\times n}{\mathbf{0}} & \cdots & \underset{n\times n}{\mathbf{0}} \\ \underset{n\times n}{\mathbf{0}} & \Omega^{-1} & \cdots & \underset{n\times n}{\mathbf{0}} \\ \vdots & \vdots & \ddots & \vdots \\ \underset{n\times n}{\mathbf{0}} & \underset{n\times n}{\mathbf{0}} & \cdots & \Omega^{-1} \end{bmatrix}.$$

Moreover:

$$\begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_T \end{bmatrix} = \varepsilon = Z - X \cdot \beta = \begin{bmatrix} Z_1 \\ \vdots \\ Z_T \end{bmatrix} - \begin{bmatrix} X_1 \\ \vdots \\ X_T \end{bmatrix} \cdot \beta,$$

[17] See Ord (1975)

therefore,:

$$\begin{aligned}\sum_{t=1}^{T}\varepsilon_t'\cdot\Omega^{-1}\varepsilon_t &= (Z-X\cdot\beta)'\cdot\Sigma^{-1}\cdot(Z-X\cdot\beta)=\\ &= Z'\cdot\Sigma^{-1}\cdot Z-2\cdot Z'\cdot\Sigma^{-1}\cdot X\cdot\beta+\beta'\cdot X\cdot\Sigma^{-1}\cdot X\cdot\beta\end{aligned}$$

At this point it can be verified that:

$$\text{(FOC)}\quad \frac{\partial\log\mathcal{L}(\rho,\beta,\Omega|\cdot)}{\partial\beta}=X'\cdot\Sigma^{-1}\cdot Z-X'\cdot\Sigma^{-1}\cdot X\cdot\beta=0$$

$$\beta=(X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Z.$$

The above estimator is the GLS estimator. The result is not surprising, since we have simply solved a standard squared deviation minimization problem.
Furthermore, we need to estimate the variance of the model to fully concentrate the likelihood in order to simply solve a two variable maximization problem.

$$\begin{aligned}\frac{\partial\log\mathcal{L}(\rho,\beta,\Omega|\cdot)}{\partial\Omega} &= -\frac{T}{2}\cdot\frac{\partial(\ln(|\Omega|))}{\partial\Omega}-\frac{1}{2}\cdot\sum_{t=1}^{T}\frac{\partial(\varepsilon_t'\cdot\Omega^{-1}\cdot\varepsilon_t)}{\partial\Omega}=\\ &= -\frac{T}{2}\cdot(\Omega')^{-1}-\frac{1}{2}\cdot\sum_{t=1}^{T}(-\Omega^{-1}\cdot\varepsilon_t\cdot\varepsilon_t'\cdot\Omega^{-1})\\ &= \frac{1}{2}\cdot\Omega^{-1}\cdot\big[(\sum_{t=1}^{T}\varepsilon_t\cdot\varepsilon_t')\cdot\Omega^{-1}-T\big]=0.\quad\text{(FOC)}\end{aligned}$$

From the FOC it follows that:

$$\Omega=\frac{\sum_{t=1}^{T}\varepsilon_t\cdot\varepsilon_t'}{T}=\frac{1}{T}\cdot\begin{bmatrix}\sum_{t=1}^{T}\varepsilon_{1,t}^2 & \sum_{t=1}^{T}\varepsilon_{1,t}\cdot\varepsilon_{2,t} & \cdots & \sum_{t=1}^{T}\varepsilon_{1,t}\cdot\varepsilon_{n,t}\\ & \sum_{t=1}^{T}\varepsilon_{2,t}^2 & \cdots & \sum_{t=1}^{T}\varepsilon_{2,t}\cdot\varepsilon_{n,t}\\ & & \ddots & \vdots\\ & & & \sum_{t=1}^{T}\varepsilon_{n,t}^2\end{bmatrix}$$

Since we assume Ω to be diagonal, we are only interested in the variances of the sectors:

$$\Omega=diag(\sigma_1^2,\ldots,\sigma_n^2)$$

$$\sigma_i^2 = \frac{1}{T} \cdot \sum_{t=1}^{T} \varepsilon_{i,t}^2$$

In order to pass to Feasible GLS estimator, we need to use the OLS residuals:

$$\hat{\varepsilon}_{i,t} = Z_{i,t} - X_{i,t} \cdot \underbrace{(X'X)^{-1}X'Z}_{\beta_{OLS}}.$$

Since every equation has 7 parameters (the industry fixed effect plus the 6 fiscal shocks coefficients) we have:

$$\hat{\sigma}_i^2 = \frac{1}{T-7} \cdot \sum_{t=1}^{T} \hat{\varepsilon}_{i,t}^2,$$

and finally we have:

$$\hat{\Omega} = diag(\hat{\sigma}_1^2, \ldots, \hat{\sigma}_n^2)$$
$$\hat{\beta} = (X'\hat{\Sigma}^{-1}X)^{-1}X'\hat{\Sigma}^{-1}Z$$

At this point it is simply a matter of solving the following problem:

$$\max_{\rho_1,\rho_2} \quad \log \mathcal{L}(\rho_1, \rho_2, \hat{\Omega}, \hat{\beta} \mid \cdot\,) \quad \text{s.t. } \rho_1 \in (\lambda_{\max}^{-1}, \lambda_{\max}^{-1}) \text{ and } \rho_2 \in (\mu_{\min}^{-1}, \mu_{\max}^{-1})$$

Once estimated the coefficients of model

$$\Delta y_{i,t} = c_i + \left(\beta^{down} \cdot \Delta y_{i,t}^d + \delta^u \cdot e_t^u + \delta^a \cdot e_{t,t}^a + \delta^f \cdot e_t^f\right) \cdot TB_t +$$
$$+ \left(\beta^{up} \cdot \Delta y_{i,t}^u + \gamma^u \cdot e_{i,t}^u + \gamma^a \cdot e_{i,t,t}^a + \gamma^f \cdot e_{i,t}^f\right) \cdot EB_T.$$

we proceeded with computing the standard errors of the estimates. In order to do that, we computed analytically the elements of the Fisher Information Matrix ($\mathcal{I}$). In fact recall that:

$$\sqrt{n} \cdot (\hat{\theta}_0 - \theta) \xrightarrow{d} \mathcal{N}(0, \mathcal{I}^{-1})$$

In order to derive the Fisher Information Matrix we firstly need to obtain the total gradient of the log-likelihood function. Let's start with the spatial coefficient ρ_1:

$$\frac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1} = T_1 \frac{1}{|I_n - \rho_1 W_1|} \frac{\partial |I_n - \rho_1 W_1|}{\partial \rho_1} - \frac{1}{2} \sum_{t=1}^{T} \frac{\partial (Z_t' \Omega^{-1} Z_t)}{\partial \rho_1} - 2 \frac{\partial (Z_t' \Omega^{-1} X_t \beta)}{\partial \rho_1}.$$

By some matrix algebra, it is possible to show that:

$$\begin{aligned}\frac{\partial(Z_t'\Omega^{-1}Z_t)}{\partial\rho_1} &= -TB_t\cdot\Delta y_t'\cdot\Omega^{-1}\cdot W_1\cdot\Delta y_t - TB_t\cdot\Delta y_t'\cdot W_1'\Omega^{-1}\cdot\Delta y_t\\ &+2\rho_1\cdot TB_t^2\cdot\Delta y_t'\cdot W_1\cdot\Omega^{-1}\cdot W_1\cdot\Delta y_t' + 2\rho_2\cdot TB_t\cdot EB_t\cdot\Delta y_t'\cdot W_1\cdot\Omega^{-1}\cdot W_2\cdot\Delta y_t'\end{aligned}$$

Since our fiscal adjustment plans are mutually exclusive, we have that $TB_t\cdot EB_t = 0$ for all t. Moreover, by rearranging the above expression, we get:

$$\frac{\partial(Z_t'\Omega^{-1}Z_t)}{\partial\rho_1} = -2\cdot TB_t\cdot\Delta y_t'\cdot(I_n-\rho_1\cdot W_1')\cdot\Omega^{-1}\cdot W_1\cdot\Delta y_t$$

After other matrix algebra, we get:

$$-2\cdot\frac{\partial(Z_t\cdot\Omega^{-1}X_t\beta)}{\partial\rho_1} = 2\cdot TB_t\cdot\Delta y_t'\cdot W_1'\cdot\Omega^{-1}\cdot X_t\cdot\beta$$

Wrapping up all together, and employing the notation introduced earlier: $(I_n - \rho_1 W_1)^{-1} = H_\tau$, we have:

$$\begin{aligned}\frac{\partial\log\mathcal{L}(\theta|\Delta y,X)}{\partial\rho_1} &= T_1\frac{1}{|I_n-\rho_1W_1|}\frac{\partial|I_n-\rho_1W_1|}{\partial\rho_1}+\\ &+\sum_{t\in t_1}^{T_1}\Big[\Delta y_t'\cdot(I_n-\rho_1\cdot W_1')\cdot\Omega^{-1}\cdot W_1\cdot\Delta y_t - \Delta y_t'\cdot W_1'\cdot\Omega^{-1}\cdot X_t\cdot\beta\Big] =\\ &= T_1\frac{1}{|I_n-\rho_1W_1|}\cdot|I_n-\rho_1W_1|\cdot Tr\Big((I_n-\rho_1W_1)^{-1}\cdot(-W_1)\Big)+\\ &+\sum_{t\in t_1}^{T_1}\Big[\big((I_n-\rho_1\cdot W_1)\cdot\Delta y_t\big)'\cdot\Omega^{-1}\cdot W_1\cdot\Delta y_t - \beta'\cdot X_t'\cdot\Omega^{-1}\cdot W_1\cdot\Delta y_t\Big]\\ &= -T_1\cdot Tr\Big(H_\tau\cdot W_1\Big)+\sum_{t\in t_1}^{T_1}\Big[\big(Z_t-X_t\beta\big)'\cdot\Omega^{-1}\cdot W_1\cdot\Delta y_t\Big]\\ &= \sum_{t\in t_1}^{T_1}\big(\varepsilon_t'\cdot\Omega^{-1}\cdot W_1\cdot\Delta y_t\big) - T_1\cdot Tr\big(H_\tau\cdot W_1\big).\end{aligned}$$

By simmetry we have that:

$$\frac{\partial\log\mathcal{L}(\theta|\Delta y,X)}{\partial\rho_2} = \sum_{t\in t_2}^{T_2}\big(\varepsilon_t'\cdot\Omega^{-1}\cdot W_2\cdot\Delta y_t\big) - T_2\cdot Tr\big(H_\gamma\cdot W_2\big),$$

with $H_\gamma = (I_n - \rho_2 W_2)^{-1}$, from the previous notation.
As far as concern the derivative with respect to β, we have already seen when concentrating the log-likelihood that:

$$\begin{aligned}\frac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \beta} &= X' \cdot \Sigma^{-1} \cdot Z - X' \cdot \Sigma^{-1} \cdot X \cdot \beta \\ &= X' \cdot \Sigma^{-1} \cdot (Z - X \cdot \beta) = \\ &= X' \cdot \Sigma^{-1} \cdot \varepsilon = \\ &= \sum_{t=1}^{T} X_t' \cdot \Omega^{-1} \cdot \varepsilon_t.\end{aligned}$$

Concerning the derivatives with respect to σ_i^2, we need firstly to acknowledge that:

$$\sum_{t=1}^{T} \varepsilon_t' \cdot \Omega^{-1} \cdot \varepsilon_t = \sum_{t=1}^{T} \sum_{i=1}^{n} \frac{\varepsilon_{i,t}^2}{\sigma_i^2} = \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \sum_{t=1}^{T} \varepsilon_{i,t}^2,$$

and that:

$$\ln(|\Omega|) = \ln(\prod_{i=1}^{n} \sigma_i^2) = \sum_{i=1}^{n} \ln(\sigma_i^2).$$

Therefore, we have that:

$$\begin{aligned}\frac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \sigma_i^2} &= -\frac{T}{2} \frac{\partial \ln(|\Omega|)}{\partial \sigma_i^2} - \frac{1}{2} \cdot \frac{\partial}{\partial \sigma_i^2} \sum_{t=1}^{T} \varepsilon_t' \cdot \Omega^{-1} \cdot \varepsilon_t \\ &= -\frac{T}{2 \cdot \sigma_i^2} + \frac{1}{2 \cdot \sigma_i^4} \cdot \sum_{t=1}^{T} \varepsilon_{i,t}^2.\end{aligned}$$

We now have all the elements to write down the gradient of the log-likelihood:

$$\nabla \log \mathcal{L}(\theta|\Delta y, X) = \underset{38\times 1}{\begin{bmatrix} \dfrac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1} \\ \\ \dfrac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_2} \\ \\ \dfrac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \beta} \\ \\ \dfrac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \sigma_1^2} \\ \\ \vdots \\ \\ \dfrac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \sigma_n^2} \end{bmatrix}} = \begin{bmatrix} \sum_{t\in t_1}^{T_1} \big(\varepsilon_t' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\big) - T_1 \cdot Tr\big(H_\tau \cdot W_1\big) \\ \\ \sum_{t\in t_2}^{T_2} \big(\varepsilon_t' \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t\big) - T_2 \cdot Tr\big(H_\gamma \cdot W_2\big) \\ \\ \sum_{t=1}^{T} X_t' \cdot \Omega^{-1} \cdot \varepsilon_t \\ \\ -\dfrac{T}{2\cdot\sigma_1^2} + \dfrac{1}{2\cdot\sigma_1^4} \cdot \sum_{t=1}^{T} \varepsilon_{1,t}^2 \\ \\ \vdots \\ \\ -\dfrac{T}{2\cdot\sigma_n^2} + \dfrac{1}{2\cdot\sigma_n^4} \cdot \sum_{t=1}^{T} \varepsilon_{n,t}^2 \end{bmatrix}$$

The gradient contains overall 38 elements, that is, we need to estimate 38 parameters. By consequence, the Fisher Information Matrix will be a 38×38 array.
Let's start with the first row of the matrix: all the derivatives of $\dfrac{\partial \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1}$ with respect to all the parameters. To simplify notation I will refer with $\mathcal{H}_{ij}$ to the element of row i and column j of the Hessian matrix.

