# Exam criteria

Exam consists of 20 problems, the correct solution of each gives 5 points to the grade.

The exam grade can be increased by providing certificates for the following online courses:

- Coursera course "Mathematical Thinking in Computer Science" by HSE University and UCSD — 20 points.
- Coursera course "Algorithmic Toolbox" by HSE University and UCSD 15 points.
- Courses of Coursera specialization "Mathematics for Data Science" by HSE University 7 points for each course.

Each prospective student can be awarded no more than 20 points for providing online courses certificates.

If the number of points for the exam exceeds 100 points after adding points for online courses certificates, the exam grade is set to 100 points.

The exam grade is set to 100 points if the certificate of winner or prize-winner of Open Doors olympiad in "Mathematics and Artificial Intelligence" subject is provided.

## Topics covered in an entrance exam

#### Numbers and operations on them

**Numbers and remainders.** Integer numbers, divisibility. Division with remainder. Modular arithmetic. Prime numbers, decomposition of integers into primes.

**Numbers, exponentiation and roots.** Fractions, percents, rational numbers. Absolute value of a number, its properties. Rational exponents, their properties.

**Logarithms.** Logarithm operation. Logarithms of products, quotients and powers. Decimal and natural logarithms, mathematical constant e.

**Transforming closed-form expressions.** Transforming expressions with arithmetic operations, exponentiation, roots and logarithms.

**Sequences of numbers.** Sequences of numbers, their properties. Periodic sequences. Recurrence relations. Real-life problems with recurrence relations. Fibonacci numbers. Arithmetic and geometric progressions.

## Equations and inequalities

**Equations.** Linear and quadratic equations. Exponential equations. Logarithmic equations. Equivalence of equations. Systems of equations. Systems of linear equations in two variables. Using graphs and properties to solve equations. Representing solution sets of systems of equations in two variables as subsets of the plane. Application of mathematical reasoning to solving problems in various areas of science and real life.

**Inequalities.** Linear and quadratic inequalities. Rational inequalities. Exponential inequalities. Logarithmic inequalities. Systems of linear inequalities. Systems of inequalities in one variable. Equivalence of inequalities and systems of inequalities. Using graphs and their properties to solve inequalities. Solving rational inequalities. Solutions of inequalities of two variables and their systems as sets in the coordinate system.

## Functions

**The notion of a function and its graph.** Function, its domain and range. Graph of a function. Examples of functional dependencies in real life. Inverse function, its graph. Transformation of graphs: shifts, symmetries, dilation.

**Basic properties of functions. Monotone functions**. Intervals of increase and decrease. Odd and even functions. Periodic functions. Bounded functions. Extremum points of a function (local and global).

**Main elementary functions.** Linear function, its graph. The reciprocal (multiplicative inverse) function, its graph. Quadratic function, its graph. Power functions with natural exponent, their graphs. Exponential function, its graph. Logarithmic function, its graph.

**Derivative.** The notion of the derivative of a function, its geometric interpretation. Physical interpretation of the derivative, finding the speed of a process given by a formula or a graph. The equation of the tangent line to the graph of a function. Derivative of a sum, difference, product, and quotient. Derivatives of main elementary functions. The second derivative, its physical interpretation. Applying derivatives to analyze functions and plot their graphs.

#### Information and information processing

**Representation of information. Modeling.** Description of a real life object or a process in terms of information. Types of descriptions: schemes, tables, graphs, and formulas.

**Numerical systems.** Positional systems. Binary representation of the information. Translation between decimal and binary representations. Arithmetic operations in binary system. **Logic and algorithms.** Propositions, logical operations, quantifiers, truth value of a proposition. The notion of an algorithm. Conditional operators and cycles. Implementation and construction of basic algorithms in math and real-life related problems.

## References:

- 1. Edward A. Scheinerman, Mathematics: A Discrete Introduction, 3rd Edition, Cengage Learning, 2012
- 2. Robert F. Blitzer, Precalculus, 6th Edition, Pearson, 2017
- 3. Jay Abramson, Precalculus, 12th Media Services, 2016
- 4. Eric Lehman, F Thomson Leighton, Albert R Meyer, Mathematics for Computer Science, Samurai Media Limited, 2017
- 5. Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, 3rd Edition, The MIT Press, 2009