$$\begin{aligned} \mathcal{H}_{1,1} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1^2} &= \sum_{t\in t_1}^{T_1} \big(\frac{\partial \varepsilon_t'}{\partial \rho_1} \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\big) - T_1 \cdot \frac{\partial Tr\big(H_\tau \cdot W_1\big)}{\partial \rho_1} \\ &= \sum_{t\in t_1}^{T_1} \big((-\Delta y_t' \cdot W_1') \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\big) - T_1 \cdot Tr\big(\frac{\partial H_\tau}{\partial \rho_1} \cdot W_1\big) = \\ &= -\sum_{t\in t_1}^{T_1} \big(\Delta y_t' \cdot W_1' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\big) - T_1 \cdot Tr\big((-H_\tau \cdot (-W_1) \cdot H_\tau) \cdot W_1\big) = \\ &= -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau\big) - \sum_{t\in t_1}^{T_1} \big(\Delta y_t' \cdot W_1' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\big) \end{aligned}$$

Symmetrically we have:

$$\mathcal{H}_{2,2} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_2^2} =$$
$$= -T_2 \cdot Tr\big(W_2 \cdot H_\gamma \cdot W_2 \cdot H_\gamma\big) - \sum_{t \in t_2}^{T_2} \big(\Delta y_t' \cdot W_2' \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t\big)$$

Going back to the first row, we now calculate the cross derivative with respect to $\rho 2$. Before doing so, recall that, being the log-likelihood a continuously diffirentiable function, the Schwarz's theorem applies and the Hessian matrix is symmetric.

$$\mathcal{H}_{1,2} = \mathcal{H}_{2,1} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1 \partial \rho_2} = 0.$$

Going on with the calculation we have:

$$\begin{aligned}
\mathcal{H}_{1,3:1,23} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_1 \partial \beta} &= \sum_{t \in t_1}^{T_1} \big(\frac{\partial \varepsilon_t'}{\partial \beta} \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\big) \\
&= -\sum_{t \in t_1}^{T_1} X_t' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t \\
&= -X_\tau' \cdot \underbrace{(I_{T_1} \otimes \Omega^{-1})}_{\Sigma_\tau^{-1}} \cdot (I_{T_1} \otimes W_1) \cdot \Delta y_\tau
\end{aligned}$$

where $\mathcal{H}_{1,3:1,23}$ means all the elements of the first row, from column 3 up to column 23. X_τ and Δy_τ represent X and Δy but for the only years when a tax based fiscal adjustment occur:

$$X_\tau = \underset{T_1 n \times k}{\begin{bmatrix} X_1 \\ \vdots \\ X_t \\ \vdots \\ X_{T_1} \end{bmatrix}} \quad \text{and} \quad \Delta y_\tau = \underset{T_1 n \times k}{\begin{bmatrix} \Delta y_1 \\ \vdots \\ \Delta y_t \\ \vdots \\ \Delta y_{T_1} \end{bmatrix}} \quad \text{with } t \in t_1,$$

Symmetrically:

$$\begin{aligned}
\mathcal{H}_{2,3:2,23} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \rho_2 \partial \beta} &= \sum_{t\in t_2}^{T_2} \big(\frac{\partial \varepsilon_t'}{\partial \beta} \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t\big) \\
&= -\sum_{t\in t_2}^{T_2} X_t' \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t \\
&= -X_\gamma' \cdot \underbrace{(I_{T_2} \otimes \Omega^{-1})}_{\Sigma_\gamma^{-1}} \cdot (I_{T_2} \otimes W_2) \cdot \Delta y_\gamma,
\end{aligned}$$

with:

$$X_\gamma = \underbrace{\begin{bmatrix} X_1 \\ \vdots \\ X_t \\ \vdots \\ X_{T_2} \end{bmatrix}}_{T_2 n \times k} \quad \text{and} \quad \Delta y_\gamma = \underbrace{\begin{bmatrix} \Delta y_1 \\ \vdots \\ \Delta y_t \\ \vdots \\ \Delta y_{T_2} \end{bmatrix}}_{T_2 n \times k} \quad \text{with } t \in t_2,$$

$$\begin{aligned}
\mathcal{H}_{3,3:23,23} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \beta^2} &= \frac{\partial}{\partial \beta^2}\left(\sum_{t=1}^{T} X_t' \cdot \Omega^{-1} \cdot \varepsilon_t\right) \\
&= \sum_{t=1}^{T} X_t' \cdot \Omega^{-1} \cdot \frac{\partial (Z_t - X_t \cdot \beta)}{\partial \beta^2} \\
&= \sum_{t=1}^{T} X_t' \cdot \Omega^{-1} \cdot X_t \\
&= -X' \cdot \Sigma^{-1} \cdot X.
\end{aligned}$$

$$\mathcal{H}_{3,24:23,38} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial \beta \partial \sigma^2} = \sum_{t=1}^{T} X_t' \cdot \frac{\partial \Omega^{-1}}{\partial \sigma^2} \cdot \varepsilon_t$$

The generic element of the above matrix is a $k \times 1$ vector:

$$-\sigma_1^{-4} \cdot \sum_{t=1}^{T} X_{1,t}' \cdot \varepsilon_{i,t}.$$

Going on with the calculation:

$$\mathcal{H}_{i,i|i\in[24,38]} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial(\sigma_i^2)^2} = \frac{T}{2} \cdot \frac{1}{\sigma_i^4} \cdot \Bigg(1 - \frac{2}{T \cdot \sigma_i^2} \cdot \sum_{t=1}^{T} \varepsilon_{i,t}^2\Bigg).$$

$$\mathcal{H}_{23+i,23+j|i,j\in[1,n]} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial\sigma_i^2 \partial\sigma_j^2} = 0 \quad \forall i \neq j.$$

$$\begin{aligned}
\mathcal{H}_{1,24:1,38} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial\rho_1 \partial\sigma_i^2} &= \frac{\partial}{\partial\sigma_i^2}\Big(\sum_{t\in t_1}^{T_1} \varepsilon_t' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\Big) \\
&= \frac{\partial}{\partial\sigma_i^2}\Big(\sum_{t\in t_1}^{T_1} Tr\big(\varepsilon_t' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\big)\Big) \\
&= \frac{\partial}{\partial\sigma_i^2}\Big(Tr\big((\sum_{t\in t_1}^{T_1} \Delta y_t \cdot \varepsilon_t') \cdot \Omega^{-1} \cdot W_1\big)\Big) \\
&= Tr\Big((\sum_{t\in t_1}^{T_1} \Delta y_t \cdot \varepsilon_t') \cdot \frac{\partial\Omega^{-1}}{\partial\sigma_i^2} \cdot W_1\Big)
\end{aligned}$$

Note that

$$\frac{\partial\Omega^{-1}}{\partial\sigma_i^2} = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & -\sigma_i^{-4} & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} = diag(0, \cdots, 0, -\sigma_i^{-4}, 0, \cdots, 0)$$

Symmetrically:

$$\mathcal{H}_{2,24:2,38} = \frac{\partial^2 \log \mathcal{L}(\theta|\Delta y, X)}{\partial\rho_2 \partial\sigma_i^2} = Tr\Big((\sum_{t\in t_2}^{T_2} \Delta y_t \cdot \varepsilon_t') \cdot \frac{\partial\Omega^{-1}}{\partial\sigma_i^2} \cdot W_2\Big)$$

At this point we have all the elements to construct the Hessian matrix of the log-likelihood.
To sum up, first row:

- $\mathcal{H}_{1,1} = -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau\big) - \sum_{t\in t_1}^{T_1} \big(\Delta y_t' \cdot W_1' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\big)$

- $\mathcal{H}_{1,2} = 0$

- $\mathcal{H}_{1,3:1,23} = -\sum_{t \in t_1}^{T_1} X_t' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t$

- $\mathcal{H}_{1,24:1,38} = Tr\Big(\big(\sum_{t \in t_1}^{T_1} \Delta y_t \cdot \varepsilon_t'\big) \cdot \dfrac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1\Big).$

Second row:

- $\mathcal{H}_{2,1} = 0$

- $\mathcal{H}_{2,2} = -T_2 \cdot Tr\big(W_2 \cdot H_\gamma \cdot W_2 \cdot H_\gamma\big) - \sum_{t \in t_2}^{T_2} \big(\Delta y_t' \cdot W_2' \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t\big)$

- $\mathcal{H}_{2,3:2,23} = -\sum_{t \in t_2}^{T_2} X_t' \cdot \Omega^{-1} \cdot W_2 \cdot \Delta y_t$

- $\mathcal{H}_{2,24:2,38} = Tr\Big(\big(\sum_{t \in t_2}^{T_2} \Delta y_t \cdot \varepsilon_t'\big) \cdot \dfrac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_2\Big).$

From row 3 to row 23:

- $\mathcal{H}_{3,1:23,1} = \mathcal{H}_{1,3:1,23}'$

- $\mathcal{H}_{3,2:23,2} = \mathcal{H}_{2,3:2,23}'$

- $\mathcal{H}_{3,3:23,23} = \sum_{t=1}^{T} X_t' \cdot \Omega^{-1} \cdot X_t$

- $\mathcal{H}_{3,24:23,38} = \sum_{t=1}^{T} X_t' \cdot \dfrac{\partial \Omega^{-1}}{\partial \sigma^2} \cdot \varepsilon_t$

From row 24 to the last row (number 38):

- $\mathcal{H}_{24,1:38,1} = \mathcal{H}_{1,24:1,38}'$

- $\mathcal{H}_{24,2:38,2} = \mathcal{H}_{2,24:2,38}'$

- $\mathcal{H}_{24,3:38,23} = \mathcal{H}'_{3,24:23,38}$

- $\mathcal{H}_{23+i,23+j|i,j\in[1,n]} = \begin{cases} \dfrac{T}{2} \cdot \dfrac{1}{\sigma_i^4} \cdot \left(1 - \dfrac{2}{T \cdot \sigma_i^2} \cdot \displaystyle\sum_{t=1}^{T} \varepsilon_{i,t}^2\right) & \forall i = j \in [1, n] \\ \\ 0 \quad \forall i \neq j \end{cases}$

The last step we have to make to finally obtain the Fisher Information Matrix is taking expectations of every element.

$$
\begin{aligned}
E[\mathcal{H}_{1,1}] &= -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau\big) - \sum_{t \in t_1}^{T_1} E\big[\Delta y_t' \cdot W_1' \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\big] = \\
&= -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau\big) - \sum_{t \in t_1}^{T_1} E\big[Tr\big(W_1 \cdot \Delta y_t \cdot \Delta y_t' \cdot W_1' \cdot \Omega^{-1}\big)\big] = \\
&= -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau\big) - \sum_{t \in t_1}^{T_1} Tr\big(W_1 \cdot E\big[\Delta y_t \cdot \Delta y_t'\big] \cdot W_1' \cdot \Omega^{-1}\big) = \\
&= -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau\big) - \sum_{t \in t_1}^{T_1} Tr\big(W_1 \cdot E\big[H_\tau \cdot X_t \cdot \beta \cdot \varepsilon_t' \cdot H_\tau' + \\
&+ H_\tau \cdot X_t \cdot \beta \cdot \beta' \cdot X_t' \cdot H_\tau' + H_\tau \cdot \varepsilon_t \cdot \varepsilon_t' \cdot H_\tau' \cdot \varepsilon_t \cdot \beta' \cdot X_t' \cdot H_\tau'\big] \cdot W_1' \cdot \Omega^{-1}\big) = \\
&= -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau\big) - \sum_{t \in t_1}^{T_1} Tr\Big(W_1 \cdot \Big[H_\tau \cdot X_t \cdot \beta \cdot E[\varepsilon_t'] \cdot H_\tau' + \\
&+ H_\tau \cdot X_t \cdot \beta \cdot \beta' \cdot X_t' \cdot H_\tau' + H_\tau \cdot E[\varepsilon_t \cdot \varepsilon_t'] \cdot H_\tau' + E[\varepsilon_t] \cdot \beta' \cdot X_t' \cdot H_\tau'\Big] \cdot W_1' \cdot \Omega^{-1}\Big) = \\
&= -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau\big) - \\
&- \sum_{t \in t_1}^{T_1} Tr\Big(W_1 \cdot \Big[H_\tau \cdot X_t \cdot \beta \cdot \beta' \cdot X_t' \cdot H_\tau' + H_\tau \cdot \Omega \cdot H_\tau'\Big] \cdot W_1' \cdot \Omega^{-1}\Big) = \\
&= -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau\big) - \\
&- \sum_{t \in t_1}^{T_1} Tr\big(W_1 \cdot H_\tau \cdot X_t \cdot \beta \cdot \beta' \cdot X_t' \cdot H_\tau' \cdot W_1' \cdot \Omega^{-1} + W_1 \cdot H_\tau \cdot \Omega \cdot H_\tau' \cdot W_1' \cdot \Omega^{-1}\big) = \\
&= -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau + H_\tau' \cdot W_1' \cdot \Omega^{-1} \cdot W_1 \cdot H_\tau \cdot \Omega\big) - \\
&- \sum_{t \in t_1}^{T_1} Tr\big(\beta' \cdot X_t' \cdot H_\tau' \cdot W_1' \cdot \Omega^{-1} \cdot W_1 \cdot H_\tau \cdot X_t \cdot \beta\big) =
\end{aligned}
$$

Setting $M_1^\tau = H'_\tau \cdot W'_1 \cdot \Omega^{-1} \cdot W_1 \cdot H_\tau$ we can rewrite the above identity as:

$$\begin{aligned} E[\mathcal{H}_{1,1}] &= -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau + M_1^\tau \cdot \Omega\big) - \sum_{t \in t_1}^{T_1} \beta' \cdot X'_t \cdot M_1^\tau \cdot X_t \cdot \beta = \\ &= -T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau + M_1^\tau \cdot \Omega\big) - \beta' \cdot X'_\tau \cdot \big(I_{T_1} \otimes M_1^\tau\big) \cdot X_\tau \cdot \beta. \end{aligned}$$

Simmetrically:

$$E[\mathcal{H}_{2,2}] = -T_2 \cdot Tr\big(W_2 \cdot H_\gamma \cdot W_2 \cdot H_\gamma + M_1^\gamma \cdot \Omega\big) - \beta' \cdot X'_\gamma \cdot \big(I_{T_2} \otimes M_1^\gamma\big) \cdot X_\gamma \cdot \beta.$$

with $M_1^\gamma = H'_\gamma \cdot W'_2 \cdot \Omega^{-1} \cdot W_2 \cdot H_\gamma$.

Going on with the calculation:

$$\begin{aligned} E[\mathcal{H}_{1,3:1,23}] &= E\Big[-\sum_{t \in t_1}^{T_1} X'_t \cdot \Omega^{-1} \cdot W_1 \cdot \Delta y_t\Big] = \\ &= -\sum_{t \in t_1}^{T_1} X'_t \cdot \Omega^{-1} \cdot W_1 \cdot E\Big[H_\tau \cdot X_t \cdot \beta + H_\tau \cdot \varepsilon_t\Big] = \\ &= -\sum_{t \in t_1}^{T_1} X'_t \cdot \Omega^{-1} \cdot W_1 \cdot H_\tau \cdot X_t \cdot \beta \\ &= X'_\tau \cdot (I_{T_1} \otimes M_2^\tau) \cdot X_\tau \cdot \beta \end{aligned}$$

with $M_2^\tau = \Omega^{-1} \cdot W_1 \cdot H_\tau$.

Simmetrically:

$$E[\mathcal{H}_{2,3:2,23}] = X'_\gamma \cdot (I_{T_2} \otimes M_2^\gamma) \cdot X_\gamma \cdot \beta$$

with $M_2^\gamma = \Omega^{-1} \cdot W_2 \cdot H_\gamma$.

Next step:

$$
\begin{aligned}
E[\mathcal{H}_{1,24:1,38}] &= Tr\Big(\big(\sum_{t\in t_1}^{T_1} E\big[\Delta y_t \cdot \varepsilon_t'\big]\big) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1\Big) = \\
&= Tr\Big(\big(\sum_{t\in t_1}^{T_1} E\big[\Delta y_t \cdot \varepsilon_t'\big]\big) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1\Big) = \\
&= Tr\Big(\big(\sum_{t\in t_1}^{T_1} H_\tau \cdot E\big[\varepsilon_t \cdot \varepsilon_t'\big]\big) \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1\Big) = \\
&= T_1 \cdot Tr\Big(H_\tau \cdot \Omega \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1\Big) = \\
&= T_1 \cdot Tr\Big(\Omega \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^2} \cdot W_1 \cdot H_\tau\Big),
\end{aligned}
$$

Notice that

$$
\Omega \cdot \frac{\partial \Omega^{-1}}{\partial \sigma_i^{-2}} = -\sigma_i^2 \cdot I_{ii}
$$

where the generic element of matrix I_{ii} is given by

$$
\omega_{s,t} = \begin{cases} 1 & s = i, j = i \\ 0 & \text{otherwise} \end{cases}
$$

Therefore

$$
\begin{aligned}
E[\mathcal{H}_{1,23+i}] &= T_1 \cdot \sigma_i^{-2} \cdot Tr\Big(I_{ii} \cdot W_1 \cdot H_\tau\Big) = \\
&= T_1 \cdot \sigma_i^{-2} \cdot \Big(W_1 \cdot H_\tau\Big)_{ii}
\end{aligned}
$$

Finally we have that:

$$
E[\mathcal{H}_{1,24:1:38}] = T_1 \cdot diag\Big(\Omega^{-1} \cdot W_1 \cdot H_\tau\Big) = T_1 \cdot diag(M_2^\tau).
$$

Simmetrically:

$$
E[\mathcal{H}_{2,24:2:38}] = T_2 \cdot diag\Big(\Omega^{-1} \cdot W_2 \cdot H_\gamma\Big) = T_2 \cdot diag(M_2^\gamma).
$$

Going on:

$$E[\mathcal{H}_{3,3:23,23}] = E[\sum_{t=1}^{T} X_t' \cdot \Omega^{-1} \cdot X_t] = \sum_{t=1}^{T} X_t' \cdot \Omega^{-1} \cdot X_t = X' \cdot \Sigma^{-1} \cdot X$$

$$\begin{aligned} E[\mathcal{H}_{3,24:23,38}] &= E[\sum_{t=1}^{T} X_t' \cdot \frac{\partial \Omega^{-1}}{\partial \sigma^2} \cdot \varepsilon_t] \\ &= \sum_{t=1}^{T} X_t' \cdot \frac{\partial \Omega^{-1}}{\partial \sigma^2} \cdot E[\varepsilon_t] \\ &= \underset{k\times n}{\mathbf{0}} \end{aligned}$$

$$E[\mathcal{H}_{23+i,23+j|i,j\in[1,n]}] = \begin{cases} \frac{T}{2} \cdot \frac{1}{\sigma_i^4} \cdot \left(1 - \frac{2}{T \cdot \sigma_i^2} \cdot \sum_{t=1}^{T} E[\varepsilon_{i,t}^2]\right) & \forall i = j \in [1,n] \\ \\ 0 \quad \forall i \neq j \end{cases}$$

$$= \begin{cases} -\frac{T}{2} \cdot \frac{1}{\sigma_i^4} & \forall i = j \in [1,n] \\ 0 \quad \forall i \neq j \end{cases}$$

$$= -\frac{T}{2} \cdot \begin{bmatrix} \sigma_1^{-4} & 0 & \cdots & 0 \\ 0 & \sigma_2^{-4} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ o & 0 & \cdots & \sigma_n^{-4} \end{bmatrix} = -\frac{T}{2} \cdot V$$

We finally have all the elements of the Fisher Information Matrix for our panel (with dummy variables) spatial model:

$$\mathcal{I} =$$

$$
\begin{bmatrix}
\begin{matrix}-T_1 \cdot Tr\big(W_1 \cdot H_\tau \cdot W_1 \cdot H_\tau + M_1^\tau \cdot \Omega\big) - \\ - \beta' \cdot X_\tau' \cdot \big(I_{T_1} \otimes M_1^\tau\big) \cdot X_\tau \cdot \beta\end{matrix} & 0 & \big(X_\tau' \cdot (I_{T_1} \otimes M_2^\tau) \cdot X_\tau \cdot \beta\big)' & T_1 \cdot diag(M_2^\tau)' \\
0 & \begin{matrix}-T_2 \cdot Tr\big(W_2 \cdot H_\gamma \cdot W_2 \cdot H_\gamma + M_1^\gamma \cdot \Omega\big) - \\ - \beta' \cdot X_\gamma' \cdot \big(I_{T_2} \otimes M_1^\gamma\big) \cdot X_\gamma \cdot \beta\end{matrix} & \big(X_\gamma' \cdot (I_{T_2} \otimes M_2^\gamma) \cdot X_\gamma \cdot \beta\big)' & T_2 \cdot diag(M_2^\gamma)' \\
X_\tau' \cdot (I_{T_1} \otimes M_2^\tau) \cdot X_\tau \cdot \beta & X_\gamma' \cdot (I_{T_2} \otimes M_2^\gamma) \cdot X_\gamma \cdot \beta & X' \cdot \Sigma^{-1} \cdot X & \underset{k \times n}{\mathbf{0}} \\
T_1 \cdot diag(M_2^\tau) & T_2 \cdot diag(M_2^\gamma) & \underset{n \times k}{\mathbf{0}} & -\frac{T}{2} \cdot V
\end{bmatrix}
$$

We now have all the elements to introduce our simulation procedure: since the ML estimator is asymptotically normally distributed around the true parameters, we ran a Monte-Carlo experiment by drawing the coefficients (indicated with $\tilde{\theta}$) from the following distribution:

$$\tilde{\theta} \sim \mathcal{N}(\hat{\theta}_{MLE}, \mathcal{I}(\hat{\theta}_{MLE})).$$

The term $\mathcal{I}(\hat{\theta}_{MLE})$ represents the estimated analytical Fisher Information Matrix.

At every draw we calculated the effect of tax and expenditure shock. Iterating this procedure 10,000 times allowed us to obtain a distribution of a tax and expenditure shock, thus closely mimicing the procedure adopted by Romer and Romer(2010), when they construct the confidence bands of their impulse response functions.

5.3.2 Bayesian MCMC

The model's parameters have also been estimated by the Bayesian MCMC, introduced by LeSage(1997) to provide a heteroscedastic robust estimator of the parameters of the SAR models. A Bayesian framework has been introduced since a Maximum Likelihood Heterscedasticity robust estimator was not possible to derive, because of the single-dimensional nature of the data usually employed in spatial econometrics problems. The reason to adopt such a methodology in our problem is twofold: first, we provide an alternative estimation procedure (robustness); second, we seek to improve the efficiency of the MLE estimates. In fact, LeSage(1997) shows through an experiment that the Bayesian MCMC delivers slightly more significant estimates than the ML estimator, within the homoscedastic framework. We therefore verify this fact, in our heteroscedastic, panel case:

$$
\begin{aligned}
&A_t \cdot \underset{n\times 1}{\Delta y_t} = \underset{n\times(n+6)}{X_t} \cdot \beta + \varepsilon_t \\
&A_t = I_n - \beta^{down} \cdot A_0 \cdot TB_t + \beta^{up} \cdot \hat{A}_0' \cdot EB_t \\
&\varepsilon_t \sim \mathcal{N}(0, \Omega), \forall t \in \{1, ..., T\} \\
&\Omega = \sigma^2 \cdot V \quad \text{with } V = diag(v_1, ..., v_n) \\
&\varepsilon_t \perp \varepsilon_{t+i}, \quad \forall t \in \{1, ..., T\}, \quad \forall i \in \mathcal{Z} \\
&\pi(\beta) \propto constant \\
&\pi(\sigma^2) \propto \frac{1}{\sigma^2} \\
&\pi(\frac{r}{v_i}) \overset{iid}{\sim} \chi^2_{(r)}, \quad \forall i \in \{1, ..., n\} \\
&\beta^{down} \sim Beta(d, d) \\
&\beta^{up} \sim Beta(d, d).
\end{aligned}
$$

Notice that we add prior information on the spatial coefficients: rather than letting them vary from λ_{min}^{-1} to 1, we draw it from a Beta whose support is (0,1). Such a prior was introduced by LeSage and Parent(2007); basically we rule out the possibility to have negative spatial coefficients, which is a reasonable assumption in our case, where we expect the network to be positively correlated with the dependent variable. Moreover, setting the parameter d to be close to 1, makes the Beta prior to resemble a uniform(0,1) distribution, with the advantage of putting less density on the boundaries: recall that when the spatial coefficients approach 1 (which coincide with λ_{max}^{-1}), matrix A_t becomes not invertible, and the model becomes unstable, which we believe it is a very unlikely result.
Secondly, we model the heteroscedastic terms as done in LeSage(1997); however, unlike him, we set the hyperparameter r to be equal to 3 rather than 4, as he suggests. This is because reducing the magnitude of r implies more confidence on heteroscedasticity, which is in line with our prior belief.
The Bayesian MCMC is developed independently, thus avoiding the “griddy Gibbs” procedure adopted by LeSage and Pace to overcome the huge dimension problem of standard spatial econometrics. Deriving all the formulae analytically allows to obtain more precise results, as LeSage and Pace point out. We use the standard “Metropolis within Gibbs” algorithm, and we obtain an approximation of the posterior densities for every parameter of the model. Eventually, we draw from the posteriors the parameters, like a usual MonteCarlo simulation and we use them to

construct the empirical distribution of the average total, direct and indirect effects of a tax and expenditure shock, as done before for the Maximum Likelihood Estimation.

In particular, all the steps of the procedures are:

1. Set up initial values for the parameters: $\beta_{(0)}, \sigma^2_{(0)}, V_{(0)}, \rho_{1,(0)}, \rho_{2,(0)}$.

2. Draw $\beta_{(1)}$ from the conditional posterior distribution:

$$P(\beta_{(0)}|\mathcal{D}, \sigma^2_{(0)}, V_{(0)}, \rho_{1,(0)}, \rho_{2,(0)}) = \mathcal{N}(c^*, L^*) \propto \mathcal{L}(\theta|\mathcal{D}) \cdot \mathcal{N}(c, L)$$

$$c^* = \frac{1}{T} \cdot (\sum_{t=1}^{T} X_t' \cdot V_{(0)}^{-1} \cdot X_t + \frac{\sigma^2_{(0)}}{T} \cdot L^{-1})^{-1} \cdot (\frac{1}{T} \cdot \sum_{t=1}^{T} X_t' \cdot V_{(0)}^{-1} \cdot H_t \cdot \Delta y_t + \frac{\sigma^2_{(0)}}{T} \cdot L^{-1} \cdot c)$$

$$L^* = \frac{\sigma^2_{(0)}}{T} \cdot (\sum_{t=1}^{T} X_t' \cdot V_{(0)}^{-1} \cdot X_t + \frac{\sigma^2_{(0)}}{T} \cdot L^{-1})^{-1}$$

   Notice that, setting the diagonal elements of matrix L (the prior varcov matrix) to tend to infinity (we set them up equal to 1 billion), it is like assuming a non informative prior distribution on parameter β. Notice in fact, that the parameters of the distribution tend to be equal to the FGLS estimator and its variance.

3. Draw $\sigma^2_{(1)}$ from the conditional posterior distribution:

$$P(\sigma^2_{(1)}|\mathcal{D}, \beta_{(1)}, V_{(0)}, \rho_{1,(0)}, \rho_{2,(0)}) = \Gamma^{-1}(\frac{\theta_1}{2}, \frac{\theta_2}{2}) \propto \mathcal{L}(\theta|\mathcal{D}) \cdot \Gamma^{-1}(a, b)$$

$$\theta_1 = nT + 2a \quad \theta_2 = \sum_{t=1}^{T} \varepsilon_t' \cdot V_{(0)}^{-1} \cdot \varepsilon_t + 2b$$

   In practice we draw $\sigma^2_{(1)}$ from

$$\frac{\theta_2}{\chi_{\theta_1}}$$

   Notice that, setting a and b (the prior parameters) equal to 0, is like putting a Jefferey's prior on σ^2, which is exaclty what we do.

4. Draw $v_{i,(1)}$ from the following conditional posterior distribution:

$$P(v_{i,(1)}|\mathcal{D},\sigma^2_{(1)},\beta_{(1)},\rho_{1,(0)},\rho_{2,(0)}) = \Gamma^{-1}(\frac{q_1}{2},\frac{q_2}{2}) \propto \mathcal{L}(\theta|\mathcal{D})\cdot\Gamma^{-1}(\frac{r}{2},\frac{r}{2})$$

$$q_1 = r + T \quad q_2 = \frac{1}{\sigma^2_{(1)}}\cdot\sum_{t=1}^{T}\varepsilon^2_{i,t} + r$$

In practice we draw $v_{i,(1)}$ from:

$$\frac{q_2}{\chi_{q_1}}$$

As anticipated above in the paper, since we are confident on the heteroskedastic behavior of industry value added, we set our prior hyperparameter r to be equal to 3 rather than 4, as done by LeSage&Pace (2009).
Replicating this procedure n times, we get a first simulation of matrix $V_{(1)}$.

5. We now need to draw the spatial coefficients. However we cannot apply simple Gibbs Sampling, since the conditional posterior distribution is not defined for them. Therefore, we apply the Metropolis Hastings algorithm:

   (a) Draw ρ_1^c (where the c superscript stands for "candidate") from the proposal distribution:

   $$\rho_1^c = \rho_{1,(0)} + c_1\cdot\mathcal{N}(0,1)$$

   (b) Run a bernoulli experiment to determine the updated value of ρ_1:

   $$\rho_{1,(1)} = \begin{cases}\rho_1^c & \pi \quad \text{(accept)}\\ \rho_{1,(0)} & 1-\pi \quad \text{(reject)}\end{cases}$$

   Where π is equal to

   $$\pi = \min\{1,\psi_{MH_1}\}$$

   Setting: $A_\tau(\rho_1) = I_n - \rho_1\cdot W_1$ we have:

   $$\psi_{MH_1} = \frac{|A_\tau(\rho_1^c)|}{|A_\tau(\rho_{1,(0)})|}\cdot\exp\Big\{-\frac{1}{2\sigma^2_{(1)}}\cdot\sum_{t\in t_1}^{T_1}\Big[\Delta y_t'\cdot\big(A_\tau(\rho_1^c)'\cdot V_{(1)}^{-1}\cdot A_\tau(\rho_1^c)-$$
   $$-A_\tau(\rho_{1,(0)})'\cdot V_{(1)}^{-1}\cdot A_\tau(\rho_{1,(0)})\big)\cdot\Delta y_t-$$
   $$-2\beta'\cdot X_t'\cdot V_{(1)}^{-1}\big(A_\tau(\rho_1^c)-A_\tau(\rho_{1,(0)})\big)\cdot\Delta y_t\Big]\Big\}\cdot$$
   $$\cdot\left[\frac{\rho_1^c\cdot(1-\rho_1^c)}{\rho_{1,(0)}\cdot(1-\rho_{1,(0)})}\right]^{d-1}\cdot\mathbf{1}\big(0\le\rho_1^c\le 1\big)$$

Basically, we compute the probability to accept the candidate value from the proposal distribution, and then we update the value of ρ_1 by running the bernoulli experiment with such a probability of success. Notice that if we draw a value of ρ_1 outside the support of the beta prior, $\psi_{MH_1} = 0$ and then $\pi = 0$ and we clearly reject the candidate value. Eventually, notice that d is the parameter of the beta prior that we set equal to 1.1, on both ρ_1 and ρ_2; this is to resemble a uniform(0,1) but with less density on its boundary values.

(c) Once updated ρ_1, we replicate the procedure for ρ_2. Setting $A_\gamma(\rho_2) = I_n - \rho_2 \cdot W_2$ we have:

$$\psi_{MH_2} = \frac{|A_\gamma(\rho_2^c)|}{|A_\gamma(\rho_{2,(0)})|} \cdot \exp\Big\{ - \frac{1}{2\sigma^2_{(1)}} \cdot \sum_{t \in t_2}^{T_2} \Big[\Delta y_t' \cdot \big(A_\gamma(\rho_2^c)' \cdot V_{(1)}^{-1} \cdot A_\gamma(\rho_2^c) - \\ - A_\gamma(\rho_{2,(0)})' \cdot V_{(1)}^{-1} \cdot A_\gamma(\rho_{2,(0)}) \big) \cdot \Delta y_t - \\ - 2\beta' \cdot X_t' \cdot V_{(1)}^{-1} \big(A_\gamma(\rho_2^c) - A_\gamma(\rho_{2,(0)}) \big) \cdot \Delta y_t \Big] \Big\} \cdot \\ \cdot \left[\frac{\rho_2^c \cdot (1 - \rho_2^c)}{\rho_{2,(0)} \cdot \big(1 - \rho_{2,(0)}\big)} \right]^{d-1} \cdot \mathbf{1}\big(0 \leq \rho_2^c \leq 1\big)$$

6. At this point we need to update the variance of the proposal distributions: if the acceptance rate (number of acceptances over number of iterations of the Markov Chain) of the first parameter ρ_1 falls below 40% we need to reduce the value of c_1, the so called tuning parameter, which regulate the variance of the proposal distribution. We reduce the variance in this way:

$$c_1' = \frac{c_1}{1.1}.$$

In this way, we are able to draw values closer to the current value of ρ_1, and therefore, we expect to increase the acceptance rate. On the contrary, if the acceptance rate rises above 60%, we need to increase the tuning parameter, in order to draw values far from the current value, in this way we increase the chance to explore low density parts of the distribution, thus reducing the probability of accepting the candidate value and, by consequence, the acceptance rate:

$$c_1' = 1.1 \cdot c_1.$$

Clearly we replicate this procedure also for ρ_2.

7. Once updated all the values, we replicate procedure 2-6, 35,000 times.

8. We drop (*Burn-in phase*) the first 10% of the iterations, thus obtaining a vector of 31,500 observations for each of the parameters, which account for the simulated posterior distributions.

In order to obtain the distributions of the simulated fiscal plans, we simply draw the value of the parameters from the posteriors, and we calculate the effects of a simulated fiscal plan, as described in the paper.

6 Robustness

6.1 Inverted propagation channels

So far we have brought the theoretical mechanism to the data by considering TB adjustments as supply shocks and EB adjustments as demand shocks. An obvious way to assess the validity of the theoretical model is to conduct robustness analysis by putting the theoretically "wrong" labels to EB and TB adjustments. What happens empirically when a model is estimated in which TB adjustments propagate "wrongly" upstream and EB adjustments propagate "wrongly" downstream?
As anticipated in Section 3, we conduct this experiment by "switching the dummies":

$$\begin{aligned}\Delta y_{i,t} = c_i + \left(\beta^{up} \cdot \Delta y_{i,t}^{up} + \delta^u \cdot e_t^u + \delta^a \cdot e_{t,0}^a + \delta^f \cdot e_t^f\right) \cdot TB_t + \\ + \left(\beta^{down} \cdot \Delta y_{i,t}^{down} + \gamma^u \cdot e_t^u + \gamma^a \cdot e_{t,0}^a + \gamma^f \cdot e_t^f\right) \cdot EB_t. \qquad (35)\end{aligned}$$

Equation (35) is estimated via MLE and Bayesian MCMC, as done for the standard "theoretically correct" specification. Results are reported in Table VIII-IX:

Insert Table VIII-IX here

As before, the two methodologies yield same results. Notice that, unlike the standard specification, β^{down} is now less significant than β^{up}, suggesting that during TB year the industrial network is more active than during EB years.

6.1.1 Tax Hikes in the Inverted Model

As done before, we carry out the analogue fiscal shock simulation. Tables X-XI report the simulated output effects of a TB adjustment using the "inverted" model:

Insert Table X-XI here

Table X and XI show that the share of the effect of a TB adjustment attributable to the network does not change (still more than 50%), but the magnitude of the effect is now reduced: from an average total effect of almost -2%, we move to -1.54%. The statistical significance of the effect is also reduced.

6.1.2 Spending Cuts in the Inverted Model

Tables XII-XIII report the results for EB plans. They show that the share of total network effect decreases from 27% to 21%, the magnitude of the total effect passes from -0.71% to -0.59% and the statistical significance is now slightly lower:

Insert Table XII-XIII here

6.1.3 Vuong Test

In order to provide a more quantitative assessment of the relative performance of the "theory-consistent" versus the "inverted" models, we carry out a Vuong test (see Vuong 1989) for non-nested models. We obtain a pvalue of 24%, which does not lead to the rejection of the null "superiority of the "theory-consistent" model against the inverted".

As the Vuong test suggests, there is still evidence of propagation of the fiscal shocks in the industrial network when we force shocks to propagate in the theoretically wrong way. This is consistent with the fact that tax are not necessarily pure supply shocks, and they also propagate upward in the supply chain. At the same time, expenditure fiscal plans also propagate downward. This can be partially explained by the more heterogeneous structure of EB fiscal plans (see again Figure 2): since there is a tax-component in EB plans, they still propagate through the industrial network when we force them to move in the downward direction.

Overall we can conclude that there is stronger propagation when we apply the theoretically correct model, even though we cannot totally statistically rule out the inverted model.

6.2 Placebo

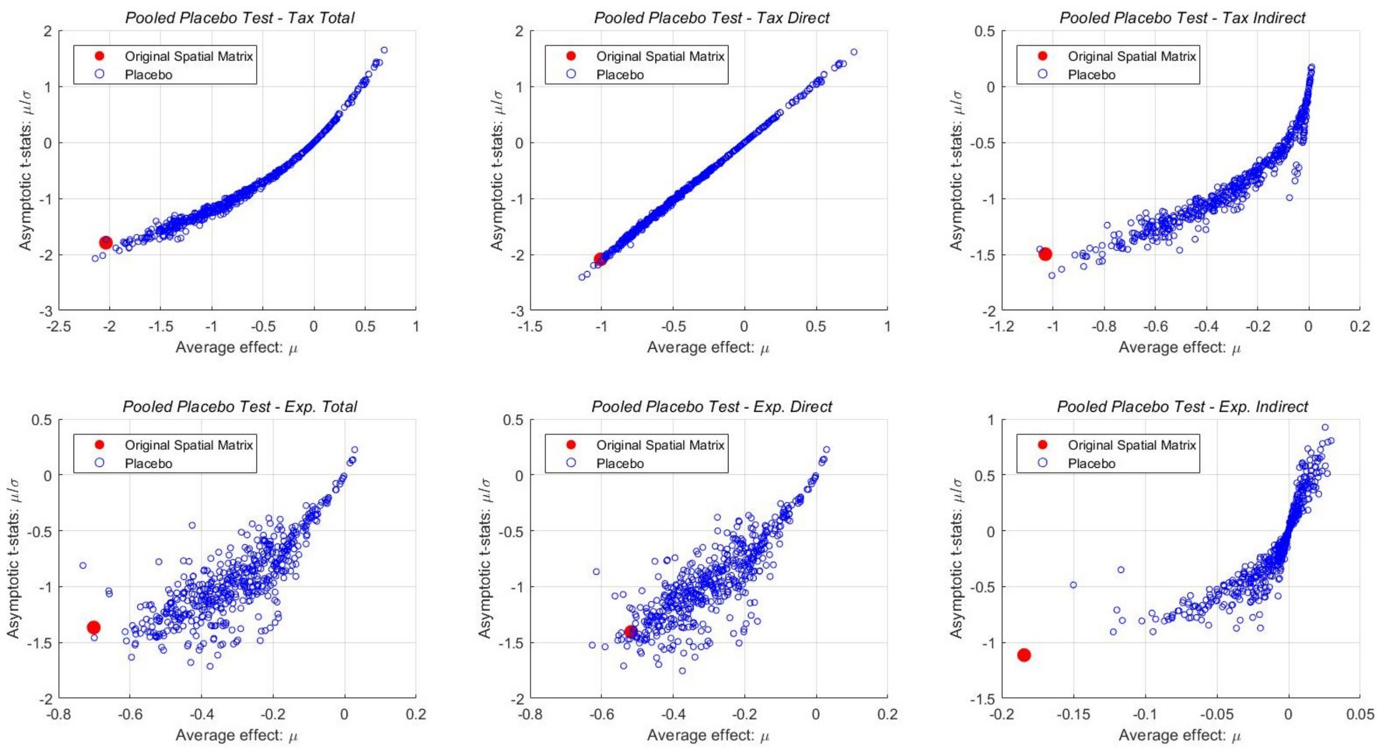
Ozdagli and Weber 2017 point out that a natural question that arises from the estimation of a SAR model, is the possibility that the estimation captures a spurious correlation (between β^{down}, β^{up} and the dependent variables), rather than a real underlying economic network. To answer this question we apply a slightly modified

(more conservative) version of the placebo test proposed by these authors(Ozdagli and Weber 2017). Basically, the idea is to: 1) simulate new spatial matrices; 2) carry out the same analysis done in our paper; 3) repeat step 2 several times; 4) compare the results with the original one.
If the original results are driven by spurious correlation, we expect to obtain similar results, in terms of statistical significance, to the placebo experiments. On the other hand, if the network, really matters, the weight given by the original spatial matrix should be the "real" one, and therefore should deliver superior results.

In practice, we simulated several new spatial matrices by: 1) permuting the elements of the rows of the original one; 2) permuting the elements of its columns; 3) reshuffling all its elements; 4) drawing new components from a uniform with support 0-0.4 in two different ways.[18] For each simulated spatial matrix, we carried out a separate analysis, as we had done with the original data. We overall collected 500 placebo experiments, whose results have been summarized by a pair: the mean and the asymptotic t-statistic of the average effect of the simulated fiscal plan distributions. We compare every placebo experiment with the original result through a scatter plot: the mean lies on the horizontal axis while the asymptotic t-stat lies on the vertical axis. Results are shown in Figure 9.

[18] see next sub-section for further details.

Figure 9: Pooled Placebo Test (500 simulations)



If the original spatial matrix is capturing a structural network, we expect the original results to have a stronger effect both in absolute terms (strongly negative mean) and statistical terms (strongly negative asymptotic t-statistic). We therefore expect the original spatial matrix results' to be located in the most south-west part of the panel.

Notice, that in Figure 9, the red dot, which accounts for the original data simulation, always stands in the bottom-left part of the panel, that is, it delivers stronger results both in terms of mean and asymptotic t-statistic. In particular, the average indirect effect is located in the most south-west position on the graphs.[19]

[19]The greatest dispersion of the placebo experiments in the EB plans, is due to the fact that shocks are industry specific, and therefore are weighed by random elements of the simulated spatial matrix.

6.2.1 Details on Placebo

As anticipated above, we conducted several "Placebo" Tests, where we simulated the spatial matrix in different ways:

1. *Row Shuffling*: we permutated the elements within the rows of the original spatial matrix.

2. *Column Shuffling*: we permutated the elements within the columns of the original spatial matrix.

3. *Total Shuffling*: we shuffled all the elements of the original matrix.

4. *Half Randomization*: we constructed an artificial spatial matrix by drawing its elements from a Uniform distribution 0-0.4; since most of the elements of the original matrix are containd in such an interval. However, matrix $\hat{A}$ has been constructed by adopting the original data transformation, starting from the artificial matrix A.

5. *Full Randomization*: same as half randomization, but in this, case, we simulated also matrix $\hat{A}$.

In our procedure, we conducted 100 simulations for each Placebo Test. At every iteration we stored in a 3D array, the mean and the asymptotic t-statistics (mean over standard deviation) for each component of the Average Effect of a fiscal shock (total, direct and indirect for both taxes and expenditure shocks).
In a second stance, we plotted the results in a graph which has on the horizontal axe the mean of the Average Effect, while on the vertical axe, the asymptotic t-stats. Therefore, every siulation is summarized by a couple: the mean and asyptotic t-stats of the average effect, which in our 2D graph, represents a point. The graphs we obtain are therefore 6 scatter plots (each for shock type and component - e.g. Average Direct Effect for and Expenditure Shock), where 100 points (shown in small blue dots) represent the 100 simulations, plus the original result (shown with a big red dot).
Since the average effects are negative, we expect to see the big red dot in the bottom left part of the graph, while, we expect the see a blue cloud of small dots shifted up and rightward: which means a weaker and less statistically significant shock effect. From the figures below we got exactly this result in all the Placebo Tests we conducted.

Figure 10: Row Shuffle

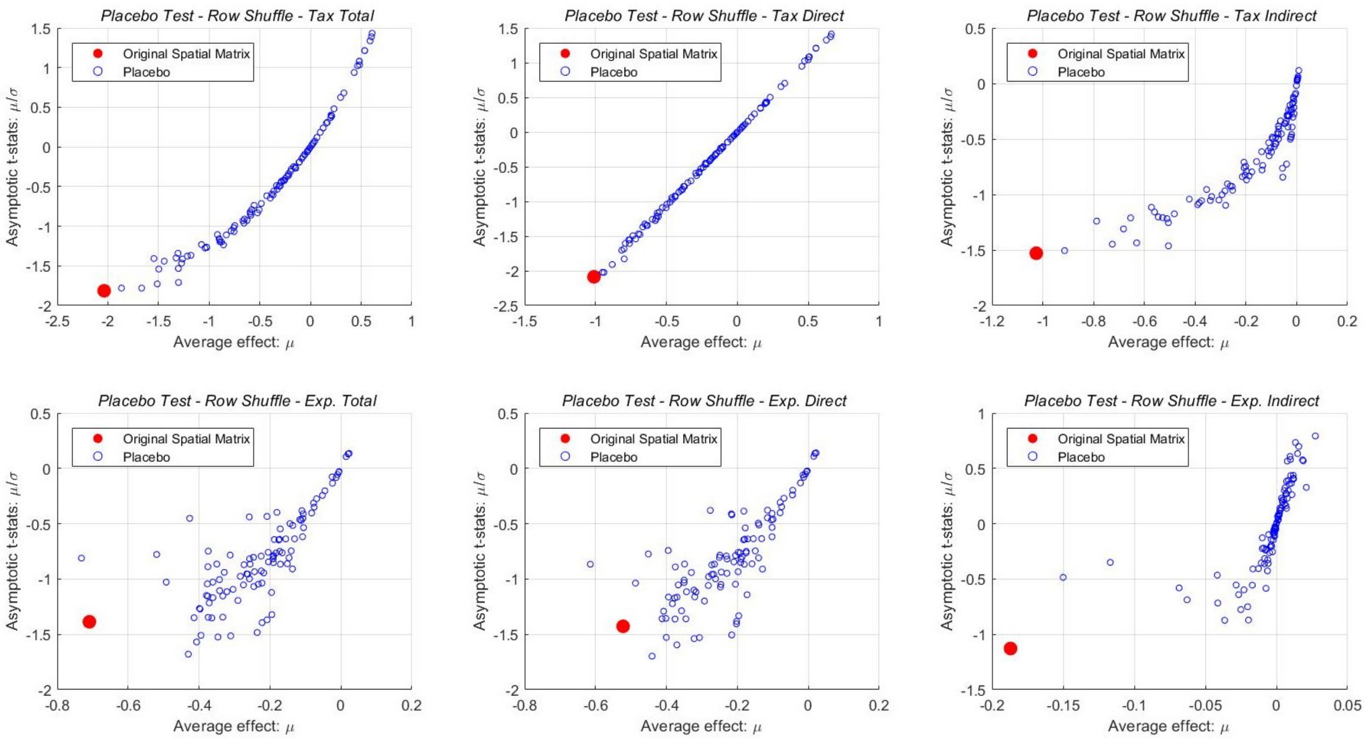


Figure 11: Column Shuffle

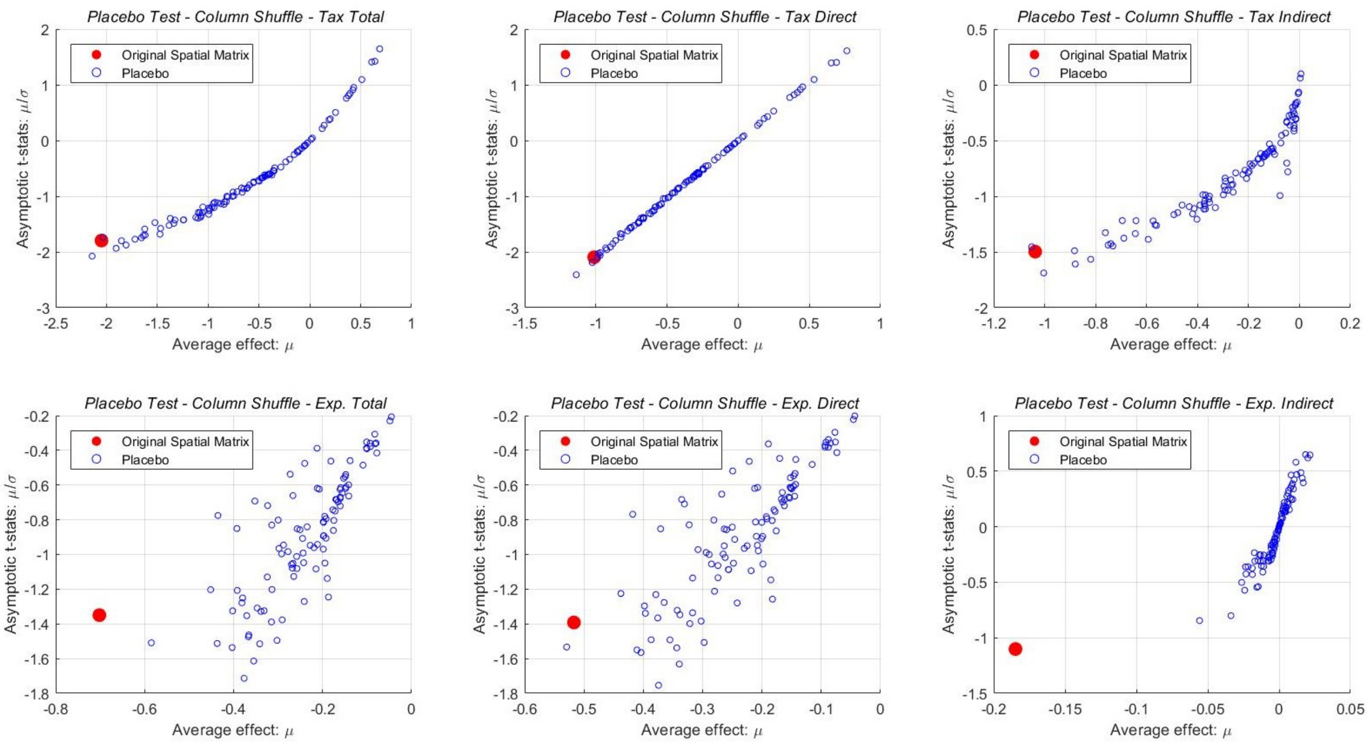


Figure 12: Total Shuffle

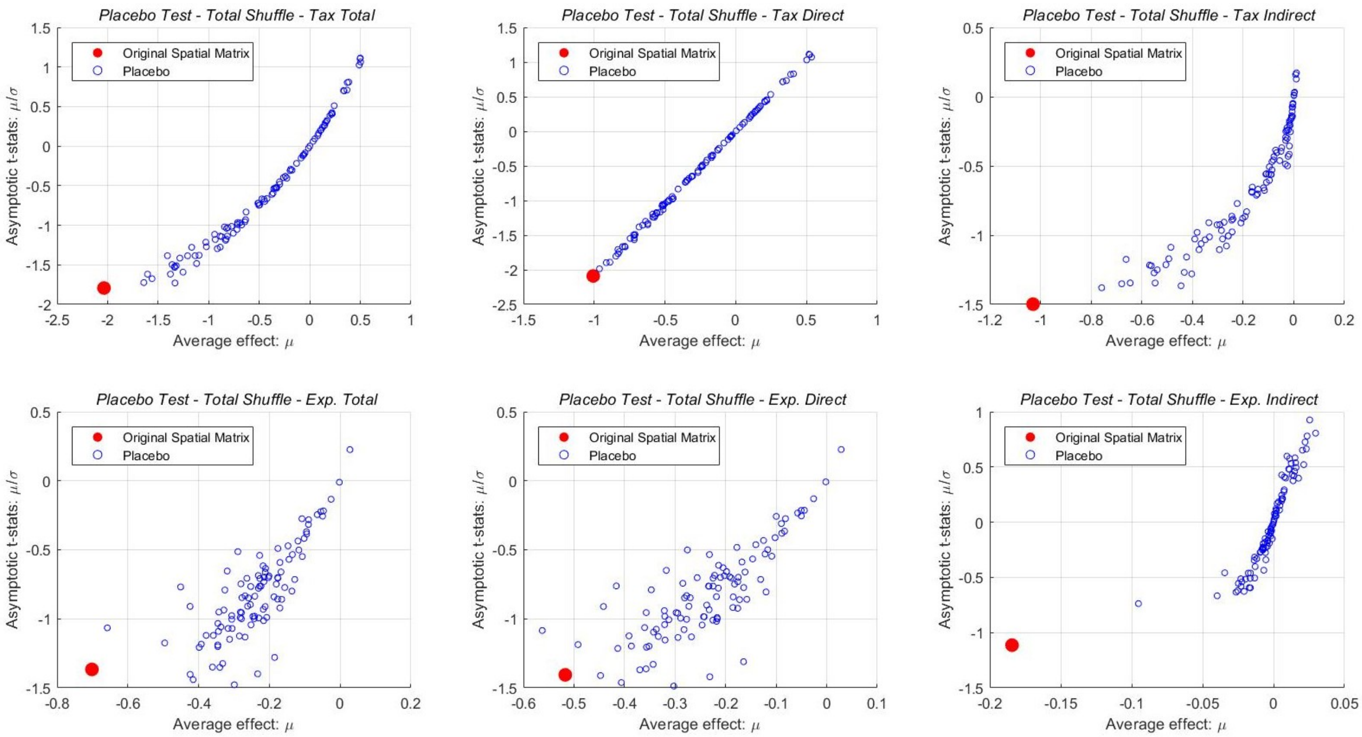


Figure 13: Half Randomization

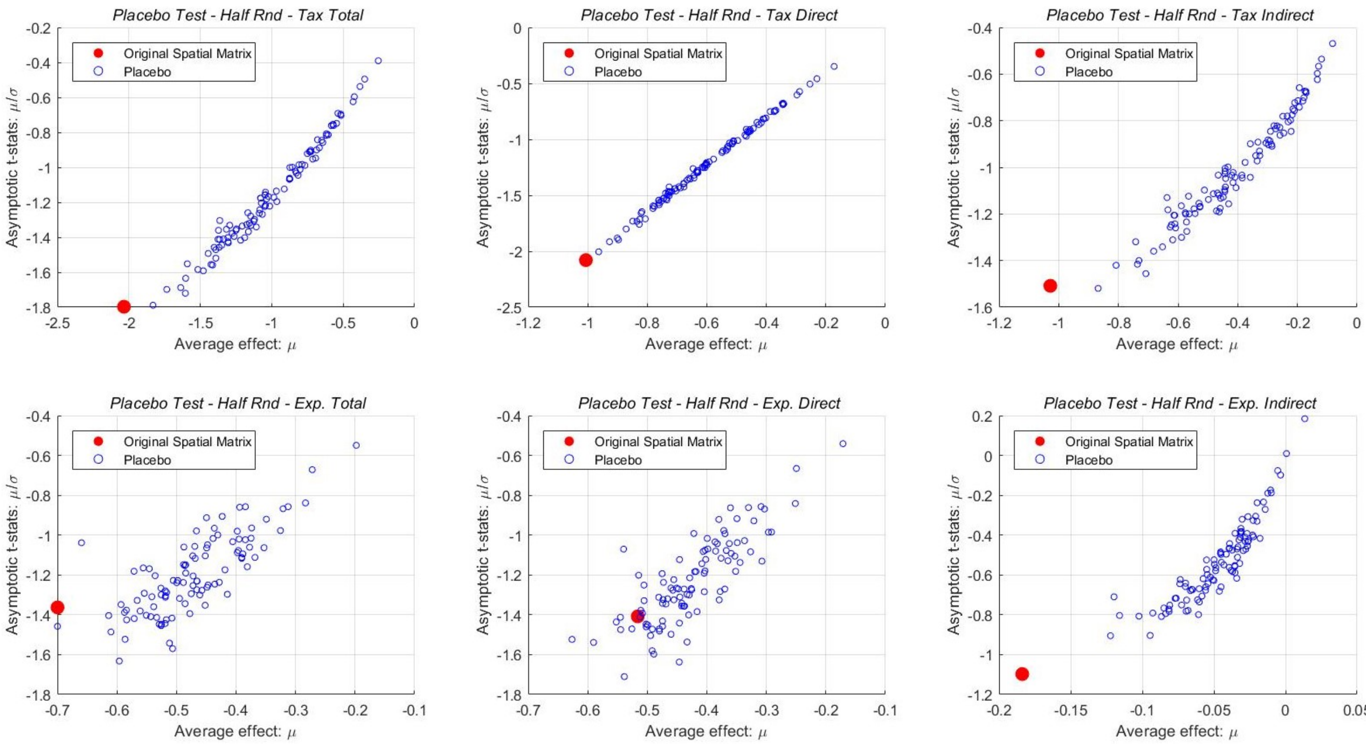
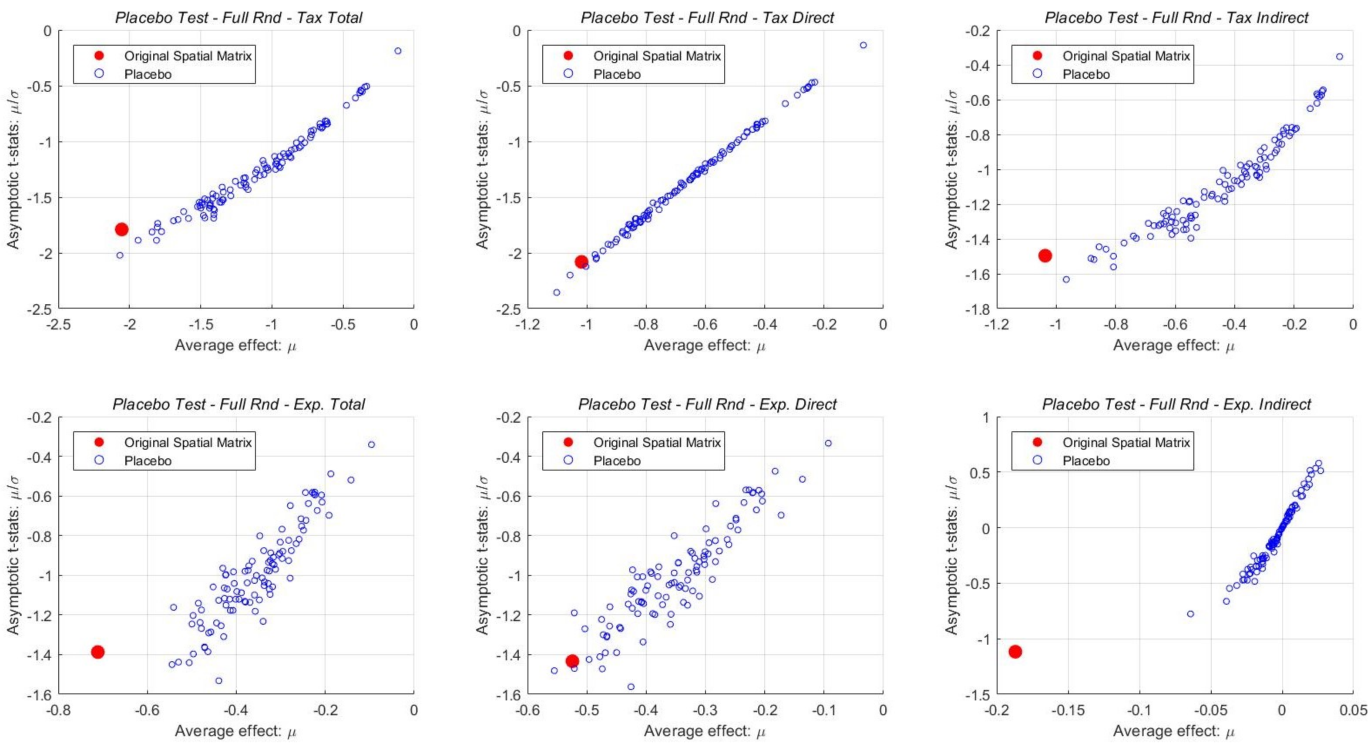


Figure 14: Full Randomization



Notice the following facts:

- The red dots always stand in the bottom left part of the graphs, as expected.
- The indirect effect, which is the one which reflects the network, always comes out weakened when using simulated data.
- The higher volatility of the expenditure shock Placebo Test, is due to the fact that the spending shocks' weights are simulated too, and therefore increase the degree of uncertainty with respect to tax shocks.

6.3 Non-Row Normalized Data

In this section we show that applying row-normalization - which is a standard practice in spatial econometric frameworks - does not affect at all our results. First of all, notice that the row-normalization changes the interpretation of the elements of the weight matrix, A (and by consequence, also of matrix $\hat{A}$). In particular, replicating what done in Section 4.2.1 of the paper, we can show the economic interpretation for each element of the weight matrix, which we denote here with $\tilde{A}$ (the row-normalized version of the I-O matrix, A). We have:

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{11} = \dfrac{\text{SALES}_{1\to 1}}{\text{INPUT}_1} & \tilde{a}_{12} = \dfrac{\text{SALES}_{2\to 1}}{\text{INPUT}_1} & \tilde{a}_{13} = \dfrac{\text{SALES}_{3\to 1}}{\text{INPUT}_1} \\ \\ \tilde{a}_{21} = \dfrac{\text{SALES}_{1\to 2}}{\text{INPUT}_2} & \tilde{a}_{22} = \dfrac{\text{SALES}_{2\to 2}}{\text{INPUT}_2} & \tilde{a}_{23} = \dfrac{\text{SALES}_{3\to 2}}{\text{INPUT}_2} \\ \\ \tilde{a}_{31} = \dfrac{\text{SALES}_{1\to 3}}{\text{INPUT}_3} & \tilde{a}_{32} = \dfrac{\text{SALES}_{2\to 3}}{\text{INPUT}_3} & \tilde{a}_{33} = \dfrac{\text{SALES}_{3\to 3}}{\text{INPUT}_3} \end{bmatrix}$$

where

$$\text{INPUT}_i = \text{SALES}_{1\to i} + \text{SALES}_{2\to i} + \text{SALES}_{3\to i}.$$

Since the coefficients change, also the interpretation of the network coefficient β^{down} (β^{up} for $\hat{A}$). However, working with non-row normalized spatial matrices, does not affect our results.
We report in XVIII the results of the Monte Carlo simulation (starting from ML estimates), conducted with non-row-normalized spatial matrices:

Insert Table XVIII here.

At the same time, running the inverted model using a non-row-normalized weight matrix, delivers the following results, summarized in XIX:

Insert Table XIX here.

7 Conclusions

We have investigated how fiscal plans propagate through the industrial network in the US economy, in order to explain a stronger output effect of TB adjustment plans than EB adjustment plans. In particular, the average total effect of a 1% of GDP tax-based adjustment plan is estimated to an average contraction of 2% in GDP while an EB plan is estimated to lead to a much smaller contraction of 0.72% in GDP. These estimates are very close to those found in a multi-country study of OECD economies by Alesina, Favero, and Giavazzi 2015. The different network transmission mechanism of EB and TB adjustments offers an interesting new explanations for their estimated heterogeneous effects. Interestingly the industrial propagation network effect accounts for half of the total impact of TB based adjustment on output while in case of EB adjustments the share of the propagation through the industrial network is of about half of that observed for TB plans.

Our robustness analysis shows that forcing the TB adjustment to flow upstream rather than downstream as predicted by the theory on the network effects of supply-side adjustments produces a statistically weaker empirical model. Even stronger evidence in the same direction is obtained when EB adjustments are forced to flow downstream in the industrial network. Overall our results on the US economy illustrate that the heterogenous impact of TB and EB adjustments on output growth could be explained theoretically and empirically by the different network propagation mechanism of demand and supply adjustments.

8 Tables

Table I: Correlation matrix of Fiscal Adjustments

	τ_t^u	$\tau_{t,0}^a$	τ_t^f	g_t^u	$g_{t,0}^a$	g_t^f
τ_t^u	1	0.0413	0.5702	0.5962	-0.1260	0.1047
$\tau_{t,0}^a$	0.0413	1	0.0376	0.0981	0.3606	0.3105
τ_t^f	0.5702	0.0376	1	-0.0469	0.0192	0.1805
g_t^u	0.5962	0.0981	-0.0469	1	-0.0498	0.0140
$g_{t,0}^a$	-0.1260	0.3606	0.0192	-0.0498	1	0.7816
g_t^f	0.1047	0.3105	0.1805	0.0140	0.7816	1

*: Where τ_t^u is the surprise increase in taxes announced at time t and implemented in the same year, and g_t^u is the surprise reduction in government expenditure announced at time t and implemented in the same year. Instead $\tau_{t,0}^a$ and $g_{t,0}^a$ are anticipated tax and expenditure changes announced by the fiscal authorities in advance and executed at time t, while τ_t^f and g_t^f are anticipated tax and expenditure changes announced by the fiscal authorities in advance to be executed in the future.

Table II: ML estimates for the baseline model.

	MLE	Std Dev	Tstat*	Pvalue
β^{down}	0.499	0.075	6.676	0.000
β^{up}	0.259	0.083	3.135	0.001
δ_u	-0.241	1.617	-0.149	0.441
γ_u	-1.463	1.871	-0.782	0.217
δ_a	-1.938	1.270	-1.526	0.064
γ_a	0.195	1.037	0.188	0.426
δ_f	-1.063	0.544	-1.954	0.025
γ_f	-0.283	0.468	-0.605	0.273

*: The t-statistics can be interpreted as an asymptotic t-statistics. In fact, keep in mind that the normality of the ML estimator is an asymptotic result.

Table III: Bayesian MCMC estimates for the baseline model.

	Mean	Std Dev	Tstat	1%	5%	10%	16%	50%	84%	90%	95%	99%
β^{down}	0.495	0.088	5.600	0.281	0.345	0.380	0.407	0.497	0.584	0.607	0.635	0.687
β^{up}	0.279	0.090	3.105	0.072	0.131	0.163	0.189	0.280	0.369	0.395	0.426	0.483
δ^u	-0.235	1.543	-0.152	-3.807	-2.751	-2.207	-1.776	-0.232	1.300	1.742	2.309	3.357
γ^u	-1.384	1.744	-0.794	-5.456	-4.252	-3.629	-3.112	-1.374	0.334	0.842	1.477	2.680
δ^a	-1.926	1.264	-1.524	-4.830	-4.009	-3.547	-3.180	-1.921	-0.676	-0.305	0.161	1.019
γ^a	0.241	0.997	0.242	-2.065	-1.388	-1.037	-0.754	0.241	1.223	1.512	1.868	2.575
δ^f	-1.016	0.565	-1.800	-2.311	-1.936	-1.738	-1.578	-1.020	-0.455	-0.290	-0.077	0.310
γ^f	-0.306	0.433	-0.705	-1.328	-1.017	-0.860	-0.737	-0.306	0.128	0.250	0.401	0.702

Table IV: The Output Effects of TB plan

	ML			Bayesian MCMC		
	Tax Tot	Tax Dir	Tax Ind	Tax Tot	Tax Dir	Tax Ind
Point Estim.	-1.959	-1.018	-0.941	-	-	-
Mean	-2.046	-1.014	-1.032	-1.919	-0.971	-0.948
Std Dev	1.145	0.490	0.689	1.079	0.503	0.630
$Pr(x < 0)$	98.08%	98.08%	98.08%	97.41%	97.41%	97.41%
1%	-5.361	-2.191	-3.237	-4.850	-2.158	-2.942
5%	-4.095	-1.823	-2.328	-3.765	-1.793	-2.086
10%	-3.522	-1.644	-1.907	-3.283	-1.607	-1.745
16%	-3.121	-1.496	-1.643	-2.933	-1.468	-1.506
50%	-1.937	-1.011	-0.917	-1.856	-0.969	-0.858
84%	-0.941	-0.522	-0.404	-0.886	-0.469	-0.382
90%	-0.685	-0.385	-0.283	-0.617	-0.330	-0.267
95%	-0.371	-0.212	-0.151	-0.276	-0.147	-0.122
99%	0.182	0.110	0.071	0.387	0.205	0.185

Table V: The Output Effect of TB plans: a Decomposition

ML - MC			Bayesian MCMC		
Direct		NIE	Direct		NIE
-1.014 (49.56%)		-1.032 (50.44%)	-0.971 (50.59%)		-0.948 (49.41%)
ϕ_δ	NDE	-	ϕ_δ	NDE	-
-0.968 (47.31%)	-0.045 (2.20%)	-	-0.926 (48.25%)	-0.045 (2.34%)	-
-	NTE		-	NTE	
-	-1.077 (52.64%)		-	-0.993 (51.75%)	

Table VI: The Output Effect of EB plans

	ML			Bayesian MCMC		
	Exp Tot	Exp Dir	Exp Ind	Exp Tot	Exp Dir	Exp Ind
Point Estim.	-0.772	-0.582	-0.190	-	-	-
Mean	-0.711	-0.523	-0.188	-0.716	-0.519	-0.198
Std Dev	0.519	0.369	0.170	0.478	0.337	0.166
$Pr(x < 0)$	92.36%	92.36%	92.35%	94.06%	94.06%	94.06%
1%	-2.025	-1.370	-0.742	-1.907	-1.310	-0.722
5%	-1.594	-1.131	-0.513	-1.520	-1.076	-0.502
10%	-1.383	-1.001	-0.405	-1.331	-0.955	-0.413
16%	-1.220	-0.896	-0.339	-1.186	-0.857	-0.349
50%	-0.690	-0.520	-0.157	-0.700	-0.515	-0.169
84%	-0.195	-0.149	-0.036	-0.247	-0.182	-0.050
90%	-0.068	-0.054	-0.012	-0.121	-0.089	-0.024
95%	0.102	0.081	0.020	0.038	0.028	0.008
99%	0.424	0.324	0.092	0.354	0.260	0.096

Table VII: The Output Effect of EB plans: a Decomposition

ML - MC			Bayesian MCMC		
Direct		NIE	Direct		NIE
-0.523 (73.56%)		-0.188 (26.44%)	-0.519 (72.49%)		-0.198 (27.51%)
ϕ_γ	NDE	-	ϕ_γ	NDE	-
-0.517 (72.71%)	-0.006 (0.85%)	-	-0.519 (72.49%)	-0.000 (0.00%)	-
-	NTE		-	NTE	
-	-0.189 (27.29%)		-	-0.198 (27.51%)	

Table VIII: Robustness: ML Estimates of "Inverted" Model

	MLE	Std Dev	Tstat	Pvalue
β^{down}	0.146	0.082	1.782	0.037
β^{up}	0.499	0.084	5.928	0.000
δ_u	-0.090	1.625	-0.056	0.478
γ_u	-1.105	1.871	-0.591	0.277
δ_a	-1.355	1.297	-1.045	0.148
γ_a	0.683	1.044	0.654	0.257
δ_f	-0.854	0.554	-1.540	0.062
γ_f	-0.297	0.468	-0.636	0.262

Table IX: Robustness:Bayesian MCMC Estimates of "inverted" model

	Mean	Std Dev	Tstat	1 %	5%	10%	16%	50 %	84%	9 %	95%	99%
β^{down}	0.179	0.080	2.211	0.013	0.048	0.074	0.097	0.175	0.261	0.284	0.317	0.376
β^{up}	0.504	0.083	6.040	0.305	0.364	0.394	0.420	0.504	0.587	0.610	0.639	0.693
δ^{u}	-0.130	1.551	-0.084	-3.768	-2.682	-2.117	-1.676	-0.117	1.399	1.850	2.403	3.472
γ^{u}	-1.070	1.733	-0.617	-5.095	-3.923	-3.288	-2.794	-1.063	0.643	1.128	1.771	3.029
δ^{a}	-1.325	1.210	-1.094	-4.141	-3.319	-2.862	-2.525	-1.327	-0.130	0.218	0.660	1.516
γ^{a}	0.671	0.979	0.685	-1.599	-0.944	-0.584	-0.300	0.676	1.644	1.919	2.284	2.955
δ^{f}	-0.819	0.542	-1.510	-2.083	-1.710	-1.515	-1.360	-0.821	-0.279	-0.123	0.073	0.444
γ^{f}	-0.315	0.434	-0.726	-1.340	-1.027	-0.870	-0.745	-0.314	0.114	0.238	0.394	0.690

Table X: The Output Effect of TB plans in the Inverted Model

	ML			Bayesian MCMC		
	Tax Total	Tax Direct	Tax Indirect	Tax Total	Tax Direct	Tax Indirect
Point Estim.	-1.528	-0.711	-0.817	-	-	-
Mean	-1.652	-0.797	-0.855	-1.537	-0.768	-0.769
Std Dev	1.184	0.497	0.722	1.014	0.478	0.573
$Pr(x < 0)$	94.80%	94.80%	94.80%	94.60%	94.60%	94.60%
1%	-5.159	-1.980	-3.285	-4.185	-1.881	-2.493
5%	-3.797	-1.627	-2.222	-3.267	-1.553	-1.806
10%	-3.189	-1.440	-1.765	-2.832	-1.382	-1.498
16%	-2.733	-1.287	-1.473	-2.513	-1.247	-1.288
50%	-1.516	-0.788	-0.711	-1.487	-0.766	-0.694
84%	-0.545	-0.305	-0.228	-0.559	-0.289	-0.252
90%	-0.287	-0.166	-0.118	-0.301	-0.154	-0.133
95%	0.019	0.011	0.009	0.036	0.019	0.016
99%	0.574	0.334	0.233	0.673	0.340	0.329

Table XI: The Output Effect of TB plans in the Inverted Model: a Deconmposition

ML - MC			Bayesian MCMC		
Direct		Network Indirect	Direct		Network Indirect
-0.797 (48.25%)		-0.855 (51.75%)	-0.768 (49.97%)		-0.769 (50.03%)
Instantaneous	Network Direct	-	Instantaneous	Network Direct	-
-0.766 (96.11%)	-0.031 (3.89%)	-	-0.740 (96.23%)	-0.029 (3.77%)	-
-	Total Network Effect		-	Total Network Effect	
-	-0.886 (53.63%)		-	-0.798 (51.92%)	

Table XII: The Output Effect of EB plans in the Inverted Model

	ML			Bayesian MCMC		
	Exp Total	Exp Direct	Exp Indirect	Exp Total	Exp Direct	Exp Indirect
Point Estim.	-0.548	-0.458	-0.089	-	-	-
Mean	-0.564	-0.462	-0.101	-0.588	-0.466	-0.122
Std Dev	0.455	0.364	0.114	0.429	0.333	0.120
$Pr(x < 0)$	89.77%	89.77%	87.14%	91.9 %	91.9%	91.9%
1%	-1.711	-1.290	-0.474	-1.655	-1.255	-0.520
5%	-1.326	-1.063	-0.319	-1.310	-1.019	-0.347
10%	-1.147	-0.925	-0.252	-1.140	-0.893	-0.276
16%	-1.014	-0.820	-0.202	-1.006	-0.798	-0.227
50%	-0.555	-0.466	-0.073	-0.577	-0.467	-0.098
84%	-0.111	-0.095	-0.005	-0.170	-0.137	-0.020
90%	0.006	0.005	0.005	-0.051	-0.041	-0.005
95%	0.158	0.141	0.024	0.104	0.081	0.016
99%	0.449	0.382	0.074	0.396	0.318	0.081

Table XIII: The Output Effect of EB plans in the Inverted Model: a Decomposition

ML - MC			Bayesian MCMC		
Direct		Network Indirect	Direct		Network Indirect
-0.462 (81.91%)		-0.101 (18.09%)	-0.466 (79.25%)		-0.122 (20.75%)
Instantaneous	Network Direct	-	Instantaneous	Network Direct	-
-0.457 (98.92%)	-0.005 (1.08%)	-	-0.464 (99.57%)	-0.002 (0.43%)	-
-	Total Network Effect		-	Total Network Effect	
-	-0.106 (18.79%)		-	-0.124 (21.09%)	

8.1 Additional Tables

Table XIV: Table of exogenous fiscal shocks

United States: Budgetary Impact of OBRA-90[20]															
Original Data		CBO:The 1990 Budget Agreement							1992 Budget of the US Government						
			1991	1992	1993	1994	1995	1991-1995		1991	1992	1993	1994	1995	1991-1995
CUMULATIVE CHANGE															
Tax			18	33	31	36	38	156		22.5	35.2	32.7	37.5	38.6	166.5
Spending			17	35	49	79	97	277							
CHANGES															
Tax			18	15	-2	5	2	38		22.5	12.7	0	0	0	35.2
Spending			17	18	14	30	18	97							
Reclassification by Calendar Year (January-December)															
		DeVries et al.							Romer&Romer						
	1989	1990	1991	1992	1993	1994	1995	1990-1995	1990	1991Q1	1992	1993	1994	1995	1990-1995
CHANGES															
Tax		4.5	17.25	10.75	−0.25	4.25	1.5	38		35.2					
Spending		4.25	17.25	17	18	27	13.5	97							
Change in percent of GDP															
Tax		0.080	0.305	0.190	−0.004	0.075	0.027	0.67		0.59					
Spending		0.075	0.305	0.300	0.318	0.477	0.239	1.71							
		0.155	0.610	0.490	0.314	0.552	0.266	2.3							
Nominal GDP	5657.7	5979.6	6174.1	6539.3	6878.7	7308.8	7664.1			5888					

*: Billions of U.S. dollars, by Fiscal Year (October-September)

Table XV: US OBRA-90 in percent of GDP

	Revenue adjustments									Expenditure adjustments							
		IMF	τ_t^u	$\tau_{t,0}^a$	$\tau_{t,1}^a$	$\tau_{t,2}^a$	$\tau_{t,5}^a$	IMF	g_t^u	$g_{t,0}^a$	$g_{t,1}^a$	$g_{t,2}^a$	$g_{t,3}^a$	$g_{t,4}^a$	$g_{t,5}^a$		
1990	0.00	0.080	0.080	0.00	0.305	0.190	0.00	0.075	0.027	0.075	0.075	0.00	0.305	0.300	0.318	0.477	0.239
1991	0.590	0.305	0.00	0.305	0.190	0.00	0.075	0.027	0.00	0.305	0.00	0.305	0.300	0.318	0.477	0.239	0.00
1992	0.00	0.190	0.00	0.190	0.00	0.075	0.027	0.00	0.00	0.300	0.00	0.300	0.318	0.477	0.239	0.00	0.00

Table XVI: descriptive statistics of estimated fixed effects for the **baseline** model. In the left panel of the table, you can see estimates for industry fixed effects, while on the right panel, you can find the estimated variances for each of the 15 sectors.

	Fixed Effects					*Variances*			
	MLE	Std Dev	t-stat	p-value		MLE	Std Dev	t-stat	p-value
c_1	1.260	2.327	0.542	0.294	σ_1^2	200.129*	46.529	4.301	0.000
c_2	4.522	2.903	1.558	0.060	σ_2^2	311.043*	72.316	4.301	0.000
c_3	1.535	0.845	1.817	0.035	σ_3^2	25.436	5.914	4.301	0.000
c_4	2.336	1.099	2.125	0.017	σ_4^2	44.005	10.231	4.301	0.000
c_5	0.802	0.619	1.296	0.097	σ_5^2	13.415	3.119	4.301	0.000
c_6	2.188	0.599	3.651	0.000	σ_6^2	12.803	2.977	4.301	0.000
c_7	1.464	0.519	2.822	0.002	σ_7^2	9.530	2.216	4.301	0.000
c_8	1.906	0.672	2.835	0.002	σ_8^2	15.600	3.627	4.301	0.000
c_9	2.783	0.673	4.136	0.000	σ_9^2	15.274	3.551	4.301	0.000
c_{10}	3.092	0.421	7.352	0.000	σ_{10}^2	6.039	1.404	4.301	0.000
c_{11}	4.648	0.587	7.924	0.000	σ_{11}^2	11.614	2.700	4.301	0.000
c_{12}	4.078	0.381	10.711	0.000	σ_{12}^2	4.961	1.153	4.301	0.000
c_{13}	3.141	0.464	6.770	0.000	σ_{13}^2	7.424	1.726	4.301	0.000
c_{14}	2.343	0.609	3.845	0.000	σ_{14}^2	12.604	2.930	4.301	0.000
c_{15}	1.923	0.308	6.253	0.000	σ_{15}^2	3.041	0.707	4.301	0.000

*: The first two sectors (Agriculture and Mining) have very high variances. This is consistent with the extreme volatile nature of output in those two sectors.

Table XVII: descriptive statistics of estimated fixed effects and variances for the **inverted** model.

	Fixed Effects					*Variances*			
	MLE	Std Dev	t-stat	p-value		MLE	Std Dev	t-stat	p-value
c_1	1.323	2.319	0.571	0.284	σ_1^2	198.653	46.187	4.301	0.000
c_2	4.553	2.905	1.567	0.059	σ_2^2	311.541	72.432	4.301	0.000
c_3	1.591	0.842	1.890	0.029	σ_3^2	25.228	5.865	4.301	0.000
c_4	2.220	1.098	2.021	0.022	σ_4^2	43.972	10.224	4.301	0.000
c_5	0.776	0.609	1.274	0.101	σ_5^2	12.968	3.016	4.299	0.000
c_6	2.324	0.593	3.918	0.000	σ_6^2	12.446	2.894	4.301	0.000
c_7	1.603	0.539	2.972	0.001	σ_7^2	10.216	2.375	4.301	0.000
c_8	1.890	0.674	2.804	0.003	σ_8^2	15.690	3.648	4.301	0.000
c_9	2.752	0.672	4.095	0.000	σ_9^2	15.226	3.540	4.301	0.000
c_{10}	3.201	0.423	7.560	0.000	σ_{10}^2	6.101	1.419	4.301	0.000
c_{11}	4.626	0.591	7.824	0.000	σ_{11}^2	11.815	2.747	4.301	0.000
c_{12}	4.175	0.390	10.699	0.000	σ_{12}^2	5.197	1.208	4.301	0.000
c_{13}	3.080	0.477	6.459	0.000	σ_{13}^2	7.997	1.859	4.301	0.000
c_{14}	2.279	0.609	3.743	0.000	σ_{14}^2	12.583	2.926	4.301	0.000
c_{15}	1.990	0.301	6.621	0.000	σ_{15}^2	2.908	0.676	4.300	0.000

Table XVIII: TB and EB adjustments average effects - Baseline model - Non-Row Normalized weight matrices

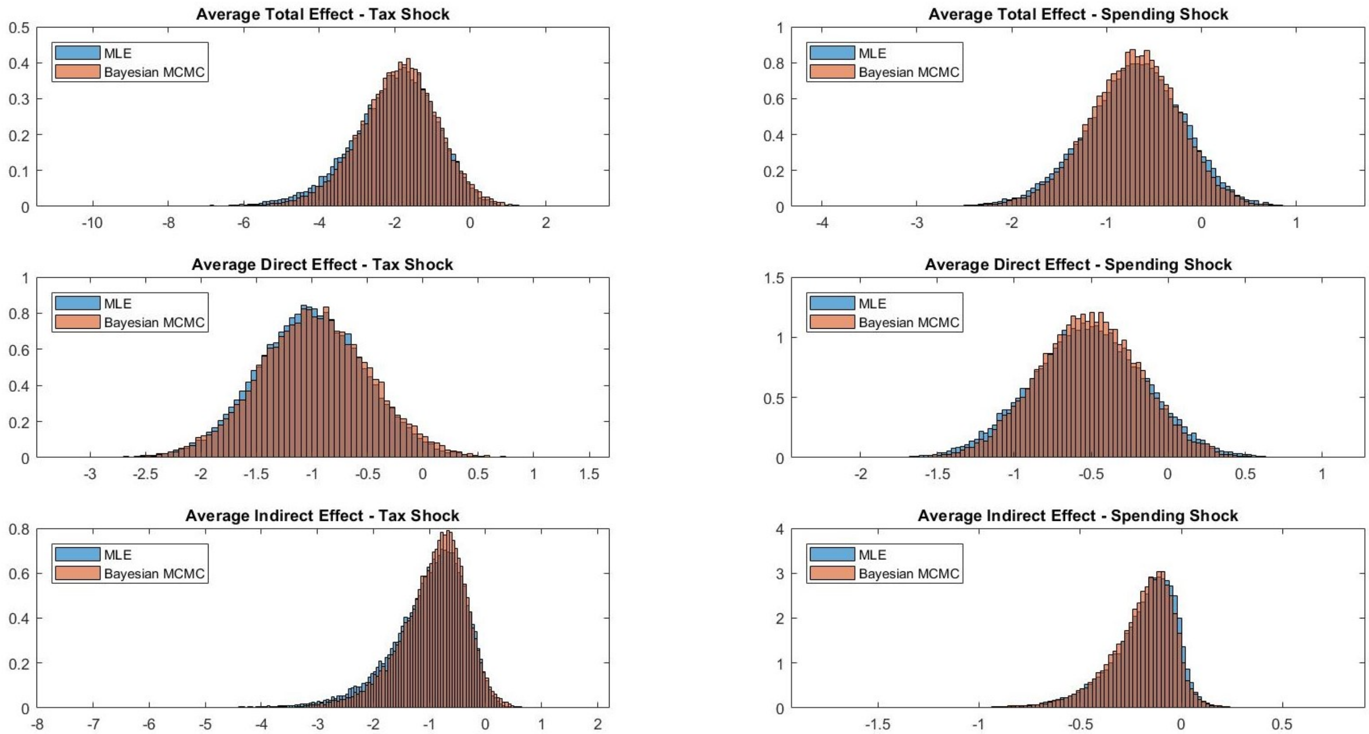
	Tax Tot	Tax Dir	Tax Ind	Exp Tot	Exp Dir	Exp Ind
Point Estim.	-1.776	-0.962	-0.815	-0.886	-0.568	-0.317
Mean	-1.856	-0.965	-0.891	-0.919	-0.573	-0.346
Std Dev	1.023	0.466	0.586	0.621	0.367	0.278
$Pr(x < 0)$	98.22%	98.22%	98.22%	94.13%	94.13%	94.12%
1%	-4.706	-2.094	-2.768	-2.500	-1.426	-1.194
5%	-3.678	-1.738	-2.006	-2.000	-1.178	-0.868
10%	-3.198	-1.562	-1.678	-1.719	-1.040	-0.706
16%	-2.817	-1.419	-1.414	-1.523	-0.937	-0.598
50%	-1.777	-0.962	-0.798	-0.887	-0.572	-0.300
84%	-0.870	-0.501	-0.351	-0.319	-0.208	-0.094
90%	-0.631	-0.366	-0.243	-0.157	-0.104	-0.045
95%	-0.344	-0.204	-0.135	0.049	0.033	0.013
99%	0.173	0.103	0.065	0.399	0.264	0.128

Table XIX: T Band EB adjustments average effects - Inverted model - Non-row Normalized weight matrices

	Tax Tot	Tax Dir	Tax Ind	Exp Tot	Exp Dir	Exp Ind
Point Estim.	-0.528	-0.272	-0.256	-0.475	-0.413	-0.062
Mean	-0.602	-0.298	-0.303	-0.484	-0.411	-0.073
Std Dev	0.945	0.483	0.482	0.444	0.370	0.099
$Pr(x < 0)$	73.08%	73.08%	73.08%	87.00%	87.00%	81.07%
1%	-3.189	-1.423	-1.876	-1.599	-1.275	-0.427
5%	-2.257	-1.097	-1.183	-1.243	-1.022	-0.265
10%	-1.815	-0.919	-0.918	-1.059	-0.888	-0.199
16%	-1.501	-0.777	-0.718	-0.914	-0.775	-0.156
50%	-0.523	-0.295	-0.217	-0.467	-0.409	-0.046
84%	0.300	0.186	0.111	-0.052	-0.047	0.003
90%	0.539	0.325	0.196	0.070	0.063	0.013
95%	0.810	0.506	0.303	0.223	0.197	0.031
99%	1.269	0.811	0.519	0.492	0.450	0.084

9 Additional Figures

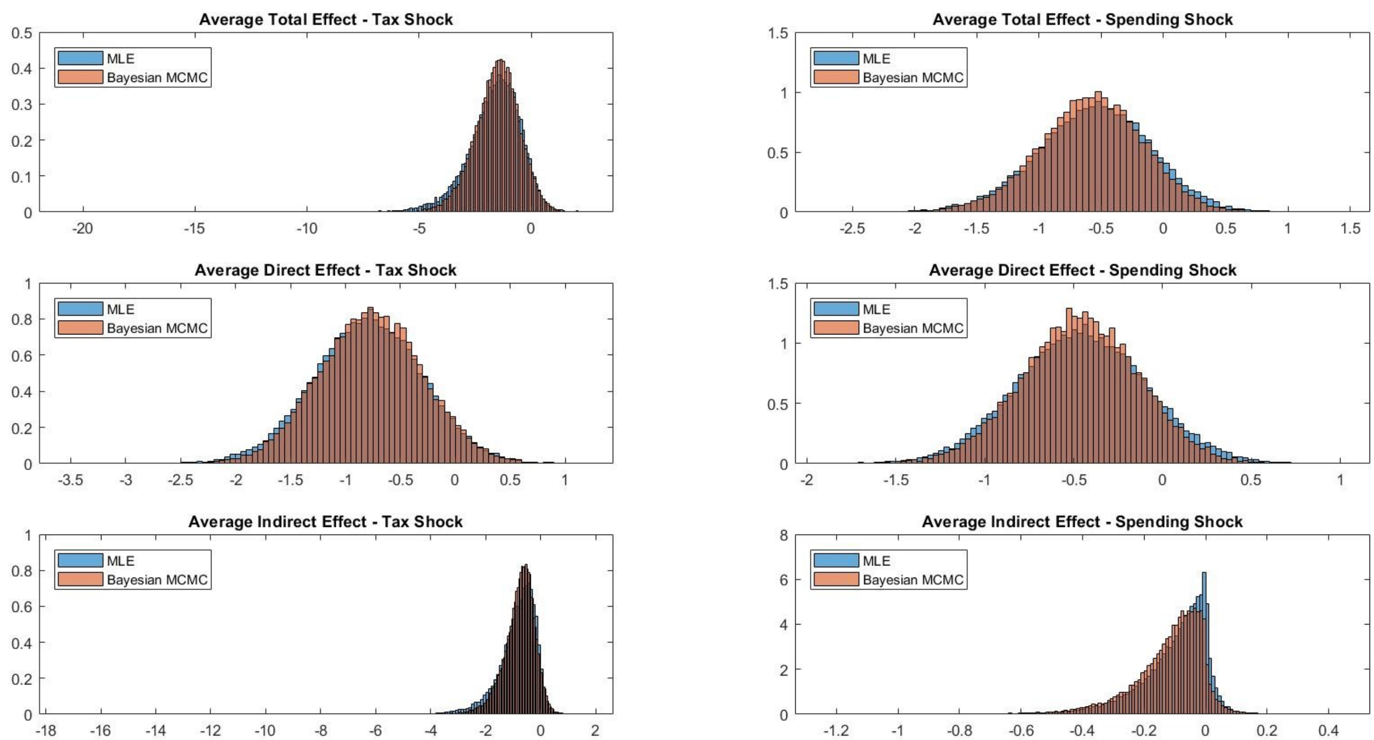
Figure 15: Average Effects - Baseline Model



$$\Delta y_{i,t} = c_i + \beta^{down} \cdot \Delta y_{i,t}^{down} + \left(+ \delta^u \cdot e_t^u + \delta^a \cdot e_{t,0}^a + \delta^f \cdot e_t^f \right) \cdot TB_t + \\ + \left(\gamma^u \cdot e_{i,t}^u + \gamma^a \cdot e_{i,t,0}^a + \gamma^f \cdot e_{i,t}^f \right) \cdot EB_t. \tag{36}$$

$$\Delta y_{i,t} = c_i + \beta^{up} \cdot \Delta y_{i,t}^{up} + \left(+ \delta^u \cdot e_t^u + \delta^a \cdot e_{t,0}^a + \delta^f \cdot e_t^f \right) \cdot TB_t + \\ + \left(\gamma^u \cdot e_{i,t}^u + \gamma^a \cdot e_{i,t,0}^a + \gamma^f \cdot e_{i,t}^f \right) \cdot EB_t. \tag{37}$$

Figure 16: Average Effects - Inverted Model



References

[1] Daron Acemoglu, Ufuk Akcigit, and William Kerr. "Networks and the macroeconomy: An empirical exploration." In: *NBER Macroeconomics Annual* 30.1 (2016), pp. 273–335. DOI: 10.1086/685961.

[2] Daron Acemoglu, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. "The network origins of aggregate fluctuations." In: *Econometrica* 80.5 (2012), pp. 1977–2016. DOI: 10.3982/ECTA9623.

[3] Alberto Alesina, Omar Barbiero, Carlo Favero, Francesco Giavazzi, and Matteo Paradisi. "The effects of fiscal consolidations: Theory and evidence." In: (2017). DOI: 10.3386/w23385.

[4] Alberto Alesina, Carlo Favero, and Francesco Giavazzi. *Austerity. When it works and when it doesn't*. Princeton University Press, 2019.

[5] Alberto Alesina, Carlo Favero, and Francesco Giavazzi. "The output effect of fiscal consolidation plans." In: *Journal of International Economics* 96 (2015), S19–S42. DOI: 10.1016/j.jinteco.2014.11.003.

[6] Alan J Auerbach, Yuriy Gorodnichenko, and Daniel Murphy. *Local Fiscal Multipliers and Fiscal Spillovers in the United States*. Tech. rep. National Bureau of Economic Research, 2019. DOI: 10.3386/w25457.

[7] Vasco M. Carvalho, Makoto Nirei, Yukiko U. Saito, and Tahbaz-Salehi Alireza. "Supply Chain Disruptions: Evidence from the Great East Japan Earthquake." In: *Cambridge Working Paper Economics: 1670* (2016). DOI: 10.17863/CAM.7827.

[8] Lawrence Christiano, Martin Eichenbaum, and Sergio Rebelo. "When is the government spending multiplier large?" In: *Journal of Political Economy* 119.1 (2011), pp. 78–121. DOI: 10.1086/659312.

[9] Alexander Chudik and M Hashem Pesaran. "Theory and practice of GVAR modelling." In: *Journal of Economic Surveys* 30.1 (2016), pp. 165–197. DOI: 10.1111/joes.12095.

[10] Peter A Diamond. "Aggregate demand management in search equilibrium." In: *Journal of political Economy* 90.5 (1982), pp. 881–894. DOI: 10.1086/261099.

[11] Xavier Gabaix. "The granular origins of aggregate fluctuations." In: *Econometrica* 79.3 (2011), pp. 733–772. DOI: 10.3982/ECTA8769.

[12] Jaime Guajardo, Daniel Leigh, and Andrea Pescatori. "Expansionary austerity? International evidence." In: *Journal of the European Economic Association* 12.4 (2014), pp. 949–968. DOI: 10.1111/jeea.12083.

[13] James P LeSage. "Bayesian estimation of spatial autoregressive models." In: *International Regional Science Review* 20.1-2 (1997), pp. 113–129. DOI: 10.1177/016001769702000107.

[14] James P LeSage and Olivier Parent. "Bayesian model averaging for spatial econometric models." In: *Geographical Analysis* 39.3 (2007), pp. 241–267. DOI: 10.1111/j.1538-4632.2007.00703.x.

[15] James LeSage and Robert Kelley Pace. "Introduction to spatial econometrics." In: (2009). DOI: 10.1111/j.1538-4632.2010.00797.x.

[16] John B Long Jr and Charles I Plosser. "Real business cycles." In: *Journal of political Economy* 91.1 (1983), pp. 39–69. DOI: 10.1086/261128.

[17] Keith Ord. "Estimation methods for models of spatial interaction." In: *Journal of the American Statistical Association* 70.349 (1975), pp. 120–126. DOI: 10.1080/01621459.1975.10480272.

[18] Ali Ozdagli and Michael Weber. "Monetary policy through production networks: Evidence from the stock market." In: (2017). DOI: 10.3386/w23424.

[19] Andrea Pescatori, Mr Daniel Leigh, Jaime Guajardo, and Mr Pete Devries. "A new action-based dataset of fiscal consolidation." In: 11-128 (2011). DOI: 10.5089/9781462346561.001.A001.

[20] Valerie A Ramey. "Ten Years after the Financial Crisis: What Have We Learned from the Renaissance in Fiscal Research?" In: *Journal of Economic Perspectives* (2018).

[21] Christina D Romer and David H Romer. "A narrative analysis of postwar tax changes." In: *Unpublished paper, University of California, Berkeley (June)* (2009). DOI: 10.1.1.370.4762.

[22] Christina D Romer and David H Romer. "The macroeconomic effects of tax changes: estimates based on a new measure of fiscal shocks." In: *American Economic Review* 100.3 (2010), pp. 763–801. DOI: 10.1257/aer.100.3.763.

[23] Quang H. Vuong. "Likelihood ratio tests for model selection and non-nested hypotheses." In: *Econometrica* (1989). DOI: 10.2307/1912557.